AP Calculus Exam Prep Assignment #12 page 2 KEY

11) $\lim_{x\to 0}\frac{\sin 2x}{x} = \lim_{x\to 0}\frac{2\cos 2x}{1} = 2$ **B)**

12) $\lim_{x\to 0}\left(x\sin\frac{1}{x}\right)$ = ______.

B) 0

13) $\lim_{x\to\infty}\frac{\sqrt{x}-4}{4-3\sqrt{x}} = \lim_{x\to\infty}\frac{\frac{1}{2\sqrt{x}}}{\frac{-3}{2\sqrt{x}}} = -\frac{1}{3}$ **A)**

14) $\lim_{x\to 0}\frac{e^{2x}-1}{\tan x} = \lim_{x\to 0}\frac{2e^{2x}}{\sec^2 x} = \lim_{x\to 0}\left(2e^{2x}\cos^2 x\right) = 2$ **D)**

15) $\lim_{h\to 0}\frac{1}{h}\ln\left(\frac{2+h}{2}\right) = \lim_{h\to 0}\frac{\ln\left(\frac{2+h}{2}\right)}{h} = \lim_{h\to 0}\frac{\left(\frac{1}{1+\frac{h}{2}}\right)\frac{1}{2}}{1} = \frac{1}{2}$ **C)**

16) $\lim_{x\to\infty}\frac{e^x}{x^{50}} = \lim_{x\to\infty}\frac{e^x}{50!} = \infty$ **D)**

17) $\lim_{a\to\infty} a^{\left[\frac{1}{2a}\right]} =$

$u = a^{\left[\frac{1}{2a}\right]} \Rightarrow \ln u = \ln a^{\left[\frac{1}{2a}\right]} = \frac{1}{2a}\ln a = \frac{\ln a}{2a}$

A)

$\lim_{a\to\infty}\frac{\ln a}{2a} = \lim_{a\to\infty}\frac{\frac{1}{a}}{2} = 0 \quad u = e^0 = 1$

Problems

18) Find the limit: $\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^x$

$u=\left(1+\frac{2}{x}\right)^x \quad \ln u=\ln\left(1+\frac{2}{x}\right)^x=x\ln\left(1+\frac{2}{x}\right)=\frac{\ln\left(1+\frac{2}{x}\right)}{\frac{1}{x}}$

$$\lim_{x\to\infty}\left[\frac{\ln\left(1+\frac{2}{x}\right)}{\frac{1}{x}}\right]=\lim_{x\to\infty}\left[\frac{\frac{1}{1+\frac{2}{x}}\left(\frac{-2}{x^2}\right)}{\frac{-1}{x^2}}\right]=2 \quad e^2$$

19) Find the maximum volume of a box that can be made by cutting off squares from the corners of an 8-inch by 15-inch rectangular sheet of cardboard and folding up the sides. Justify your answer**.**

$V=x(15-2x)(8-2x)=120x-46x^2+4x^3$

$V'=120-92x+12x^2=0\Rightarrow 4\left(3x^2-23x+30\right)=0\Rightarrow 4(3x-5)(x-6)=0$

$x=6,\frac{5}{3}$

Eliminate 6 (extraneous)

Max Volume $=\left(\frac{5}{3}\right)\left(\frac{35}{3}\right)\left(\frac{14}{3}\right)=\frac{2450}{27}$ cubic inches