Calculus Chapter 7 Part 1 Review Solutions

1) $\int \frac{x}{3x^2-6}dx \qquad \begin{matrix} u = 3x^2-6 \\ du = 6xdx \end{matrix} \qquad \frac{1}{6}\int \frac{du}{u} = \frac{1}{6}\ln\left|3x^2-6\right| + C$

2) $\int \cot^5(5x)\csc^2(5x)dx \qquad \begin{matrix} u = \cot(5x) \\ du = -5\csc^2(5x)dx \end{matrix} \qquad -\frac{1}{5}\int u^5 du = -\frac{\cot^6(5x)}{30} + C$

3)

$$\int x\sin(10x)dx \qquad \begin{matrix} u = x & dv = \sin(10x)dx \\ du = dx & v = -\frac{1}{10}\cos(10x) \end{matrix} \qquad -\frac{x}{10}\cos(10x) + \frac{1}{10}\int \cos(10x)dx$$

$$= -\frac{x}{10}\cos(10x) + \frac{1}{10}\left(\frac{1}{10}\sin(10x)\right) + C = -\frac{x}{10}\cos(10x) + \frac{1}{100}\sin(10x) + C$$

4)

$$\int \frac{x+7}{x^2+4x}dx \qquad x+7 = A(x+4) + Bx$$

$$x = 0 : 7 = 4A \Rightarrow A = \frac{7}{4} \qquad x = -4 : 3 = -4B \Rightarrow B = -\frac{3}{4}$$

$$\frac{7}{4}\int \frac{dx}{x} - \frac{3}{4}\int \frac{dx}{x+4} = \frac{7}{4}\ln|x| - \frac{3}{4}\ln|x+4| + C$$

5) $\int \csc^2 x\left(1+\cot^2 x\right)dx = \int \csc^2 x\,dx + \int \csc^2 x\cot^2 x\,dx = -\cot x - \frac{\cot^3 x}{3} + C$

6) $\int_{-\pi}^{0} \cos^4\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)dx \qquad \begin{matrix} u = \cos\left(\frac{x}{2}\right) \\ du = -\frac{1}{2}\sin\left(\frac{x}{2}\right) \end{matrix} \qquad -2\int_0^1 u^4 du = -2\left[\frac{u^5}{5}\right]_0^1 = -\frac{2}{5}$

7)

$$\int e^{7x}\sin x\,dx \qquad \begin{matrix} u = e^{7x} & dv = \sin x\,dx \\ du = 7e^{7x}dx & v = -\cos x \end{matrix}$$

$$\int e^{7x}\sin x\,dx = -e^{7x}\cos x + 7\int e^{7x}\cos x\,dx \qquad \begin{matrix} u = e^{7x} & dv = \cos x\,dx \\ du = 7e^{7x}dx & v = \sin x \end{matrix}$$

$$\int e^{7x}\sin x\,dx = -e^{7x}\cos x + 7\left[e^{7x}\sin x - 7\int e^{7x}\sin x\,dx\right]$$

$$50\int e^{7x}\sin x\,dx = -e^{7x}\cos x + 7e^{7x}\sin x \Rightarrow \int e^{7x}\sin x\,dx = \frac{-e^{7x}\cos x + 7e^{7x}\sin x}{50} + C$$

8) $\int x^3 \cos 8x \, dx$

x^3	$\cos 8x$
$3x^2$	$\frac{1}{8}\sin 8x$
$6x$	$-\frac{1}{64}\cos 8x$
6	$-\frac{1}{512}\sin 8x$
0	$\frac{1}{4096}\cos 8x$

$$\int x^3 \cos 8x \, dx = \frac{1}{8}x^3 \sin 8x + \frac{3}{64}x^2 \cos 8x - \frac{3}{256}x \sin 8x - \frac{3}{2048}\cos 8x + C$$