AP Calculus BC Summer Work

All work is done **NEATLY** on a separate paper. You must show all work. **DO NOT** try and save paper by cramming your solutions on top of each other.

Limits and Their Properties (A calculator may not be used)

√1. Evaluate the limit, if it exist: $\lim_{x\to 2}\frac{x^2+x-6}{2-x}$.

2. Evaluate the limit, if it exist: $\lim_{x\to 9}\frac{\sqrt{x-5}-2}{x-9}$.

3. Evaluate the limit, if it exist: $\lim_{x\to 2}\frac{\frac{1}{x}-\frac{1}{2}}{x-2}$.

4. Evaluate the limit, if it exist: $\lim_{x\to 1}\frac{\tan^{-1}x}{\sin^{-1}x+1}$.

5. Given the function $f(x)=\begin{cases}\sin 2x, & x\le\pi\\ 2x+k, & x>\pi\end{cases}$ what value of k will make this piecewise function continuous?

6. Evaluate the limit, if it exist: $\lim_{x\to 0}x\left(e^x+\frac{1}{x}\right)$.

7. Identify the vertical asymptotes for $f(x)=\frac{x^2+3x-4}{x^2+x-2}$.

8. How many vertical asymptotes exist for the function $f(x)=\frac{1}{2\sin^2x-\sin x-1}$ in the open interval $0<x<\pi$?

9. Evaluate the limit, if it exist: $\lim_{x\to\infty}\frac{\sin x}{e^x+\cos x}$.

10. For what value of k is the function $f(x)=\begin{cases}\frac{2x^2+5x-3}{x^2-9}, & x\ne-3\\ k, & x=-3\end{cases}$ continuous at x = -3?

Differentiation

1. What does the statement $\lim_{x\to 1}\frac{\ln(x+1)-\ln 2}{x-1}$ represent?

2. Find the derivative of the function $y = \frac{4}{x^3}$.

3. Find y' if $3xy = 4x + y^2$.

4. Find y'' if $(x) = (2x+3)^4$.

5. For what values of a and c is the piecewise function $f(x) = \begin{cases} ax^2 + \sin x, x \le \pi \\ 2x - c, x > \pi \end{cases}$ differentiable?

6. Selected function and derivative values for the differentiable functions f(x) and g(x) are given in the table below. If p(x) = xf(x) - g(3x-2), then find p'(2).

x	f(x)	g(x)	f'(x)	g'(x)
1	3	1	-2	4
2	5	3	1	-4
3	2	1	-2	1
4	4	-3	2	-1

7. When the height of a cylinder is 12cm and the radius is 4cm, the circumference of the cylinder is increasing at a rate of $\frac{\pi}{4}$ cm/min, and the height of the cylinder is increasing four times faster than the radius. How fast is the volume of the cylinder changing?

8. Find the derivative of the function $y = \frac{(3x^2-2x)(x+2)}{x^3-2x+1}$.

9. Find the derivative of the function $y = (4x+2)^3(x^2-3x+1)^2$.

10. Find the derivative of the function $y = \frac{4x+1}{4x^3-3x}$.

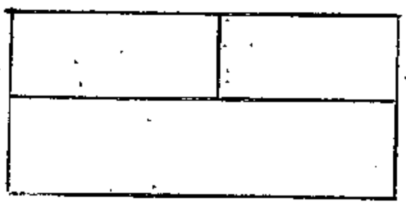
Applications of Derivatives

1. What value of c in the open interval (0,4) satisfies the Mean Value Theorem f(t)or $f(x)\sqrt{3x+4}$ +2

2. If $f'(x) = \frac{x^2(x+1)}{(x-1)^{1/3}}$, then on which interval(s) is the continuous function f(x) increasing?

3. Evaluate: $\lim_{x\to\infty} \frac{\sqrt{x^2-14}}{3-2x}$.

4. A farmer has 100 yards of fencing to form two identical rectangular pens and a third pen that is twice as long as the other two pens, as shown in the diagram. All three pens have the same width x. What value of y produces the maximum total fenced area?



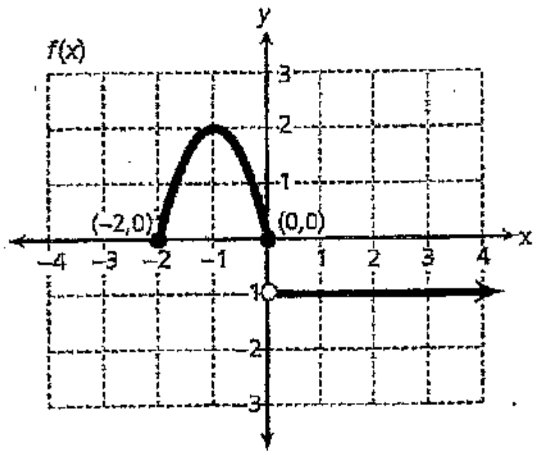
5. For the function $f(x) = 12x^5 - 5x^4$, how many of the inflection points of the function are also extrema?

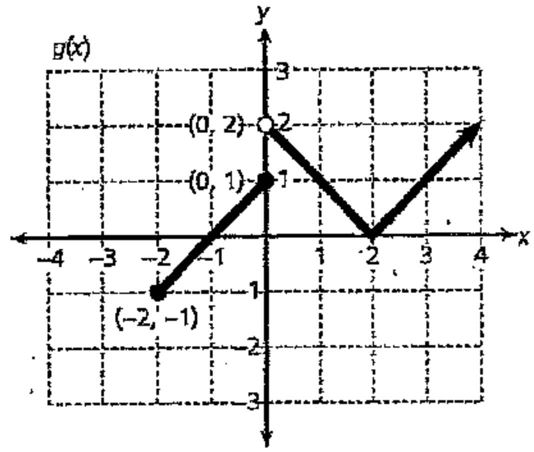
6. The position of an object moving along a straight line for $t \geq 0$ is given by $s_1(t) = t^3 + 2$, and the position of a second object moving along the same line is given by $s_2(t) = t^2$. If both objects begin at t = 0, at what time is the distance between the objects a minimum?

7. For time $0 \leq t \leq 10$, a particle moves along the x-axis with the position given $x(t) = t^3 - 7x^2 + 8t + 5$. During what time intervals is the speed of the particle increasing? What is the position of the particle when it is farthest to the left?

A calculator may not be used for this question.

2. Use the graphs of $f(x)$ and $g(x)$ given below to answer the following questions:



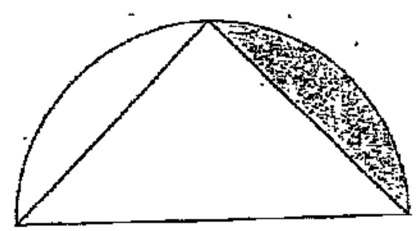


a. Is $f[g(x)]$ continuous at $x = 0$? Explain why or why not.
b. Is $g[f(x)]$ continuous at $x = 0$? Explain why or why not.
c. What is $\lim_{x \to +\infty} f[g(x)]$? Explain your reasoning.
d. If $h(x) = \begin{cases} f(x)+g(x), & -2 \le x \le 0 \\ k+g(x)f(x), & x > 0 \end{cases}$, what is k so that $h(x)$ is continuous at $x = 0$?

FREE-RESPONSE QUESTION

A calculator may be used for this question.

3. An isosceles triangle is inscribed in a semicircle, as shown in the diagram, and it continues to be inscribed as the semicircle changes size. The area of the semicircle is increasing at the rate of 1 cm^2/sec when the radius of the semicircle is 3 cm.

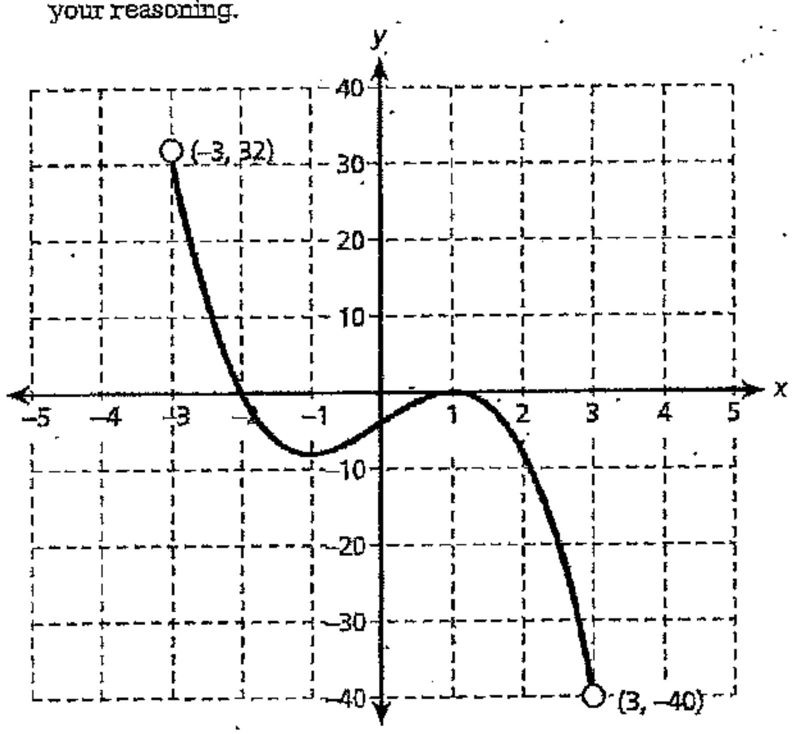


a. How fast is the radius of the semicircle increasing when the radius is 3 cm? Include units in your answer.
b. How fast is the perimeter of the semicircle increasing when the radius is 3 cm? Include units in your answer.
c. How fast is the area of the isosceles triangle increasing when the radius is 3 cm? Include units in your answer.
d. How fast is the shaded region increasing when the radius is 3 cm? Include units in your answer.

FREE-RESPONSE QUESTION

This question does not require the use of a calculator.

1. The function $f(x)$ is defined as $f(x) = -2(x+2)(x-1)^2$ on the open interval (−3, 3) as illustrated in the graph shown.
   a. Determine the coordinates of the relative extrema of $f(x)$ in the open interval (−3, 3).
   b. Let $g(x)$ be defined as $g(x) = |f(x)|$ in the open interval (−3, 3). Determine the coordinate(s) of the relative maxima of $g(x)$ in the open interval. Explain your reasoning.
   c. For what values of x is $g'(x)$ not defined? Explain your reasoning.
   d. Find all values of x for which $g(x)$ is concave down. Explain your reasoning.



$f(x)$