AP Calculus Chapter 7 Part 1 Review Day 2 SOLUTIONS

Evaluate the integral

1)

$$\int x^4\sqrt{x^5+9}\,dx \qquad \begin{matrix} u = x^5+9 \\ du = 5x^4dx \end{matrix}$$

$$\frac{1}{5}\int u^{1/2}\,du = \frac{1}{5}\frac{u^{3/2}}{3/2}+C = \frac{2}{15}\left(x^5+9\right)^{3/2}+C$$

2)

$$\int \sqrt[7]{\cot t}\csc^2 t\,dt \qquad \begin{matrix} u = \cot x \\ du = -\csc^2 x\,dx \end{matrix}$$

$$-\int u^{1/7}\,du = -\frac{u^{8/7}}{8/7}+C = -\frac{7}{8}\left(\cot x\right)^{8/7}+C$$

3)

$$\int t^{1/8}\sin\left(t^{9/8}-3\right)dt \qquad \begin{matrix} u = t^{9/8}-3 \\ du = \frac{9}{8}t^{1/8}dt \end{matrix}$$

$$\frac{8}{9}\int \sin u\,du = -\frac{8}{9}\cos u+C = -\frac{8}{9}\cos\left(t^{9/8}-3\right)+C$$

Use the given trig identity to set up a *u*-substitution and then evaluate the indefinite integral.

4)

$$\int \cos^3 7x\,dx,\ \cos^2 7x = 1-\sin^2 7x$$

$$\int \cos 7x\left(1-\sin^2 7x\right)dx = \int \cos 7x\,dx - \int \cos 7x\sin^2 7x\,dx \qquad \begin{matrix} u = \sin 7x \\ du = 7\cos 7x\,dx \end{matrix}$$

$$= \frac{1}{7}\sin 7x - \frac{1}{7}\int u^2\,du = \frac{1}{7}\sin 7x - \frac{1}{7}\left(\frac{u^3}{3}\right)+C = \frac{1}{7}\sin 7x - \frac{1}{21}\sin^3 7x+C$$

Evaluate the definite integral by making a *u*-substitution and integrating from *u*(*a*) to *u*(*b*).

5)

$$\int_0^1 \frac{6x\,dx}{\sqrt{16+3x^2}} \qquad \begin{matrix} u = 16+3x^2 \\ du = 6x\,dx \end{matrix}$$

$$\int_{16}^{19} u^{-1/2}\,du = \left[\frac{u^{1/2}}{1/2}\right]_{16}^{19} = 2\sqrt{19}-2\sqrt{16}$$

6)

$$\int_{-5}^{6} \frac{x\,dx}{4x^2+3} \qquad \begin{matrix} u = 4x^2+3 \\ du = 8x\,dx \end{matrix}$$

$$\frac{1}{8}\int_{103}^{147} \frac{du}{u} = \frac{1}{8}\Big[\ln|u|\Big]_{103}^{147} = \frac{1}{8}\big(\ln 147 - \ln 103\big)$$

7) Evaluate using Parts:

$$\int -4x\cos 3x\,dx \qquad \begin{matrix} u = -4x & dv = \cos 3x\,dx \\ du = -4dx & v = \frac{1}{3}\sin 3x \end{matrix}$$

$$= -\frac{4}{3}x\sin 3x - \int\left(-\frac{4}{3}\right)\sin 3x\,dx = -\frac{4}{3}x\sin 3x - \frac{4}{9}\cos 3x + C$$

8) Evaluate using Tabular: $\int x^3 e^{2x}\,dx$

x^3	e^{2x}
$3x^2$	$\frac{1}{2}e^{2x}$
$6x$	$\frac{1}{4}e^{2x}$
6	$\frac{1}{8}e^{2x}$
0	$\frac{1}{16}e^{2x}$

$$\int x^3 e^{2x}\,dx = \frac{1}{2}x^3e^{2x} - \frac{3}{4}x^2e^{2x} + \frac{3}{4}xe^{2x} - \frac{3}{8}e^{2x} + C$$

9) Evaluate using Partial Fractions: $\int \frac{7x-17}{x^2-3x-40}\,dx$

$$\frac{7x-17}{x^2-3x-40} = \frac{7x-17}{(x-8)(x+5)} = \frac{A}{x-8} + \frac{B}{x+5}$$

$$7x-17 = A(x+5) + B(x-8)$$

$$x = -5: -52 = -13B \Rightarrow B = 4$$

$$x = 8: \quad 39 = 13A \Rightarrow A = 3$$

$$\int \frac{7x-17}{x^2-3x-40}dx = 3\int \frac{dx}{x-8} + 4\int \frac{dx}{x+5} = 3\ln|x-8| + 4\ln|x+5| + C$$