# AP Calculus Exam Prep Assignment #11 KEY

*Problems*

12) Consider the curve given by $xy^2 - x^3y = 6$

A)

$$\frac{d}{dx}\left(xy^2 - x^3y = 6\right) = x\left(2y\frac{dy}{dx}\right) + 1\left(y^2\right) - \left[x^3\left(\frac{dy}{dx}\right) + 3x^2y\right] = 0$$

$$\frac{dy}{dx}\left(2xy - x^3\right) = 3x^2y - y^2$$

$$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$$

B)

$$(1)y^2 - (1)^3y = 6 \Rightarrow y^2 - y - 6 = 0 \Rightarrow \left(y - 3\right)\left(y + 2\right) = 0 \Rightarrow y = 3, -2$$

$\left(1,3\right)$ and $\left(1,-2\right)$

$$\left(1,3\right): \frac{dy}{dx} = \frac{3(1)(3) - 3^2}{2(1)(3) - 1^3} = 0 \quad \text{Tangent equation : } \quad y = 3$$

$$\left(1,-2\right): \frac{dy}{dx} = \frac{3(1)(-2) - (-2)^2}{2(1)(-2) - 1^3} = \frac{-10}{-5} = 2 \quad \text{Tangent equation : } \quad y = 2\left(x - 1\right) - 2$$

C) Find the *x*-coordinate of each point on the curve where the tangent line is vertical.

Vertical tangent $\Rightarrow \frac{dy}{dx}$ is undefined $\Rightarrow$ denominator = 0

$$2xy - x^3 = 0 \Rightarrow x\left(2y - x^2\right) = 0 \Rightarrow x = 0, \pm\sqrt{2y}$$

$$x = \pm\sqrt{2y} \Rightarrow y = \frac{x^2}{2} \quad x\left(\frac{x^2}{2}\right)^2 - x^3\left(\frac{x^2}{2}\right) = 6 \Rightarrow \frac{x^5}{4} - \frac{x^5}{2} = 6 \Rightarrow x^5 - 2x^5 = 24 \Rightarrow x = \sqrt[5]{-24}$$

13) Let *h* be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of *h* is given by $h'(x) = \frac{x^2 - 2}{x}$ for all $x \neq 0$.

A) Find all values of *x* for which the graph of *h* has a horizontal tangent, and determine whether *h* has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

$$h'(x) = 0 \Rightarrow x^2 - 2 = 0 \Rightarrow x = \pm\sqrt{2}$$

$$h''(x) = \frac{x(2x) - (x^2 - 2)}{x^2} = \frac{x^2 + 2}{x^2} \quad h''(x) = 0 \Rightarrow x^2 + 2 = 0, \text{ which is impossible.}$$

$$h''\left(\pm\sqrt{2}\right) = 2, \text{ so local minimum at } x = \pm\sqrt{2}$$

B) On what intervals, if any, is the graph of $h$ concave up? Justify your answer.
Concave up when $h''(x) > 0$. As $h''(x) > 0$ for all $x \neq 0$, the graph is concave up for the entire domain.

C) Write an equation for the line tangent to the graph of $h$ at $x = 4$.
$h'(4) = \frac{7}{2}$. $y = \frac{7}{2}(x-4) - 3$

D) Does the line tangent to the graph of $h$ at $x = 4$ lie above or below the graph of $h$ for $x > 4$? Why?
Below, as the second derivative of $h$ indicates that the graph is concave upward, thus the tangent line at any point on the graph will lie below the graph.

14) Consider the curve by $-8x^2 + 5xy + y^3 = -149$

A) Find $dy/dx$.
$$-16x + 5x\frac{dy}{dx} + 5y + 3y^2\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = \frac{16x - 5y}{5x + 3y^2}$$

B) Write an equation for the line tangent to the curve at the point (4,-1).
$\frac{dy}{dx} = \frac{64+5}{20+3} = 3$ $y = 3(x-4) - 1$

C) There is a number $k$ so that the point $(4.2,k)$ is on the curve. Using the tangent line found in part (B), approximate the value of $k$.
$k = 3(4.2-4) - 1 = -0.4$

D) Write an equation that can be solved to find the actual value of $k$ so that the point $(4.2,k)$ is on the curve. Do not solve the equation at this point.
$-8(4.2)^2 + 5(4.2)k + k^3 = -149$

E) Solve the equation found in part (D) for the value of $k$. (Calculator) $-0.373$