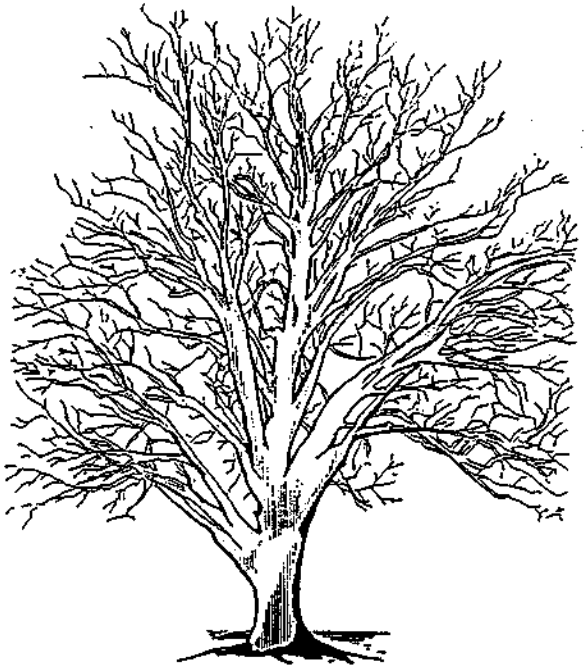
Monroe Township Schools



Curriculum Management System

AP Calculus BC

Grade 12

December 2008

*** For adoption by all regular education programs as specified and for adoption or adaptation by all Special Education Programs in accordance with Board of Education Policy # 2220.**

Board Approved: January 2009

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MONROE TOWNSHIP SCHOOL DISTRICT

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Acknowledgments

The following individuals are acknowledged for their assistance in the preparation of this Curriculum Management System:

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Monroe Township Schools

Mission and Goals

Mission

The mission of the Monroe Township School District, a unique multi-generational community, is to collaboratively develop and facilitate programs that pursue educational excellence and foster character, responsibility, and life-long learning in a safe, stimulating, and challenging environment to empower all individuals to become productive citizens of a dynamic, global society.

Goals

To have an environment that is conducive to learning for all individuals.

To have learning opportunities that are challenging and comprehensive in order to stimulate the intellectual, physical, social and emotional development of the learner.

To procure and manage a variety of resources to meet the needs of all learners.

To have inviting up-to-date, multifunctional facilities that both accommodate the community and are utilized to maximum potential.

To have a system of communication that will effectively connect all facets of the community with the Monroe Township School District.

To have a staff that is highly qualified, motivated, and stable and that is held accountable to deliver a safe, outstanding, and superior education to all individuals.

INTRODUCTION, PHILOSOPHY OF EDUCATION, AND EDUCATIONAL GOALS

Philosophy

Monroe Township Schools are committed to providing all students with a quality education resulting in life-long learners who can succeed in a global society. The mathematics program, grades K - 12, is predicated on that belief and is guided by the following six principles as stated by the National Council of Teachers of Mathematics (NCTM) in the *Principles and Standards for School Mathematics, 2000.* First, a mathematics education requires equity. All students will be given worthwhile opportunities and strong support to meet high mathematical expectations. Second, a coherent mathematics curriculum will effectively organize, integrate, and articulate important mathematical ideas across the grades. Third, effective mathematics teaching requires the following: a) knowing and understanding mathematics, students as learners, and pedagogical strategies b) having a challenging and supportive classroom environment and c) continually reflecting on and refining instructional practice. Fourth, students must learn mathematics with understanding. A student's prior experiences and knowledge will actively build new knowledge. Fifth, assessment should support the learning of important mathematics and provide useful information to both teachers and students. Lastly, technology enhances mathematics learning, supports effective mathematics teaching, and influences what mathematics is taught.

As students begin their mathematics education in Monroe Township, classroom instruction will reflect the best thinking of the day. Children will engage in a wide variety of learning activities designed to develop their ability to reason and solve complex problems. Calculators, computers, manipulatives, technology, and the Internet will be used as tools to enhance learning and assist in problem solving. Group work, projects, literature, and interdisciplinary activities will make mathematics more meaningful and aid understanding. Classroom instruction will be designed to meet the learning needs of all children and will reflect a variety of learning styles.

In this changing world those who have a good understanding of mathematics will have many opportunities and doors open to them throughout their lives. Mathematics is not for the select few but rather is for everyone. Monroe Township Schools are committed to providing all students with the opportunity and the support necessary to learn significant mathematics with depth and understanding. This curriculum guide is designed to be a resource for staff members and to provide guidance in the planning, delivery, and assessment of mathematics instruction for grades K - 8.

Educational Goals

Advanced Placement Calculus BC is the fifth course of the honors college preparatory sequence. It is a rigorous and challenging course designed for the student of exceptional mathematical ability needing a strong background in mathematics for future academic work at the college level. Topics in this course include all AP Calculus AB topics and, in addition, the following topics are included: parametric, polar, and vector functions; analysis of planar curves given in parametric form, polar form, and vector form; numerical solution of differential equations using Euler's method; L'Hôpital's Rule; derivatives of parametric, polar, and vector functions; applications of integrals; antiderivatives by substitution of variables, parts, and simple partial fractions; improper integrals; solving logistic differential equations and using them in modeling; polynomial approximations and series; concepts of series; series of constants; geometric series, harmonic series, alternating series; terms of series; ratio test for convergence and divergence; comparing series to test for convergence or divergence; Taylor series; Maclaurin series; power series; radius and interval of convergence of power series; and Lagrange error bound for Taylor polynomials. The study of calculus is necessary not only for programs in mathematics and engineering, but also required for programs in science, medicine and many other fields.

New Jersey State Department of Education
Core Curriculum Content Standards

A note about Mathematics Standards and Cumulative Progress Indicators.

The New Jersey Core Curriculum Content Standards for Mathematics were revised in 2008. The Cumulative Progress Indicators (CPI's) referenced in this curriculum guide refer to these new standards and may be found in the Curriculum folder on the district servers. A complete copy of the new Core Curriculum Content Standards for Mathematics may also be found at:

http://education.state.nj.us/cccs/?_standard_matrix;c=4

Course Name Grade

Scope and Sequence

<table>
<tr><th colspan="2">Quarter I</th></tr>
<tr><td>

Big Idea: Change

I. Limits and Continuity
- a. Average and Instantaneous Rates of Change
- b. Limits
- c. Continuity
- d. Tangent Lines and the Normal to a Curve
- e. Slope of a Curve at a Given Point

</td><td>

Big Idea: Change

II. Derivatives
- a. Definition of a f'
- b. Relationship Between the Graphs of f and f'
- c. Differentiability
- d. Rules for Differentiation
- e. Velocity and Rate of Change
- f. Chain Rule and Implicit Differentiation
- g. Derivatives of Trigonometric Functions and Inverse Trigonometric, Inverse, Exponential and Logarithmic Functions

</td></tr>
<tr><td>

Big Idea: Derivatives

III. Applications of Derivatives
- a. Extreme Values of Functions
- b. Mean Value Theorem for Derivatives
- c. Analysis of Graphs Using First and Second Derivatives
- d. Modeling and Optimization
- e. Linearization and Newton's Method
- f. Differentials and Change
- g. Related Rates

</td><td>

Big Idea: Integrals

IV. Definite Integral
- a. Approximating Areas
- b. Definite Integrals and Antiderivatives
- c. Average Value of A Function
- d. Mean Value Theorem for Definite Integrals
- e. Area and Volume of a Function
- f. Connecting Differential and Integral Calculus
- g. Fundamental Theorem of Calculus

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<tr><td>

Big Idea: Derivatives

V. Differential Equations and Mathematical Modeling
- a. Slope Fields
- b. Euler's Method
- c. Antiderivatives and the Indefinite Integral
- d. Techniques of Integration
- e. Separable Differential Equations
- f. Exponential Growth and Decay
- g. Logistic Growth

</td><td></td></tr>
</table>

Quarter II

Big Idea: Mathematical Modeling

VI. Applications of Definite Integrals
- a. Net Change, Motion on a Line and Consumption Over Tine
- b. Area in the Plane
- c. Volumes Using Integral
- d. Lengths of Curves, Vertical Tangents, Corners and Cusps
- e. Surface Area of a Solid of Revolution
- f. Work and Fluid Force

Big Idea: Improper Integrals

VII. L'Hopital's Rule and Improper Integrals
- a. Indeterminate Forms Using L'Hopital's Rule and Applications
- b. Relative Rates of Growth
- c. Improper Integrals

Quarter III

Big Idea: Series

VIII. Infinite Series
- a. Geometric Series
- b. Power Series
- c. Taylor's Series, Maclaurin Series and Taylor's Theorem
- d. Lagrange Form of the Remainder
- e. Euler's Formula
- f. Test for Convergence and Divergence
- g. Harmonic Series
- h. Testing for Convergence at Endpoints
- i. Alternating Series (Leibniz's Theorem)
- j. Testing a Power Series for Convergence

Big Idea: Advanced Functions

IX. Parametric, Vector and Polar Functions
- a. Parametric Functions
- b. Slope and Concavity: First and Second Derivatives at a Point
- c. Arc Length
- d. Angle Between two Vectors
- e. Vector Addition and Scalar Multiplication
- f. Properties of Vector Operations
- g. Using Vectors to Describe Motion in a Plane
- h. Velocity, Acceleration, Speed, Displacement and Distance Traveled

Big Idea: Derivatives and Integrals

X. Formal Review of all Derivatives, Integrals and Applications
- a. Spiral Applications and Practice Problems
- b. Practice Tests from College Board Material
- c. Practice Problems from Review Sources

Quarter IV

Big Idea: Derivatives and Integrals

XI. Formal Review of all Derivatives, Integrals and Applications
- a. Spiral Applications and Practice Problems
- b. Practice Tests from College Board
- c. Practice Problems from Review Sources

Big Idea: Multivariable Calculus

XII. Three-Dimensional Space; Vectors
- a. Rectangular Coordinates in 3-Space
- b. Spheres
- c. Cylindrical Surfaces
- d. Vectors

Suggested days of Instruction	Curriculum Management System Subject/Grade Level: Course Name/Grade AP Calculus BC/12	Big Idea: Change Topic: Limits and Rates of Change Overarching Goals: (1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes. (2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems. (3) Investigate, research, and synthesize various information from a variety of media sources. Major Goal 1: The student will be able to understand and apply the concepts of limits and rates of change.	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
2	1.1. calculate average and instantaneous speed. (CPI 4.3.12.B.2)* 1.2. function values and apply the properties of limits. (CPI 4.3.12.B.2)* 1.3. use the Sandwich Theorem to find certain limits indirectly. (CPI 4.3.12.B.2)* 1.4. find and verify end behavior models for various functions. (CPI 4.5.K12.F.1, 4.5.K12.F.3)* 1.5. calculate limits as $x \to \pm\infty$ $x \to \pm\infty$ and to identify vertical and horizontal asymptotes. (CPI 4.3.12.B.2, 4.5.K12.F.3)*	• What is the difference between average and instantaneous rates of change? • What is the definition of a limit? • What methods can we use to evaluate limits? (analytical, graphical, numerical) • What is the difference between a one-sided and a two-sided limit? Example : If $f(x) = \begin{cases} 1-x, & x < 0 \\ 2, & x = 0 \\ x^2+1, & x > 0 \end{cases}$ Find each limit and $f(0)$: $\lim_{x \to 0^-} f(x)$ $\lim_{x \to 0^+} f(x)$ $\lim_{x \to 0} f(x)$ Find $f(0)$	**NOTE: The assessment models provided in this document are suggestions for the teacher. If the teacher chooses to develop his/her own model, *it must be of equal or better quality and at the same or higher cognitive levels (as noted in parentheses*).** **Depending upon the needs of the class, the assessment questions may be answered in the form of essays, quizzes, mobiles, PowerPoint, oral reports, booklets, or other formats of measurement used by the teacher.** **Learning Activities:** • The teacher and students will evaluate limits using various methods including graphical, analytical and numerical. (*Evaluation)* • Explore properties of limits of two functions by evaluating each one separately as $x \to c$ and comparing it to the limit of their product as $x \to c$ (*Evaluation)* • Explore the use of factoring to remove a discontinuity *(Analysis)*

Suggested days of Instruction	Curriculum Management System Subject/Grade Level: **Course Name/Grade** **AP Calculus BC/12**	**Big Idea: Change** **Topic: Limits and Rates of Change** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Major Goal 1: The student will be able to understand and apply the concepts of limits and rates of change.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
	1.6. identify the interval upon which a given function is continuous and understand the meaning of a continuous function. (CPI 4.3.12.B.2)* 1.7. remove removable discontinuities by extending or modifying a function. (CPI 4.3.12.B.2)* 1.8. apply the Intermediate Value Theorem and the properties of algebraic combinations and composites of continuous functions. (CPI 4.3.12.B.2)* 1.9. apply directly the definition of the slope of a curve in order to calculate slope. (CPI 4.3.12.B.2)* 1.10. find the equations of the tangent line and normal line to a curve at a given point. (CPI 4.3.12.B.2)*	• How can limits be used to identify vertical and horizontal asymptotes? • How can limits be used to determine if a function is continuous? Example : Using $f(x)$ from the previous example , is $f(x)$ continuous? Why or why not? • What is the Intermediate Value Theorem for continuous functions? • How can limits be used to find the slope of a tangent line at a given point on a curve? • How can we use our knowledge of average and instantaneous rates of change to find the slopes of secant and tangent lines to a curve?	• Quick assessment: (Concepts worksheet 2.2-2.3) Evaluating limits using end-behavior models and evaluating continuity at $x = c$. (*Evaluation)* • Appropriate practice questions from Multiple-Choice & Free-Response Questions in Preparation for the AP Calculus (BC) Examination,8th Edition, will be discussed in preparation for the Advanced Placement examination. (*Evaluation)* **Assessment Model:** Assessment questions should be open-ended and should follow the general format illustrated in the Essential Questions/Sample Conceptual Understanding section. *(Analysis)*

Suggested days of Instruction	Curriculum Management System Subject/Grade Level: **Course Name/Grade** **AP Calculus BC/12**	**Big Idea: Change** **Topic: Limits and Rates of Change** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Major Goal 1: The student will be able to understand and apply the concepts of limits and rates of change.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**

	1.11. find the average rate of change of a function. (CPI 4.3.12.B.2)*	Example: Find the slope the parabola $y = x^2$ at the point $P(2,4)$. Write an equation for the tangent to the parabola at this point.
		Write an expression for the slope of the secant line of P and a nearby point $Q(2+x,(2+x)^2)$, when $x = 2$. Write an equation for the secant line through the parabola through these two points.
		
		Example: Write an equation for the normal to the curve $f(x) = x^3 - 3$ at $x = 2$.
		
		Enduring Understandings
		• **Limits are used to describe continuity and rates of change.**
		• **End behavior models are analyzed to describe a functions limit as x approaches positive or negative infinity.**
		• **Average rate of change is the amount of change over a period of time while instantaneous rate of change is the rate of change at any time t.**
			Resources:
			• Calculus: Graphical, Numerical, Algebraic Finney, Ross L.; Demana. Franklin D.; Waits, Bert K.; Kennedy, Daniel; Pearson Education, Inc., 2007. Teachers Resources;
			  • Teacher's Guide with Answers.
			  • Teacher's AP Correlations and Preparation Guide
			  • Student Practice Workbook
			• Multiple-Choice & Free-Response Questions in Preparation for the AP Calculus (BC) Examination,8th Edition, Lederman, David, D & S Marketing Systems, Inc, 2003.
			• How to Prepare for the AP Calculus Advanced Placement Examination, 7th Edition, Barron's Educational Series, Inc, 2002.
			• Teaching AP Calculus, Lin McMulllin, D & S Marketing Systems, Inc, 2005.
			• Calculus Explorations, Foerster, Paul A., Key Curriculum Press, 1998.
			• Journey Through Calculus software, Bill, Ralph; Stewart, James.
			• *Calculus In Motion*™ software, Audrey Weeks

<table>
<tr><td rowspan="5">Suggested days of Instruction</td><td rowspan="4">Curriculum Management System
<u>Subject/Grade Level</u>:
Course Name/Grade
AP Calculus BC/12</td><td colspan="2">Big Idea: Change</td></tr>
<tr><td colspan="2">Topic: Limits and Rates of Change</td></tr>
<tr><td colspan="2"><u>Overarching Goals:</u>
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(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.
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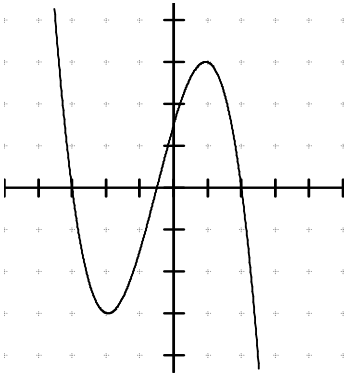
<tr><td colspan="2"><u>Major Goal 1:</u> The student will be able to understand and apply the concepts of limits and rates of change.</td></tr>
<tr><td>Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's)
The student will be able to:</td><td>Essential Questions
Sample Conceptual Understandings</td><td>Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model</td></tr>
<tr><td></td><td></td><td><u>Instructional tools</u>:
• TI-84 plus graphing calculator
• Winplot free graphing tool: www.winplot.com</td></tr>
</table>

Suggested days of Instruction	Curriculum Management System **Subject/Grade Level:** **Course Name/Grade** **AP Calculus BC/12**	**Big Idea: Change** **Topic: Derivatives** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Major Goal 2: The student will be able to find and graph derivatives of a function.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
2	2.1 calculate slopes and derivatives using the definition of a derivative. 2.2 graph a function from the graph of a derivative and graph a derivative from the graph of a function. (CPI 4.3.12.B.2, 4.5.K12.F.1, 4.5.K12.F.3)* 2.3 graph the derivative of a function given numerically with data. (CPI 4.3.12.B.2, 4.5.K12.F.4)* 2.4 find where a function is not differentiable and distinguish between corners, cusps, discontinuities, vertical and horizontal tangents. (CPI 4.3.12.B.2, 4.5.K12.F.3)*	• What is a derivative? • Define a derivative using the formal and the alternate definitions using limits. • What is the connection between derivatives and the graph of a function? Example: Given the graph of the function f below, sketch a graph of f'. 	**NOTE: The assessment models provided in this document are suggestions for the teacher. If the teacher chooses to develop his/her own model, *it must be of equal or better quality and at the same or higher cognitive levels (as noted in parentheses*).** **Depending upon the needs of the class, the assessment questions may be answered in the form of essays, quizzes, mobiles, PowerPoint, oral reports, booklets, or other formats of measurement used by the teacher.** **Learning Activities:** • Explore local linearity using a graphing calculator to "zoom in" to see if the following functions are differentiable at $x = 0$. a) $f(x) = \lvert x \rvert + 1$ b) $g(x) = \sqrt{x^2 + 0.0001} + 0.99$ *(Analysis)*

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	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
	2.5 Analyze differentiable functions using a graphing calculator to explore local linearity.] 2.6 approximate derivatives numerically and graphically. (CPI 4.3.12.B.2, 4.5.K12.F.3)* 2.7 use the rules of differentiation to calculate derivatives, including second and higher order derivatives. 2.8 use derivatives to analyze straight-line motion and solve other problems involving rates of change. 2.9 use the rules of differentiating the six basic trigonometric functions. 2.10 differentiate composite functions using the Chain Rule.	• What is a one-sided derivative? • Can a one-sided derivative differ at a point? • How might $f'(a)$ fail to exist? • How can local linearity be used to confirm differentiability of a function? • Does differentiability imply continuity? Example: True or False If a function is continuous then it is differentiable. If a function is differentiable then it is continuous. Example: Find $\frac{dy}{dx}$ if $y=(4x^2+3)(2x+1)$ comparing the use distribution rule versus the product rule. Example: Find $\frac{dy}{dx}$ if $y=\frac{2x^2+7x-2}{x^2}$ comparing simplifying using negative exponents versus the quotient rule.	• Group activity: Students will be given two sets of cards containing functions of one colored set and graphs of their derivatives on the other. In small groups, students will use their knowledge of functions and derivatives to match a function's graph with the graph of it's derivative. *(Synthesis)* • Quick assessment: (Concepts worksheet 3.1-3.3) Students will be able to make conclusions about the differentiability of functions based on information taken from tables and graphs. (*Evaluation)* • Quick assessment: (Concepts worksheet 3.4) Students will work together to describe a particle's motion using position, velocity and acceleration graphs. Students will also be able to make graphs describing a particle's motion using position, velocity and acceleration functions. *(Synthesis)*

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	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
	2.11 find derivatives using implicit differentiation. 2.12 find the derivatives using the Power Rule for rational powers of x. 2.13 calculate derivatives of functions involving the inverse trigonometric functions. 2.14 calculate derivatives of exponential and logarithmic functions.	• How are derivatives used to describe instantaneous rates of change? Example: Find the rate of change of the area of a circle with respect to its radius. Evaluate the rate of change at $r = 3$ and $r = 9$. • How can derivatives be used to describe velocity, speed and acceleration? Example: Discuss the difference between displacement, average velocity and instantaneous velocity. • How can derivatives be used to find marginal costs and marginal revenue? • How can derivatives of trigonometric functions be used to describe simple harmonic motion? • How is the Chain Rule used to compute the derivative of a composite function?	• Explore the motion of a particle moving along a horizontal line $y = 2$ whose position is given by $x(t) = 4t^3 - 16t^2 + 15t$ for $t \geq 0$ *(Analysis)* • Explore finding a derivative on an inverse graph geometrically when $f(x) = x^5 + 2x - 1$. *(Analysis)* • Quick assessment: (Concepts worksheet 3.8) Students will be able to identify inverse functions and use their knowledge to draw conclusions about the inverse function and its derivative. *(Comprehension)* • Explore the rate at which milk warms when taken from a refrigerator and left on the counter on a warm summer day. *(Analysis)*

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	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
		Example: Find $\frac{dy}{dx}$ if $y = \cos(3x^2 - 5)$ Example: Find $\frac{dy}{dx}$ parametrically if $x = \sec t$ and $y = \tan t$ where $-\frac{\pi}{2} < t < \frac{\pi}{2}$ when $t = \frac{\pi}{4}$. Find the line tangent to this function at the point $(\sqrt{2}, 1)$. • When is implicit differentiation necessary? Example: Find $\frac{dy}{dx}$ if $2y = x^2 + \sin y$ Example: Use the rules of differentiation to find $\frac{dy}{dx}$ if: $y = \sec^{-1}(6x^3)$ $y = x^x$ if $x > 0$	• Appropriate practice questions from Multiple-Choice & Free-Response Questions in Preparation for the AP Calculus (BC) Examination,8th Edition, will be discussed in preparation for the Advanced Placement examination. (*Evaluation)* **Assessment Model:** Assessment questions should be open-ended and should follow the general format illustrated in the Essential Questions/Sample Conceptual Understanding section. *(Analysis)*

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		Example: The spread of flu in a certain school is modeled by the equation $P(t)=\frac{500}{1+e^{6-t}}$ where t is the total number of students infected t days after the flu first started to spread. Estimate the initial number of students infected with the flu. How fast is the flu spreading after 6 days? When will the flu spread at its maximum rate? What is this rate?
		
		Enduring Understandings
		• **Derivatives are understood to be the instantaneous rate of change of a function at any point.**
		• **The relationship between a function and the graph of its first derivative are understood graphically as the instantaneous slope of the function.**
		• **The concept of analytical derivatives of functions is applied to describe velocity, acceleration, periodic motion, and exponential growth and decay.**
			Resources:
			• Calculus: Graphical, Numerical, Algebraic Finney, Ross L.; Demana. Franklin D.; Waits, Bert K.; Kennedy, Daniel; Pearson Education, Inc., 2007. Teachers Resources;
			  ◦ Teacher's Guide with Answers.
			  ◦ Teacher's AP Correlations and Preparation Guide
			  ◦ Student Practice Workbook
			• Multiple-Choice & Free-Response Questions in Preparation for the AP Calculus (BC) Examination,8th Edition, Lederman, David, D & S Marketing Systems, Inc, 2003.
			• How to Prepare for the AP Calculus Advanced Placement Examination, 7th Edition, Barron's Educational Series, Inc, 2002.
			• Teaching AP Calculus, Lin McMulllin, D & S Marketing Systems, Inc, 2005.
			• Calculus Explorations, Foerster, Paul A., Key Curriculum Press, 1998.
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<tr><td rowspan="5">Suggested days of Instruction</td><td rowspan="4">Curriculum Management System
<u>Subject/Grade Level</u>:
Course Name/Grade
AP Calculus BC/12</td><td colspan="2">Big Idea: Change</td></tr>
<tr><td colspan="2">Topic: Derivatives</td></tr>
<tr><td colspan="2"><u>Overarching Goals:</u>
(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.
(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.
(3) Investigate, research, and synthesize various information from a variety of media sources.</td></tr>
<tr><td colspan="2"><u>Major Goal 2:</u> The student will be able to find and graph derivatives of a function.</td></tr>
<tr><td>Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's)
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Sample Conceptual Understandings</td><td>Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model</td></tr>
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Suggested days of Instruction	Curriculum Management System Subject/Grade Level: Course Name/Grade AP Calculus BC/12	Big Idea: Derivatives Topic: Applications of Derivatives Overarching Goals: (1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes. (2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems. (3) Investigate, research, and synthesize various information from a variety of media sources. Major Goal 3: The student will be able to apply derivatives in modeling real-life situations.	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
7	3.1. determine the local or global extreme values of a function. (CPI 4.5.K12.F.3, 4.3.12.B.2)* 3.2. apply the Mean Value Theorem and find the intervals on which a function is increasing or decreasing. (CPI 4.3.12.B.4)* 3.3. use the First and Second Derivative Tests to determine the local extreme values of a function. (CPI 4.5.K12.F.3)* 3.4. determine the concavity of a function and locate the points of inflection by analyzing the second derivative. 3.5. graph a function using information about its derivative. (CPI 4.5.K12.F.3)*	• What is the difference between absolute extreme values and local extreme values? • Does the Extreme Value Theorem occur on an open or closed interval? • How are derivatives used to identify possible extreme values? Example: Determine the absolute extreme values of the function $y = x^2$ on the interval $[-1, 3]$. Example: Find the extreme values of $f(x) = \begin{cases} 5 - 2x^2, & x \le 1 \\ x + 2, & x > 1 \end{cases}$ Use graphical and analytical methods. • What is the Mean Value Theorem for Derivatives? Example: Explain how the Mean Value Theorem can be used to issue a speeding ticket using data from Easy Pass on the New Jersey Turnpike.	**NOTE: The assessment models provided in this document are suggestions for the teacher. If the teacher chooses to develop his/her own model, *it must be of equal or better quality and at the same or higher cognitive levels (as noted in parentheses*).** **Depending upon the needs of the class, the assessment questions may be answered in the form of essays, quizzes, mobiles, PowerPoint, oral reports, booklets, or other formats of measurement used by the teacher.** **Learning Activities:** • The teacher and students will explore extreme values and the derivative of a function at its extreme values using the graphing calculator. *(Analysis)* • Quick Assessment: (concepts worksheet 4.1) Students will use derivatives to identify extreme values and points of inflection, as well as intervals on which a function is increasing, decreasing, concave up or concave down. *(Comprehension)*

Suggested days of Instruction	Curriculum Management System **Subject/Grade Level:** **Course Name/Grade** **AP Calculus BC/12**	**Big Idea: Derivatives** **Topic: Applications of Derivatives** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Major Goal 3: The student will be able to apply derivatives in modeling real-life situations.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
	3.6. solve application problems involving minimum or maximum values of a function. (CPI 4.5.K12.F.4)* 3.7. find a linearization of a function. 3.8. estimate the change in a function using differentials. 3.9. solve related rate problems. (CPI 4.5.K12.F.4)*	• How are derivatives used to determine if a function is increasing or decreasing? • What is an inflection point and how are derivatives used to identify possible inflection points. Example: What does $f'(x)=0$ at every point on a given interval tell about the function? What does $f''(x)=0$ at every point on a given interval tell about the function? • How are different functions with the same derivative related? Example: Given $f(x)=x^3-\frac{27}{2}x^2+3$. Find the following: a)local extrema b) intervals on which the function is increasing and or decreasing c) inflection points d) intervals on which the function is concave up or down.	• Explore local extrema by analyzing derivative data from a table. *(Analysis)* • Quick Assessment: (concepts worksheet 4.2) Students will use graphs and tables to demonstrate their knowledge of the Mean Value Theorem. *(Synthesis)* • Quick Assessment: (concept worksheet 4.3) Students will use information about first and second derivatives to sketch the graph of a function. *(Synthesis)* • Group Activity: Students will work together to minimize fuel consumption and optimize fuel efficiency of an automobile given C(v) , a function for the rate of fuel consumption with respect to velocity. *(Synthesis)*

Suggested days of Instruction	Curriculum Management System Subject/Grade Level: **Course Name/Grade** **AP Calculus BC/12**	**Big Idea: Derivatives** **Topic: Applications of Derivatives** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Major Goal 3: The student will be able to apply derivatives in modeling real-life situations.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
		Example: A particle is moving along the x-axis with position $x(t) = t^3 - 12t + 2$. Find the velocity and acceleration of the particle and describe the motion for $t \geq 0$. Example: Sketch a possible smooth curve $y = f(x)$ through the origin with the following properties: $f'(x) < 0 \quad for \quad x \leq 2$ $f'(x) > 0 \quad for \quad x \geq 2$ $f''(x) < 0 \quad for \quad x < -1$ $f''(x) > 0 \quad for \quad x > -1$ • Describe the modeling process used to solve an optimization problem. Example: An open-top box is to be made by cutting congruent squares of side length x from the corner of an 18 by 24 inch sheet of tin by bending up the sides. How large should the squares be to make the box hold as much as possible? What is the resulting maximum volume of the box? (A graphing calculator will be necessary to complete this problem.)	• Group Project: Students will be given materials and instructed to create an open rectangular box with the largest possible volume. Students will submit all calculations to support their model. *(Synthesis)* • Explore Newton's Method for approximating a solution to the equation $f(x) = 0$ using a graphing calculator. *(Analysis)* • Appropriate practice questions from Multiple-Choice & Free-Response Questions in Preparation for the AP Calculus (BC) Examination,8th Edition, will be discussed in preparation for the Advanced Placement examination. (*Evaluation)* **Assessment Model**: Assessment questions should be open-ended and should follow the general format illustrated in the Essential Questions/Sample Conceptual Understanding section. *(Analysis)*

Suggested days of Instruction	Curriculum Management System Subject/Grade Level: **Course Name/Grade** **AP Calculus BC/12**	**Big Idea: Derivatives** **Topic: Applications of Derivatives** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Major Goal 3: The student will be able to apply derivatives in modeling real-life situations.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**

		Example: Suppose $r(x) = 3x$ and $c(x) = x^3 - 9x^2 + 18x$, where x represents thousands of units. Is there a production level that maximizes profit? If so, what is it?
		• Define linearization.
		• What is the importance of Newton's Method for approximating a solution of $f(x) = 0$?
		• Describe the strategy used to solve a related rate problem.
		• Example: Water runs into a conical tank at the rate of 12 $ft^3/\min$. The tank stands point down and has a height of 12 ft and a base radius of 9 ft. How fast is the water level rising when the water is 8 ft deep?
		Enduring Understandings
		• **The connection of a function's first and second derivative with the original function is conceptualized analytically through the first and second derivative tests, and graphically through the use of a graphing calculator.**
			Resources:
			• Calculus: Graphical, Numerical, Algebraic Finney, Ross L.; Demana. Franklin D.; Waits, Bert K.; Kennedy, Daniel; Pearson Education, Inc., 2007. Teachers Resources;
			  • Teacher's Guide with Answers.
			  • Teacher's AP Correlations and Preparation Guide
			  • Student Practice Workbook
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			• Journey Through Calculus software, Bill, Ralph; Stewart, James.
			• *Calculus In Motion*™ software, Audrey Weeks

Suggested days of Instruction	Curriculum Management System Subject/Grade Level: Course Name/Grade AP Calculus BC/12	**Big Idea:** Derivatives Topic: Applications of Derivatives Overarching Goals: (1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes. (2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems. (3) Investigate, research, and synthesize various information from a variety of media sources. Major Goal 3: The student will be able to apply derivatives in modeling real-life situations.	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) The student will be able to:	Essential Questions Sample Conceptual Understandings	Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model
		• Modeling of optimization applications can be conceptualized graphically and analytically with the use of derivatives. • The concept of related rates is understood and solved using the derivative of a function whose variables change with respect to time.	Instructional tools: • TI-84 plus graphing calculator • Winplot free graphing tool: www.winplot.com

Suggested days of Instruction	Curriculum Management System Subject/Grade Level: **Course Name/Grade** **AP Calculus BC/12**	**Big Idea: Integrals** **Topic: Definite and Indefinite Integrals** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Major Goal 4: The student will be able to develop, understand and apply indefinite and definite integrals.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
6	4.1. approximate the area under the graph of a nonnegative continuous function by using the rectangular approximation methods. CPI 4.5.K12.F.1)* 4.2. interpret the area under a graph as a net accumulation of a rate of change.* 4.3. express the area under a curve as a definite integral and as a limit of Reimann sums.* 4.4. compute the area under a curve using a numerical integration procedure. (CPI 4.5.K12.F.1)* 4.5. apply the rules for definite integrals and find the average value of a function over a closed interval.	• When a curve's concavity is in an upward direction, is a left rectangular approximation method(LRAM) an over estimate or an under estimate? • How can the approximation of the area under a curve, using rectangular approximation method (RAM), be made more accurate? • How can the rectangular approximation method be used to estimate the volume of a sphere or a cone? • How is a Riemann Sum different from a RAM? • How are Riemann sums used to define integrals? • Write the definite integral of a continuous function on $[a,b]$ using limits and again using integral notation. • What is the difference between the definite integral of a function and the total area between a curve and the x-axis on a closed interval? • Define the average value of a function over a closed interval. Example: Find the average value of $f(x)=6-x^2$ on $[1,7]$. Does f actually take on this value at some point in the given interval?	**NOTE: The assessment models provided in this document are suggestions for the teacher. If the teacher chooses to develop his/her own model, *it must be of equal or better quality and at the same or higher cognitive levels (as noted in parentheses*).** **Depending upon the needs of the class, the assessment questions may be answered in the form of essays, quizzes, mobiles, PowerPoint, oral reports, booklets, or other formats of measurement used by the teacher.** **Learning Activities:** • Explore the area of a circle by computing the limit of the area of an inscribed regular polygon as $n \to \infty$. Explain how this computation parallels the computation used in the rectangular approximation method for estimating the area under a curve. *(Analysis)*

Suggested days of Instruction	Curriculum Management System Subject/Grade Level: **Course Name/Grade** **AP Calculus BC/12**	**Big Idea: Integrals** **Topic: Definite and Indefinite Integrals** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Major Goal 4: The student will be able to develop, understand and apply indefinite and definite integrals.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
	4.6. apply the Fundamental Theorem of Calculus. 4.7. understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus. 4.8. approximate the definite integral by using the Trapezoidal Rule and Simpson's Rule. 4.9. to use Error Bounds to analyze approximations given by the Trapezoidal Rule and Simpson's Rule.	• State The Fundamental Theorem of Calculus in your own words. Example: Use the Fundamental Theorem of Calculus to evaluate $\frac{dy}{dx}$ if $y = \int_{3x}^{x^2} \frac{3}{1+e^t}\,dt$. Example: After proving the Fundamental Theorem of Calculus part 2, evaluate the integral $\int_0^5 (9-x^2)\,dx$ analytically, numerically and graphically. Example: Sketch the graph of $y = \sqrt{16-x^2}+4$ then find the integral on $[-4,4]$ using geometry. • Is the Trapezoidal Method more or less accurate than a Riemann Sum when estimating the integral of a function? Explain.	• Explore the use of the graphing calculator to graph the function $F(x) = \int_3^x \tan t\,dt + 5$. *(Synthesis)* • Explore the effect of changing a in $\int_a^x f(t)\,dt$. *(Analysis)* • Quick Assessment: (concepts worksheet 5.3-5.4) Students will sketch the graph of a function given the graph of its derivative. *(Synthesis)* • Group activity: Students will be given two sets of cards containing functions of one colored set and graphs of their derivatives on the other. In small groups, students will use their knowledge of functions and derivatives to match a function's graph with the graph of it's derivative. *(Synthesis)*

Suggested days of Instruction	Curriculum Management System Subject/Grade Level: **Course Name/Grade** **AP Calculus BC/12**	**Big Idea: Integrals** **Topic: Definite and Indefinite Integrals** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Major Goal 4: The student will be able to develop, understand and apply indefinite and definite integrals.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
		Example: The table below records the velocity of a sled at 1-second intervals for the first eight seconds of its run. Use the Trapezoidal Rule to approximate the distance the bobsled travels during the 8-second interval. (note the change in units) Time (seconds) Speed (miles / hour) 0 0 1 4 2 9 3 15 4 23 5 31 6 39 7 45 8 50 **Enduring Understandings** • **Integrals are used to describe instantaneous changes accumulate over a period of time.**	• Quick Assessment: (concepts worksheet 5.4) Students will test their understanding of Parts 1 and 2 of The Fundamental Theorem of Calculus. *(Synthesis)* • Quick Assessment: (concepts worksheet 5.5) Students will compare the Trapezoidal Rule and Simpson's Rule for evaluating integrals. *(Analysis)* • Group Activity: Students will derive the formula for the volume of a right circular cone using a Riemann Sum of the volumes of cylindrical disks. *(Synthesis)* • Appropriate practice questions from Multiple-Choice & Free-Response Questions in Preparation for the AP Calculus (BC) Examination,8th Edition, will be discussed in preparation for the Advanced Placement examination. (*Evaluation)*

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	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**

		• **The Fundamental Theorem of Calculus shows a clear connection between differential calculus and integral calculus. When problems are analyzed graphically, numerically and analytically, a visualization of each application can be analyzed for greater understanding.**
		• **When shown graphically, integrals which are used to find areas are understood to be the absolute value of the integral.**
		• **The average of the Right Rectangular Approximation Method and the Left Rectangular Approximation Method is the same as the Trapezoidal Rule. Error Analysis allows for an understanding of the accuracy of the formulas.**
			Assessment Model:
			Assessment questions should be open-ended and should follow the general format illustrated in the Essential Questions/Sample Conceptual Understanding section.
			(Analysis)
			
			Resources:
			• Calculus: Graphical, Numerical, Algebraic Finney, Ross L.; Demana. Franklin D.; Waits, Bert K.; Kennedy, Daniel; Pearson Education, Inc., 2007. Teachers Resources;
			  ◦ Teacher's Guide with Answers.
			  ◦ Teacher's AP Correlations and Preparation Guide
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			• Multiple-Choice & Free-Response Questions in Preparation for the AP Calculus (BC) Examination,8th Edition, Lederman, David, D & S Marketing Systems, Inc, 2003.
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<u>Instructional tools</u>:
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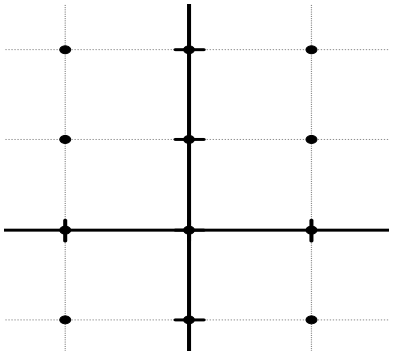
Suggested days of Instruction	Curriculum Management System Subject/Grade Level: **Course Name/Grade** **AP Calculus BC/12**	**Big Idea: Integrals** **Topic: Differential Equations** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Major Goal 5: The student will be able to solve problems using differential equations and mathematical modeling.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
10	5.1. construct antiderivatives using the Fundamental Theorem of Calculus. 5.2. find antiderivatives of polynomials, e^{kx} and selected trigonometric functions of kx, as well as linear combinations of these functions. 5.3. solve initial value problems of the form $\frac{dy}{dx} = f(x), y_0 = f(x_0)$.. 5.4. construct slope fields using technology and interpret slope fields as visualizations of differential equations. 5.5. compute definite and indefinite integrals by the method of substitution.	• What is a differential equation? Example: Find the particular solution to the equation $\frac{dy}{dx} = e^x - 8x^3$ through the point $(1, 2)$. • What is a slope field? Example: Construct a slope field for the differential equation $\frac{dy}{dx} = x - 3y$ using the given set of points. 	**NOTE: The assessment models provided in this document are suggestions for the teacher. If the teacher chooses to develop his/her own model, *it must be of equal or better quality and at the same or higher cognitive levels (as noted in parentheses*).** **Depending upon the needs of the class, the assessment questions may be answered in the form of essays, quizzes, mobiles, PowerPoint, oral reports, booklets, or other formats of measurement used by the teacher.** **Learning Activities:** • Group activity: Students will create a slope field for a given differential equation and plot a possible solution to the equation based on a given initial value. Students will then solve the differential equation analytically and graph it on a graphing calculator to confirm the sketch on the slope field. *(Synthesis)* • Explore different approaches to the substitution method for evaluating a definite integral. *(Analysis)*

Suggested days of Instruction	Curriculum Management System Subject/Grade Level: **Course Name/Grade** **AP Calculus BC/12**	**Big Idea: Integrals** **Topic: Differential Equations** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Major Goal 5: The student will be able to solve problems using differential equations and mathematical modeling.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
	5.6. solve differential equations of the form $\frac{dy}{dx} = f(x)$, in which the variables are separable. 5.7. using integration by parts to evaluate indefinite and definite integrals. 5.8. use tabular integration to evaluate integrals that require repeated use of integration by parts. 5.9. solve problems involving exponential growth and decay in a variety of applications. (CPI 4.3.12.C.1) * 5.10. antidifferentiate using partial fraction decompositions. 5.11. solve problems involving population using logistic differential equations. (CPI 4.3.12C.1)*	• How can slope fields be used to illustrate the solution to a differential equation? • Describe how Euler's Method is used to approximate a value of a given function. **example 9 student workbook** • What is the difference between a definite integral and an indefinite integral? • When is it necessary to use the substitution method to evaluate an integral? Example: Evaluate $\int_0^1 \frac{dx}{3(1+x)^2}$ $\int_0^1 \frac{dx}{3x^2}$ • When is it necessary to evaluate an integral using integration by parts?	• Show your knowledge of The Law of Exponential Change by making a table identifying the differential equation and initial values for each of the formulas used in the exponential growth and decay problems. *(Knowledge)* • Appropriate practice questions from Multiple-Choice & Free-Response Questions in Preparation for the AP Calculus (BC) Examination,8th Edition, will be discussed in preparation for the Advanced Placement examination. (*Evaluation)* **Assessment Model:** Assessment questions should be open-ended and should follow the general format illustrated in the Essential Questions/Sample Conceptual Understanding section. *(Analysis)*

Suggested days of Instruction	Curriculum Management System Subject/Grade Level: **Course Name/Grade** **AP Calculus BC/12**	**Big Idea: Integrals** **Topic: Differential Equations** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Major Goal 5: The student will be able to solve problems using differential equations and mathematical modeling.**	
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		Example: Evaluate $\int e^x \cos x \, dx$ (solve for the unknown integral)
		• When is tabular integration an option?
		Example: Evaluate $\int x^2 \sin 4x \, dx$
		• When is it necessary to use separation of variables to solve a differential equation?
		Example: Solve for y if $\frac{dy}{dx} = 6y^2x$ and $y = \frac{1}{25}$ when $x = 1$.
		• How can differential equations be used to show how a direct variation can lead to exponential change?
		Example: Show that differential equation $\frac{dy}{dt} = ky$ with initial condition $y = y_0$ when $t = 0$ yields $y = y_0 e^{kt}$.
			Resources:
			• Calculus: Graphical, Numerical, Algebraic Finney, Ross L.; Demana. Franklin D.; Waits, Bert K.; Kennedy, Daniel; Pearson Education, Inc., 2007. Teachers Resources;
			  • Teacher's Guide with Answers.
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Course Name/Grade
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<tr><td colspan="2">Topic: Differential Equations</td></tr>
<tr><td colspan="2"><u>Overarching Goals:</u>
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(3) Investigate, research, and synthesize various information from a variety of media sources.</td></tr>
<tr><td colspan="2"><u>Major Goal 5:</u> The student will be able to solve problems using differential equations and mathematical modeling.</td></tr>
<tr><td>Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's)
The student will be able to:</td><td>Essential Questions
Sample Conceptual Understandings</td><td>Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model</td></tr>
<tr><td></td><td>Enduring Understandings

• Slope fields and their family of functions show the visual representation to the connection of a function and its derivatives.
• The connection between a function and its antiderivative can be easily conceptualized using techniques such as substitution and integration by parts when solving complicated integrals.
• Differential equations and their family of functions can be applied to exponential growth and decay along with logistic growth in population problems to find out the limitations of each problem.</td><td><u>Instructional tools</u>:
• TI-84 plus graphing calculator
• Winplot free graphing tool: www.winplot.com</td></tr>
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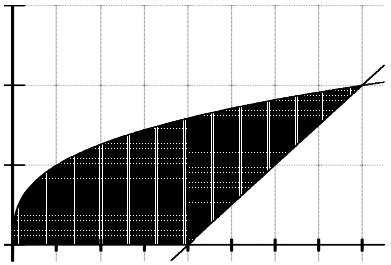
Suggested days of Instruction	Curriculum Management System Subject/Grade Level: **Course Name/Grade** **AP Calculus BC/12**	**Big Idea: Mathematical Modeling** **Topic: Real-Life Applications** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Major Goal 6: The student will be able to apply definite integrals to model real-life situations mathematically.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
9	6.1. use integration to calculate displacement, position, total distance traveled, velocity and acceleration. 6.2. use integration to calculate areas of regions in a plane. (CPI 4.5.K12.F.1)* 6.3. use integration (by the slice method) to calculate volumes of solids. (CPI 4.5.K12.F.1)* 6.4. use integration to calculate surface areas of solids of revolution. (CPI 4.5.K12.F.1)* 6.5. use integration to calculate lengths of curves in a plane. (CPI 4.5.K12.F.1)* 6.6. solve problems involving work, fluid force and fluid pressure. (CPI 4.3.12.C.1)*	• What is the difference between displacement and total distance traveled? • Explain how to use integrals to find displacement and total distance traveled. Example: The function $v(t) = 4\cos(2t)$ for $0 \le t \le \frac{\pi}{2}$ is the velocity, in m/sec, of a particle moving along the x-axis with $s(0) = 8$. a) Determine when the particle is moving to the right, to the left, and when it is stopped. b) Find the particle's displacement and its final position, for the given interval. Find the total distance traveled by the particle. • How are integrals used to compute the area between two curves? • When is it helpful to integrate with respect to the y-axis as opposed to the x-axis?	**NOTE: The assessment models provided in this document are suggestions for the teacher. If the teacher chooses to develop his/her own model, *it must be of equal or better quality and at the same or higher cognitive levels (as noted in parentheses*).** **Depending upon the needs of the class, the assessment questions may be answered in the form of essays, quizzes, mobiles, PowerPoint, oral reports, booklets, or other formats of measurement used by the teacher.** **Learning Activities:** • Quick Assessment: (Concepts worksheet 7.1-7.5) Students will use integrals to determine the net change in a quantity over a period of time. (*Evaluation)* • Explore the area of an ellipse by finding the formulas for the semi-ellipses and integrating using a graphing calculator. *(Analysis)*

Suggested days of Instruction	Curriculum Management System **Subject/Grade Level:** **Course Name/Grade** **AP Calculus BC/12**	**Big Idea: Mathematical Modeling** **Topic: Real-Life Applications** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Major Goal 6: The student will be able to apply definite integrals to model real-life situations mathematically.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
	6.7. use integrals to find the normal probability of a function. (CPI 4.4.12.A.2)* 6.8. understand and apply the normal probability density function (Gaussian curve). (CPI 4.4.12.B.5)*	Example: The shaded region below is bounded by the x-axis and the functions $y = \sqrt[3]{x}$ and $y = \frac{1}{2}x - 2$. find the area of the shaded region by integrating with respect to the x-axis and the y-axis.  • When are washer cross sections used instead of circular cross sections to find the volume of a solid of revolution?	• Quick Assessment: (Concepts worksheet 7.2) Students will demonstrate ability to determine area between two curves with respect to the x or y-axis. (*Evaluation)* • Quick Assessment: (Concepts worksheet 7.3) students will demonstrate the ability to determine the volumes of solids of revolution. (*Evaluation)* • Group Activity: Students will derive the formula for the volume of a "cone" with irregular polygonal base using a Riemann Sum of volumes of thin slabs with thickness Δx. *(Synthesis)* • Appropriate practice questions from Multiple-Choice & Free-Response Questions in Preparation for the AP Calculus (BC) Examination,8th Edition, will be discussed in preparation for the Advanced Placement examination. (*Evaluation)*

Suggested days of Instruction	Curriculum Management System Subject/Grade Level: **Course Name/Grade** **AP Calculus BC/12**	**Big Idea: Mathematical Modeling** **Topic: Real-Life Applications** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Major Goal 6: The student will be able to apply definite integrals to model real-life situations mathematically.**	
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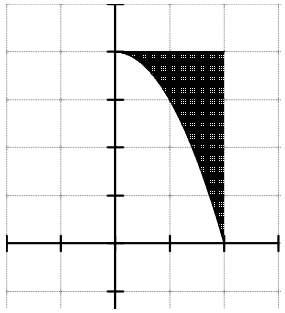
		Example: The shaded region below is bounded by the function $y = 4 - x^2$ and the lines $y = 4$ and $x = 2$.
		find the volume of the solid generated by revolving the region about the x-axis.
		 
		Example: Find the volume of the solid that lies between planes perpendicular to the x-axis at $x = -4$ and $x = 4$. The cross sections perpendicular to the x-axis between these planes are squares with one side running from the semi-circle $y = \sqrt{16 - x^2}$ and $y = -\sqrt{16 - x^2}$
			Assessment Model:
			Assessment questions should be open-ended and should follow the general format illustrated in the Essential Questions/Sample Conceptual Understanding section.
			(Analysis)
			
			Resources:
			• Calculus: Graphical, Numerical, Algebraic Finney, Ross L.; Demana. Franklin D.; Waits, Bert K.; Kennedy, Daniel; Pearson Education, Inc., 2007. Teachers Resources;
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	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
		Example: For the given curve, set up the integral for the length of the curve, graph the curve and find the length analytically and numerically. $y = -x^2 - 4x + 3$, where $-3 \le x \le 2$ **Enduring Understandings** • **Integrals can be used as a tool to calculate net change, area, and volume.** • **Applications involving accumulation in statistics and science can be solved with the use of integrals.**	• Teaching AP Calculus, Lin McMulllin, D & S Marketing Systems, Inc, 2005. • Calculus Explorations, Foerster, Paul A., Key Curriculum Press, 1998. • Journey Through Calculus software, Bill, Ralph; Stewart, James. • *Calculus In Motion*™ software, Audrey Weeks **Instructional tools:** • TI-84 plus graphing calculator • Winplot free graphing tool: www.winplot.com

Suggested days of Instruction	Curriculum Management System Subject/Grade Level: **Course Name/Grade** **AP Calculus BC/12**	**Big Idea: Improper Integrals** **Topic: Applications of Integrals, Convergence and Divergence** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Major Goal 7: The student will be able to find limits of sequences, compare rates of growth and solve improper integral applications.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
11	7.1. define and apply finite and infinite sequences as well as arithmetic and geometric sequences to application problems. (CPI 4.3.12.A.1)* 7.2. find the limit of a sequence. (CPI 4.3.12.A.2)* 7.3. Understand and apply the Absolute Value Theorem involving sequences. (CPI 4.3.12.A.1)* 7.4. find limits of indeterminate forms using L'Hopital's Rule. (CPI 4.3.12.A.2)* 7.5. compare rates of growth as $x \to \infty$. (CPI 4.3.12.A.1)* 7.6. understand the difference between a sequential search and a binary search.	• Compare and contrast an arithmetic sequence with a geometric sequence. Example: Determine whether each sequence is arithmetic or geometric; find an explicit rule and a recursive rule for the n th term of each. a) $-3, 9, -27, 81, \ldots$ b) $-9, -5, -1, 3, 7, \ldots$ • How can it be determined whether or not a sequence converges or diverges? Example: Determine whether $a_n = \frac{-5n+7}{-7n}$ converges or diverges. If it converges, find its limit. • How can limits be used to describe the behavior of a function? • Describe the indeterminate forms of l'Hopital's Rule and how they can be used to find limits.	**NOTE: The assessment models provided in this document are suggestions for the teacher. If the teacher chooses to develop his/her own model, *it must be of equal or better quality and at the same or higher cognitive levels (as noted in parentheses*).** **Depending upon the needs of the class, the assessment questions may be answered in the form of essays, quizzes, mobiles, PowerPoint, oral reports, booklets, or other formats of measurement used by the teacher.** **Learning Activities:** • Explore limits such as $\lim_{x\to\infty} \frac{\sqrt{9x+1}}{\sqrt{x+1}}$, where l'Hopital's Rule does not help find the limit. Justify your answer. Find the limit analytically and confirm graphically. *(Analysis)* • Explore the comparison of rates of growth as $x \to \infty$ with functions $f(x)=a^x$ if $a>1$, $f(x)=x^2$, and $f(x)=b^x$ if $a>b>1$. *(Analysis)*

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	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
	7.7. evaluate improper integrals. (CPI 4.5.K12.F.3)* 7.8. evaluate improper integrals with discontinuities. (CPI 4.5.K12.F.3)* 7.9. test for convergence and divergence of integrals.	Example: Evaluate $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$ using L'Hopital's Rule. • How does the rate of growth of an exponential function compare to that of a polynomial function and a logarithmic function? Example: Order the given functions from slowest-growing to fastest growing as $x \to \infty$ $x \to \infty$: $2^x, x^2, (\ln x)^x,$ and e^x Example: Find the points of discontinuity and then evaluate $\int_0^5 \frac{dx}{(x-3)^{2/3}}$. • Explain how testing for convergence and divergence can be done using the Comparison Test.	• Investigate divergence and convergence of $\int_0^1 \frac{dx}{x^p}$ when $p=1$, $p>1$, and $0<p<1$. *(Analysis)* • Explore the integrals of even and odd functions on the interval $(-\infty,\infty)$. *(Analysis)* • Appropriate practice questions from Multiple-Choice & Free-Response Questions in Preparation for the AP Calculus (BC) Examination,8th Edition, will be discussed in preparation for the Advanced Placement examination. (*Evaluation)* **Assessment Model:** Assessment questions should be open-ended and should follow the general format illustrated in the Essential Questions/Sample Conceptual Understanding section. *(Analysis)*

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		Enduring Understandings
		• **Limits can be used to describe the behavior of functions using l'Hopital's Rule.**
		• **Growth rates as x approaches infinity can be used to easily understand the behavior of functions.**
		• **Improper integrals can be described as convergent and evaluated or stated that they are divergent.**
			Resources:
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			  • Teacher's Guide with Answers.
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Suggested days of Instruction	Curriculum Management System Subject/Grade Level: **Course Name/Grade** **AP Calculus BC/12**	**Big Idea: Series** **Topic: Infinite Series** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Major Goal 8: The student will be able to solve problems involving infinite series.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
13	8.1. define geometric and infinite series. (CPI 4.3.12.A.1)* 8.2. determine whether or not a series converges or diverges.(CPI 4.3.12.A.3)* 8.3. find power series using differentiation and integration. (CPI 4.3.12.A.1)* 8.4. find the interval of convergence of a power series. 8.5. construct Taylor polynomials and Taylor series. Generated by f at $x=0$ and $x=a$. 8.6. use the Maclaurin series to construct other series. (CPI 4.5.K12.C.1)*	• How can an infinite series be identified as divergent or convergent? Example: Compute the limit of the partial sums to determine whether the series converges or diverges: $\frac{7}{10}+\frac{7}{100}+\frac{7}{1000}+...+\frac{7}{10^n}+...$ • Describe how the connection between a function and a series can be determined using differentiation and integration. Example: Given that $\frac{1}{1-x}=1+x+x^2+x^3+...+x^n+..., \ -1<x<1$ find a power series to represent $\ln(1-x)$. • Explain how the partial sums of a Taylor series can be used to approximate a function represented by a series.	**NOTE: The assessment models provided in this document are suggestions for the teacher. If the teacher chooses to develop his/her own model, *it must be of equal or better quality and at the same or higher cognitive levels (as noted in parentheses*).** **Depending upon the needs of the class, the assessment questions may be answered in the form of essays, quizzes, mobiles, PowerPoint, oral reports, booklets, or other formats of measurement used by the teacher.** **Learning Activities:** • Explore finding power series for other functions along with the interval of convergence given that $1/(1-x)=1+x+x^2+...+x^n+...$ on the interval $(-1,1)$. *(Analysis)* • Explore finding a power series for $\tan^{-1}x$ and determine if it converges at $x=1$. *(Analysis)*

Suggested days of Instruction	Curriculum Management System Subject/Grade Level: **Course Name/Grade** **AP Calculus BC/12**	**Big Idea: Series** **Topic: Infinite Series** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Major Goal 8: The student will be able to solve problems involving infinite series.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
	8.7. use Taylor's Theorem with Remainder to prove convergence of a Maclaurin series. (CPI 4.5.K12.F3)* 8.8. Understand and apply the Remainder Estimation Theorem. (CPI 4.5.K12.F3)* 8.9. Understand and apply Euler's formula. (CPI 4.5.K12.F3)* 8.10. determine convergence using the n th-Term Test, Direct Comparison Test, and Ratio Test. (CPI 4.5.K12.F3)* 8.11. solve applications of Harmonic series and p -series . (CPI 4.5.K12.F3)*	Example: Find the Taylor series generated by $f(x) = \ln x$ at the given point $a = 3$. • What is the connection between the linearization of f and Taylor polynomials? • Based on what you know about polynomial functions, explain why no Taylor polynomial of any order could actually equal $\sin x$? • Is it possible to find the interval of convergence of a geometric series by using the Ratio Test? Example: Find the radius of convergence using the Ratio Test of $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n}$. • Explain the strategy used when determining whether to use the n th-Term Test, Direct Comparison Test, or the Ratio Test when show that a series converges.	• Construct a polynomial at $x = 0$ with given behaviors for $P(0), P'(0), P''(0), P'''(0),$ and $P^4(0)$ as a discovery lesson for Taylor polynomials. *(Analysis)* • Explore finding a power series to represent $f(x) = (\sin x)/x$ using the table of Maclaurin series and determine why $f(x)$ cannot have a Maclaurin series. *(Synthesis)* • Group Activity: Within each group, have each student make up a power series with radius of convergence equal to one of the numbers $1, 2, ..., n$. Then exchange series with another group and match the other group's series with the correct radii of convergence. *(Synthesis)*

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	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
	8.12. determine convergence using the Limit Comparison Test and the Alternating Series Test. (CPI 4.5.K12.F3)* 8.13. determine absolute and conditional convergence. (CPI 4.5.K12.F3)* 8.14. test power series for intervals of convergence. (CPI 4.5.K12.F3)*	**Enduring Understandings** • **A strategy can be developed to determine which tests of convergence will ultimately determine if a series convergences to a function on its interval of convergence.**	• Explore the $p-$Series Test using the Integral Test to prove that $\sum_{n=1}^{\infty} 1/n^p$ converges if $p>1$ and diverges if $p<1$ or $p=1.$ *(Analysis)* • Group Activity: Construct a series that diverges more slowly than the harmonic series. Justify your answer and share with the class. *(Synthesis)* • Quick Assessment: (Concepts worksheet 9.1-9.5) Students will demonstrate the ability to evaluate and write a series expansion for Power Series using Taylor's Theorem. *(Evaluation)*

Suggested days of Instruction	Curriculum Management System Subject/Grade Level: **Course Name/Grade** **AP Calculus BC/12**	**Big Idea: Series** **Topic: Infinite Series** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Major Goal 8: The student will be able to solve problems involving infinite series.**	
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			• Appropriate practice questions from Multiple-Choice & Free-Response Questions in Preparation for the AP Calculus (BC) Examination,8th Edition, will be discussed in preparation for the Advanced Placement examination. (*Evaluation)* **Assessment Model:** Assessment questions should be open-ended and should follow the general format illustrated in the Essential Questions/Sample Conceptual Understanding section. *(Analysis)* **Resources:** • Calculus: Graphical, Numerical, Algebraic Finney, Ross L.; Demana. Franklin D.; Waits, Bert K.; Kennedy, Daniel; Pearson Education, Inc., 2007. Teachers Resources; ◦ Teacher's Guide with Answers. ◦ Teacher's AP Correlations and Preparation Guide ◦ Student Practice Workbook

Suggested days of Instruction	Curriculum Management System **Subject/Grade Level:** **Course Name/Grade** **AP Calculus BC/12**	**Big Idea: Series** **Topic: Infinite Series** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Major Goal 8: The student will be able to solve problems involving infinite series.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
			• Multiple-Choice & Free-Response Questions in Preparation for the AP Calculus (BC) Examination,8th Edition, Lederman, David, D & S Marketing Systems, Inc, 2003. • How to Prepare for the AP Calculus Advanced Placement Examination, 7th Edition, Barron's Educational Series, Inc, 2002. • Teaching AP Calculus, Lin McMulllin, D & S Marketing Systems, Inc, 2005. • Calculus Explorations, Foerster, Paul A., Key Curriculum Press, 1998. • Journey Through Calculus software, Bill, Ralph; Stewart, James. • *Calculus In Motion*™ software, Audrey Weeks **Instructional tools:** • TI-84 plus graphing calculator • Winplot free graphing tool: www.winplot.com

Suggested days of Instruction	Curriculum Management System Subject/Grade Level: **Course Name/Grade** **AP Calculus BC/12**	**Big Idea: Advanced Functions** **Topic: Parametric, Vector and Polar Functions** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Major Goal 9: The student will be able to solve applications of parametric, vector and polar functions.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
13	9.1. sketch parametric curves in a plane. (CPI 4.5.K12.F3)* 9.2. find the arc length of a parametrized curve. (CPI 4.5.K12.F3)* 9.3. Find the magnitude and direction of vectors. (CPI 4.5.K12.F3)* 9.4. find speed, acceleration and direction using applications of vectors. (CPI 4.5.K12.F3)* 9.5. find displacement and distance traveled using vectors. (CPI 4.5.K12.F3)* 9.6. find rectangular coordinates of a point given polar coordinates. (CPI 4.5.K12.F3)*	• Describe how an equation written in parametric form can be rewritten in an equation that relates x and y directly. • When a curve is defined parametrically, what is the procedure used to determine concavity? Example: Given a curve defined as $x = t^3 - 3$ and $y = -4\sin t$ for $0 \le t \le \pi$: b) Sketch a graph of the curve in the viewing window $[-5, 9]$ by $[-6, 2]$, indicating the direction in which it is traced. c) Find the lowest point on the curve. d) Find all points of inflection on the curve. Justify all work. • How are vectors more useful than just finding the slopes of a curve? • How can displacement and distance traveled be determined when a particle's vector velocity has been given? • Explain the usefulness of a polar equation when compared to an equation in rectangular form.	**NOTE: The assessment models provided in this document are suggestions for the teacher. If the teacher chooses to develop his/her own model, *it must be of equal or better quality and at the same or higher cognitive levels (as noted in parentheses*).** **Depending upon the needs of the class, the assessment questions may be answered in the form of essays, quizzes, mobiles, PowerPoint, oral reports, booklets, or other formats of measurement used by the teacher.** **Learning Activities** • Group Activity: Involute of a Circle: If a string wound around a fixed circle is unwound while being held taut in the plane of the circle, its end P traces an involute of the circle. The circle is the unit circle in the $xy-$ plane, and the initial position of the tracing point is the point $(1, 0)$ on the $x-$ axis. The unwound portion of the string is tangent to the circle at Q, and t is the radian measure of the angle from the positive $x-$ axis to the segment OQ.

Suggested days of Instruction	Curriculum Management System Subject/Grade Level: **Course Name/Grade** **AP Calculus BC/12**	**Big Idea: Advanced Functions** **Topic: Parametric, Vector and Polar Functions** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Major Goal 9: The student will be able to solve applications of parametric, vector and polar functions.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
	9.7. find polar coordinates of a point given rectangular coordinates. (CPI 4.5.K12.F3)* 9.8. graph a polar equation. (CPI 4.5.K12.F3)* 9.9. convert a polar equation to rectangular form. (CPI 4.5.K12.F3)* 9.10. find area of a region enclosed by polar curves. (CPI 4.5.K12.F3)* 9.11. review for Advanced Placement Examination and Actual Examination.	• Describe the procedure used to graph polar curves parametrically. • Explain the method used to convert polar equations to rectangular. Example: Given $r = 6\sin\theta$, replace the polar equation by an equivalent rectangular equation. Identify or describe the graph. • State the formula necessary to find the area between two polar curves. Example: Find the area of the region that lies inside the circle $r = 3$ and outside the cartioid $r = 3(1-\cos\theta)$. Sketch a graph and shade in the designated area. **Enduring Understandings** • **Polar equations make it possible to define curves that would be difficult to describe in rectangular form.**	Derive parametric equations fro the involute by expressing the coordinates x and y of P in terms of t for $t \geq 0$. Find the length of the involute for $0 \leq t \leq 2\pi$. *(Analysis)* • Quick Assessment: (Concepts Worksheet 10.2) Students will demonstrate the ability to relate vector functions to particle motion problems *(Evaluation)* • Quick Assessment: (Concepts Worksheet 10.3) Students will evaluate families of curves such as lines, circles, roses and lemniscates, using polar graphs. *(Evaluation)* • Explore graphing polar curves parametrically by graphing the equations: $x = \sin(6t)\cos t$ and setting an $y = \sin(6t)\sin t$ appropriate window. Graph a 12-petaled rose, a limacon and a circle as well. *(Analysis)*

Suggested days of Instruction	Curriculum Management System Subject/Grade Level: **Course Name/Grade** **AP Calculus BC/12**	**Big Idea: Advanced Functions** **Topic: Parametric, Vector and Polar Functions** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Major Goal 9: The student will be able to solve applications of parametric, vector and polar functions.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
		• **Vectors add direction to an object allowing the motion of objects to be modeled in a coordinate plane.** • **Non-linear motion in a plane can be understood and defined using parametric and polar forms.**	• Appropriate practice questions from Multiple-Choice & Free-Response Questions in Preparation for the AP Calculus (BC) Examination,8th Edition, will be discussed in preparation for the Advanced Placement examination. (*Evaluation)* **Assessment Model:** Assessment questions should be open-ended and should follow the general format illustrated in the Essential Questions/Sample Conceptual Understanding section. *(Analysis)* **Resources:** • Calculus: Graphical, Numerical, Algebraic Finney, Ross L.; Demana. Franklin D.; Waits, Bert K.; Kennedy, Daniel; Pearson Education, Inc., 2007. Teachers Resources; ◦ Teacher's Guide with Answers. ◦ Teacher's AP Correlations and Preparation Guide ◦ Student Practice Workbook

Suggested days of Instruction	Curriculum Management System Subject/Grade Level: **Course Name/Grade** **AP Calculus BC/12**	**Big Idea: Advanced Functions** **Topic: Parametric, Vector and Polar Functions** **Overarching Goals:** **(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.** **(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.** **(3) Investigate, research, and synthesize various information from a variety of media sources.** **Major Goal 9: The student will be able to solve applications of parametric, vector and polar functions.**	
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Suggested days of Instruction	Curriculum Management System Subject/Grade Level: Course Name/Grade	Big Idea: Multivariable Calculus Topic: Three-Dimensional Space; Vectors Overarching Goals: (1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes. (2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems. (3) Investigate, research, and synthesize various information from a variety of media sources. Major Goal 10: The student will be able to understand coordinates in three dimensions and study analytic geometry with respect to lines and planes.	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
17	10.1. understand coordinates in a three-dimensional rectangular system. 10.2. apply the three-dimensional rectangular system to cylindrical surfaces and spheres. 10.3. understand vectors in a coordinate system in two-dimensions and three-dimensions. 10.4. Review for final and final exam	• How is distance in three-dimensional space found? • Describe how to graph a sphere in three-dimensional space. • If an equation contains two of three variables, what does this represent in a three-dimensional space? Example: Use a graphing calculator to generate the curve $y = x^3/(1+x^2)$ in the xy-plane, and then use the graph to help sketch the surface $z = y^3/(1+y^2)$ in 3-dimensional space. • Explain the process of normalizing a vector. Example: Find the unit vector that has the same direction as $v = 2i + 2j - k$.	**NOTE: The assessment models provided in this document are suggestions for the teacher. If the teacher chooses to develop his/her own model, *it must be of equal or better quality and at the same or higher cognitive levels (as noted in parentheses*).** **Depending upon the needs of the class, the assessment questions may be answered in the form of essays, quizzes, mobiles, PowerPoint, oral reports, booklets, or other formats of measurement used by the teacher.** **Learning Activities** • Suppose that a box has its faces parallel to the coordinate planes and the points $(4,2,-2)$ and $(-6,1,1)$ are endpoints of a diagonal. Sketch the box and give the coordinates of the remaining six corners. *(Analysis)* • Interpret the graph of $x = 1$ in the contexts of a number line, 2-dimensional space, and 3-dimensional space. *(Analysis)*

<table>
<tr><td rowspan="5">Suggested days of Instruction</td><td rowspan="4">Curriculum Management System
<u>Subject/Grade Level</u>:
Course Name/Grade</td><td colspan="2">Big Idea: Multivariable Calculus</td></tr>
<tr><td colspan="2">Topic: Three-Dimensional Space; Vectors</td></tr>
<tr><td colspan="2"><u>Overarching Goals:</u>
(1) Communicate mathematical ideas in clear, concise, organized language that varies in content, format and form for different audiences and purposes.
(2) Comprehend, understand, analyze, evaluate, critique, solve, and respond to a variety of real-life, meaningful problems.
(3) Investigate, research, and synthesize various information from a variety of media sources.</td></tr>
<tr><td colspan="2"><u>Major Goal 10:</u> The student will be able to understand coordinates in three dimensions and study analytic geometry with respect to lines and planes.</td></tr>
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Sample Conceptual Understandings</td><td>Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model</td></tr>
<tr><td></td><td>Enduring Understandings
Three-Dimensional figures can be represented and described using the 8 octants.
Cylindrical surfaces can be obtained when graphing and equation that contains only two variables in a three-dimensional rectangular coordinate system.
Vectors have the ability to describe length and direction, allowing for physical quantities such as area, length, mass and temperature to be described. In addition, wind speed, direction, force and displacement can also be found with the use of vectors.</td><td><u>Assessment Model:</u>
Assessment questions should be open-ended and should follow the general format illustrated in the Essential Questions/Sample Conceptual Understanding section. (Analysis)

<u>Resources:</u>
<u>Calculus: Multivariable</u>, Anton, Howard; Bivens, Irl; Davis, Stephen: John Wiley & Sons, Inc, 2005.
<u>Teaching AP Calculus</u>, Lin McMulllin, D & S Marketing Systems, Inc, 2005.
<u>Calculus Explorations</u>, Foerster, Paul A., Key Curriculum Press, 1998.
<u>Journey Through Calculus</u> software, Bill, Ralph; Stewart, James.
Calculus In Motion™ software, Audrey Weeks
<u>Instructional tools</u>:
TI-84 plus graphing calculator
<u>Winplot</u> free graphing tool: www.winplot.com</td></tr>
</table>

AP Calculus BC/Grade 12

COURSE BENCHMARKS

1. The student will be able to understand and apply the concept of limits and rates of change.
2. The student will be able to find and graph derivatives of a function.
3. The student will be able to apply derivatives to real-life situations.
4. The student will be able to develop, understand and apply the indefinite and definite integral.
5. The student will be able to solve problems using differential equations and mathematical modeling.
6. The student will be able to apply definite integrals to model real-life situations mathematically.
7. The student will be able to find limits of sequences, compare rates of growth and solve improper integral applications.
8. The student will be able to solve problems involving infinite series.
9. The student will be able to solve applications of parametric, vector and polar functions.
10. The student will be able to understand coordinates in three dimensions and study analytic geometry with respect to lines and planes.