

AP Calculus Exam Prep Assignment #2 Name _____

Multiple Choice

1)

$$V = \pi \int_0^{2\pi} (1 - \cos \phi)^2 d\phi = \pi \int_0^{2\pi} (1 - 2\cos \phi + \cos^2 \phi) d\phi = \pi \int_0^{2\pi} \left(1 - 2\cos \phi + \frac{1 + \cos 2\phi}{2}\right) d\phi \quad \mathbf{D)}$$

$$= \pi \left[\frac{3}{2}\phi + 2\sin \phi + \frac{1}{4}\sin 2\phi \right]_0^{2\pi} = \pi [(3\pi + 0 + 0) - (0 + 0 + 0)] = 3\pi^2$$

2) $A = \int_0^{2\pi} (1 - \cos x) dx = [x - \sin x]_0^{2\pi} = (2\pi - 0) - (0 - 0) = 2\pi \quad \mathbf{E)}$

3)

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{-t^2 + 3}{3}}{e^t} = \frac{-t^2 + 3}{3e^t} \quad x - \text{int} \Rightarrow y = 0 \Rightarrow t \left(-\frac{t^2}{9} + 1 \right) = 0 \Rightarrow t = 0, \pm 3 \quad \mathbf{A)}$$

$$\frac{dy}{dx}(-3) = \frac{-9 + 3}{3e^{-3}} = -2e^3$$

4)

$$\frac{dy}{dt} = 4\cos t + 12\cos 12t \quad \frac{dy}{dt}(1) = 4\cos 1 + 12\cos 12 \approx 12.287 \quad \mathbf{E)}$$

5) Find $\frac{d^2y}{dx^2}$ if $x = 2\cos \phi$ and $y = \sin \phi$.

$$\frac{dy}{dx} = \frac{\cos \phi}{-2\sin \phi} = -\frac{1}{2}\cot \phi \quad \frac{d^2y}{dx^2} = \frac{\frac{1}{2}\csc^2 \phi}{-2\sin \phi} = -\frac{1}{4}\csc^3 \phi \quad \mathbf{A)}$$

6) The length of $r = 3\csc \phi$ from $\phi = \frac{\pi}{4}$ to $\phi = \frac{3\pi}{4}$ is:

$$x = r \cos \theta = 3\csc \theta \cos \theta = 3\cot \theta \Rightarrow \frac{dx}{d\theta} = -3\csc^2 \theta$$

$$y = r \sin \theta = 3\csc \theta \sin \theta = 3 \Rightarrow \frac{dy}{d\theta} = 0$$

$$L = \int_{\pi/4}^{3\pi/4} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} d\theta = \int_{\pi/4}^{3\pi/4} \sqrt{9\csc^4 \theta + 0} d\theta = \int_{\pi/4}^{3\pi/4} 3\csc^2 \theta d\theta = [-3\cot \theta]_{\pi/4}^{3\pi/4} = 3 - (-3) = 6 \quad \mathbf{A)}$$

7) Find the slope of the curve $r = \cos 2\phi$ at $\phi = \frac{\pi}{6}$

$$x = \cos 2\phi(\cos \phi) \quad y = \cos 2\phi(\sin \phi) \quad \frac{dy}{dx} = \frac{\cos 2\phi(\cos \phi) - 2 \sin 2\phi(\sin \phi)}{-\cos 2\phi(\sin \phi) - 2 \sin 2\phi(\cos \phi)}$$

$$\frac{dy}{dx}\left(\frac{\pi}{6}\right) = \frac{\frac{1}{2}\left(\frac{\sqrt{3}}{2}\right) - 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)}{-\frac{1}{2}\left(\frac{1}{2}\right) - 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)} = -\frac{\sqrt{3}}{4} \cdot \frac{4}{-7} = \frac{\sqrt{3}}{7} \quad \text{A)}$$

8) Find the first quadrant area inside the rose, $r = 3 \sin 2\phi$, but outside the circle, $r = 2$

$$A = \frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta \quad 3 \sin 2\phi = 2 \Rightarrow \sin 2\phi = \frac{2}{3} \Rightarrow \phi \approx 0.365, 1.206$$

D)

$$A = \frac{1}{2} \int_{0.365}^{1.206} \left[(3 \sin 2\phi)^2 - (2)^2 \right] d\phi \approx 1.328$$

9) The common area inside $r = 2 \sin \phi$ and $r = 2 \cos \phi$ is:

$$A = \frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta \quad A = 2 \left(\frac{1}{2} \int_{\pi/4}^{\pi/2} (2 \cos \phi)^2 d\phi \right) = \int_{\pi/4}^{\pi/2} 4 \cos^2 \phi d\phi = 4 \int_{\pi/4}^{\pi/2} \frac{1 + \cos 2\phi}{2} d\phi = 2 \int_{\pi/4}^{\pi/2} (1 + \cos 2\phi) d\phi$$

B)

$$2 \left[\phi + \frac{1}{2} \sin 2\phi \right]_{\pi/4}^{\pi/2} = 2 \left[\left(\frac{\pi}{2} + 0 \right) - \left(\frac{\pi}{4} + \frac{1}{2} \right) \right] = \frac{\pi}{2} - 1$$

10) The area enclosed by the rose, $r = \cos 2\phi$ is:

$$A = \frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta \quad A = \frac{1}{2} \int_0^{2\pi} (\cos 2\phi)^2 d\phi = \frac{1}{2} \int_0^{2\pi} \frac{1 + \cos 4\phi}{2} d\phi = \frac{1}{4} \int_0^{2\pi} d\phi + \frac{1}{4} \int_0^{2\pi} \cos 4\phi d\phi$$

B)

$$\frac{\pi}{2} + \frac{1}{16} [\sin 4\phi]_0^{2\pi} = \frac{\pi}{2}$$