9) $\lim_{n\to\infty}\left[\frac{|x+2|^{n+1}}{(n+1)3^{n+1}}\cdot\frac{n3^n}{|x+2|^n}\right]=\lim_{n\to\infty}\frac{n}{3(n+1)}|x+2|$

converges if $\frac{|x+2|}{3}<1\Rightarrow|x+2|<3\Rightarrow-5<x<1$

Checking the endpoints:

$\sum_{n=1}^{\infty}\frac{(-5+2)^n}{n3^n}=\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$ converges (alt. harmonic)

$\sum_{n=1}^{\infty}\frac{(1+2)^n}{n3^n}=\sum_{n=1}^{\infty}\frac{1}{n}$ diverges (harmonic)

B) [-5,1)

10) $\lim_{n\to\infty}\left[\frac{(n+1)|x|^{n+1}}{2^{n+1}}\cdot\frac{2^n}{|nx|^n}\right]=\lim_{n\to\infty}\frac{n+1}{2n}|x|$

converges if $\frac{|x|}{2}<1\Rightarrow|x|<2\Rightarrow-2<x<2$

B) 2

Problems

1) A) $f'(0)=\frac{1}{2!}=\frac{1}{2}\quad f^{(17)}(0)=\frac{1}{18}$

B) (-∞,∞)

C) $g(x)=x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots+\frac{x^{n+1}}{(n+1)!}+\ldots.$

D) Write $f(x)$ in terms of a familiar function without using series.

$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots+\frac{x^n}{n!}+\ldots$

$\frac{e^x}{x}-\frac{1}{x}=1+\frac{x}{2!}+\frac{x^2}{3!}+\frac{x^3}{4!}+\ldots=\frac{e^x-1}{x}$

Then write $g(x)$ in terms of the same familiar function.

$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots+\frac{x^{n+1}}{(n+1)!}+\ldots$

$e^x-1=x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots+\frac{x^{n+1}}{(n+1)!}+\ldots$

2) [1997 BC2] Let $P(x) = 7 - 3(x-4) + 5(x-4)^2 - 2(x-4)^3 + 6(x-4)^4$ be the fourth-degree Taylor polynomial for the function f about 4. Assume f has derivatives of all orders for all real numbers.

A) Find $f(4)$ and $f'''(4)$. $\quad f(4)=7;\quad \frac{f'''(4)}{3!}=-2\Rightarrow f'''(4)=-12$

B) Write the second-degree Taylor polynomial for $f'(x)$ about 4 and use it to approximate $f'(4.3)$. $\quad P_2(x)=-3+10(x-4)-6(x-4)^2;\quad P_2(4.3)=-3+10(0.3)-6(0.3)^2=-0.54$

C) Write the fourth-degree Taylor polynomial for $g(x)=\int_4^x f(t)\,dt$ about 4.

$g(x)=7x-\frac{3}{2}(x-4)^2+\frac{5}{3}(x-4)^3-\frac{1}{2}(x-4)^4$

D) Can $f(3)$ be determined from the information given? Justify your answer.

3) [2001 BC 6] A function f is defined by $f(x) = \frac{1}{3} + \frac{2}{3^2}x + \frac{3}{3^3}x^2 + \ldots + \frac{n+1}{3^{n+1}}x^n + \ldots$ for all x in the interval of convergence of the given power series.

A) Find the interval of convergence for this power series. Show the work that leads to your answer.

$$\text{Ratio Test: } \lim_{n\to\infty}\left[\frac{n+2|x|^{n+1}}{3^{n+2}} \cdot \frac{3^{n+1}}{(n+1)|x|^n}\right] = \frac{|x|}{3}$$

$$\text{Converges if: } \frac{|x|}{3} < 1 \Rightarrow -3 < x < 3$$

$$\text{Testing endpoints: } \frac{(n+1)}{3^{n+1}}\left(-3^n\right) = \left(-1\right)^n \frac{n+1}{3} \text{ diverges} \quad \frac{(n+1)}{3^{n+1}}\left(3^n\right) = \frac{n+1}{3} \text{ diverges}$$

Interval of convergence: (-3,3)

B) Find $\lim_{x\to 0}\frac{f(x)-\frac{1}{3}}{x}$. $\quad \frac{2}{3^2} = \frac{2}{9}$

C) Write the first three nonzero terms and the general term for an infinite series that represents $\int_0^1 f(x)\,dx$.

$$\left[\frac{1}{3}x + \frac{1}{3^2}x^2 + \frac{1}{3^3}x^3 + \ldots + \frac{1}{3^{n+1}}x^{n+1} + \ldots\right]_0^1 = \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots + \frac{1}{3^{n+1}} + \ldots$$

D) Find the sum of the series determined in part (C). $\quad \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{1}{2}$