AP Calculus BC Section 9.2 Taylor Series (Old School)

Use the definition to find the Taylor Series (centered at *c*) for the function
(Source: Calculus (6th ed.) Larson/Hostetler)

1) $f(x)=e^{2x},\quad c=0$

$f'(x)=2e^{2x} \qquad f''(x)=4e^{2x} \qquad f'''(x)=8e^{2x} \qquad f^{(4)}(x)=16e^{2x} \qquad f^{(n)}(x)=2^n e^{2x}$

$$P_n(x)=1+2x+\frac{4}{2!}x^2+\frac{8}{3!}x^3+\frac{16}{4!}x^4+\ldots+\frac{2^n}{n!}x^n$$

2) $f(x)=e^{-2x},\quad c=0$

$f'(x)=-2e^{2x} \qquad f''(x)=4e^{2x} \qquad f'''(x)=-8e^{2x} \qquad f^{(4)}(x)=16e^{2x} \qquad f^{(n)}(x)=\left(-2\right)^n e^{2x}$

$$P_n(x)=1-2x+\frac{4}{2!}x^2-\frac{8}{3!}x^3+\frac{16}{4!}x^4-\ldots+\frac{\left(-2\right)^n}{n!}x^n$$

3) $f(x)=\cos x,\quad c=\frac{\pi}{4}$

$f'(x)=-\sin x \qquad f''(x)=-\cos x \qquad f'''(x)=\sin x \qquad f^{(4)}(x)=\cos x$

$$P_n(x)=\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right)-\frac{\frac{\sqrt{2}}{2}}{2!}\left(x-\frac{\pi}{4}\right)^2+\frac{\frac{\sqrt{2}}{2}}{3!}\left(x-\frac{\pi}{4}\right)^3+\frac{\frac{\sqrt{2}}{2}}{4!}\left(x-\frac{\pi}{4}\right)^4-\ldots+\left(-1\right)^{n(n+1)/2}\frac{\sqrt{2}}{2}\left(\frac{\left(x-\frac{\pi}{4}\right)^n}{n!}\right)$$

4) $f(x)=\sin x,\quad c=\frac{\pi}{4}$

$f'(x)=\cos x \qquad f''(x)=-\sin x \qquad f'''(x)=-\cos x \qquad f^{(4)}(x)=\sin x$

$$P_n(x)=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right)-\frac{\frac{\sqrt{2}}{2}}{2!}\left(x-\frac{\pi}{4}\right)^2-\frac{\frac{\sqrt{2}}{2}}{3!}\left(x-\frac{\pi}{4}\right)^3+\frac{\frac{\sqrt{2}}{2}}{4!}\left(x-\frac{\pi}{4}\right)^4+\ldots+\left(-1\right)^{n(n-1)/2}\frac{\sqrt{2}}{2}\left(\frac{\left(x-\frac{\pi}{4}\right)^n}{n!}\right)$$

5) $f(x)=\ln x,\quad c=1$

$f'(x)=\frac{1}{x} \qquad f''(x)=-\frac{1}{x^2} \qquad f'''(x)=\frac{2}{x^3} \qquad f^{(4)}(x)=-\frac{6}{x^4}$

$$P_n(x)=0+1\left(x-1\right)-\frac{1}{2!}\left(x-1\right)^2+\frac{2}{3!}\left(x-1\right)^3-\frac{6}{4!}\left(x-1\right)^4+\ldots+\left(-1\right)^{n-1}\frac{1}{n}\left(\frac{\left(x-1\right)^n}{n!}\right)$$

$$\sum_{k=1}^{\infty}\left(-1\right)^{n-1}\frac{1}{k}\left(\frac{\left(x-1\right)^k}{k!}\right)$$

6) $f(x)=\sin 2x, \quad c=0$

$f'(x)=2\cos 2x \qquad f''(x)=-4\sin 2x \qquad f'''(x)=-8\cos 2x \qquad f^{(4)}(x)=16\sin 2x \qquad f^{(5)}(x)=32\cos 2x$

$$P_n(x)=2x-\frac{8}{3!}x^3+\frac{32}{5!}x^5-\ldots+(-1)^n\left(\frac{(2x)^{2n+1}}{(2n+1)!}\right)$$

7) $f(x)=\ln\left(x^2+1\right), \quad c=0$

$f'(x)=\dfrac{2x}{x^2+1} \qquad f''(x)=-\dfrac{2\left(x^2-1\right)}{\left(x^2+1\right)^2} \qquad f'''(x)=\dfrac{4x\left(x^2-3\right)}{\left(x^2+1\right)^3} \qquad f^{(4)}(x)=-\dfrac{12\left(x^4-6x^2+1\right)}{\left(x^2+1\right)^4}$

$$P_n(x)=0+\frac{2}{2!}x^2+\frac{12}{4!}x^4+\ldots+(-1)^n\frac{x^{2n+2}}{n+1}$$

8) $f(x)=\dfrac{1}{x}, \quad c=1$

$f'(x)=-\dfrac{1}{x^2} \qquad f''(x)=\dfrac{2}{x^3} \qquad f'''(x)=-\dfrac{6}{x^4} \qquad f^{(4)}(x)=\dfrac{24}{x^5}$

$$P_n(x)=1-(x-1)+\frac{2}{2!}(x-1)^2-\frac{6}{3!}(x-1)^3+\frac{24}{4!}(x-1)^4-\ldots+(-1)^n(x-1)^n$$

9) $f(x)=\dfrac{1}{x}, \quad c=2$

$f'(x)=-\dfrac{1}{x^2} \qquad f''(x)=\dfrac{2}{x^3} \qquad f'''(x)=-\dfrac{6}{x^4} \qquad f^{(4)}(x)=\dfrac{24}{x^5}$

$$P_n(x)=\frac{1}{2}-\frac{1}{4}(x-2)+\frac{1/4}{2!}(x-2)^2-\frac{3/8}{3!}(x-2)^3+\frac{3/4}{4!}(x-2)^4-\ldots+\frac{(-1)^n}{2^{n+1}}(x-2)^n$$

10) $f(x)=\dfrac{1}{x}, \quad c=-1$

$f'(x)=-\dfrac{1}{x^2} \qquad f''(x)=\dfrac{2}{x^3} \qquad f'''(x)=-\dfrac{6}{x^4} \qquad f^{(4)}(x)=\dfrac{24}{x^5}$

$$P_n(x)=-1-(x+1)-\frac{2}{2!}(x+1)^2-\frac{6}{3!}(x+1)^3-\frac{24}{4!}(x+1)^4-\ldots-(x+1)^n$$