

AP Calculus Exam Prep Assignment #2 page 2

11)

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = -\frac{3}{4} \frac{dx}{dt} \text{ at } (3,4) \quad \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = 2 \Rightarrow \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 4$$

$$\left(\frac{dx}{dt}\right)^2 + \left(-\frac{3}{4} \frac{dx}{dt}\right)^2 = 4 \Rightarrow \frac{25}{16} \left(\frac{dx}{dt}\right)^2 = 4 \Rightarrow \frac{dx}{dt} = \pm \frac{8}{5}, \text{ but counterclockwise} \Rightarrow \frac{dx}{dt} = -\frac{8}{5} \quad \mathbf{A)}$$

$$\frac{dy}{dt} = \frac{6}{5} \quad \mathbf{v} = -\frac{8}{5} \mathbf{i} + \frac{6}{5} \mathbf{j}$$

12) **D) $1 < t < 1$**

For problems 13-16, use the following:

$\mathbf{r} = \left(3 \cos\left(\frac{\pi}{3}t\right)\right)\mathbf{i} + \left(2 \sin\left(\frac{\pi}{3}t\right)\right)\mathbf{j}$ is the position vector from the origin to a moving point $P(x,y)$ at time t .

13) $x = 3 \cos\left(\frac{\pi}{3}t\right), y = 2 \sin\left(\frac{\pi}{3}t\right) \quad \left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = 1 \Rightarrow 4x^2 + 9y^2 = 36 \quad \mathbf{E)}$

14) $\mathbf{v} = \left\langle -\pi \sin\left(\frac{\pi}{3}t\right), \frac{2\pi}{3} \cos\left(\frac{\pi}{3}t\right) \right\rangle \quad \text{speed at } t = 3 \Rightarrow \|\mathbf{v}\| = \sqrt{0^2 + \left(-\frac{2\pi}{3}\right)^2} = \frac{2\pi}{3} \quad \mathbf{A)}$

15) $\mathbf{a} = \left\langle -\frac{\pi^2}{3} \cos\left(\frac{\pi}{3}t\right), -\frac{2\pi^2}{9} \sin\left(\frac{\pi}{3}t\right) \right\rangle \quad \text{When } t = 3, \|\mathbf{a}\| = \sqrt{\left(-\frac{\pi^2}{3}\right)^2 + 0} = \frac{\pi^2}{3} \quad \mathbf{B)}$

16) At the point where $t = \frac{1}{2} \left[\left(\frac{3\sqrt{3}}{2}, 1 \right) \right]$ the slope of the curve along which the particle moves is:

$$\frac{dy}{dx} = \frac{\frac{2\pi}{3} \cos\left(\frac{\pi}{3}t\right)}{-\pi \sin\left(\frac{\pi}{3}t\right)} = -\frac{2}{3} \cot\left(\frac{\pi}{3}t\right) \quad \text{At } t = \frac{1}{2}, \frac{dy}{dx} = -\frac{2}{3} \cot\left(\frac{\pi}{6}\right) = -\frac{2\sqrt{3}}{3} \quad \mathbf{D)}$$

17) **D) Its velocity and acceleration vectors must be perpendicular.**