

# AP Calculus BC Free Response Questions 1998-2014

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## Example of a Polar, Vector, and Parametric Problem (w/calculator)

2008 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)

CALCULUS BC  
SECTION II, Part A  
Time—45 minutes  
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

1. A particle moving along a curve in the  $xy$ -plane has position  $(x(t), y(t))$  at time  $t \geq 0$  with

$$\frac{dx}{dt} = \sqrt{3t} \quad \text{and} \quad \frac{dy}{dt} = 3\cos\left(\frac{t^2}{2}\right).$$

The particle is at position  $(1, 5)$  at time  $t = 4$ .

- (a) Find the acceleration vector at time  $t = 4$ .
- (b) Find the  $y$ -coordinate of the position of the particle at time  $t = 0$ .
- (c) On the interval  $0 \leq t \leq 4$ , at what time does the speed of the particle first reach 3.5 ?
- (d) Find the total distance traveled by the particle over the time interval  $0 \leq t \leq 4$ .

## Example of a Sequence and Series (Taylor & McLaurin) Problem (w/calc)

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$x$	$h(x)$	$h'(x)$	$h''(x)$	$h'''(x)$	$h^{(4)}(x)$
1	11	30	42	99	18
2	80	128	$\frac{488}{3}$	$\frac{448}{3}$	$\frac{584}{9}$
3	317	$\frac{753}{2}$	$\frac{1383}{4}$	$\frac{3483}{16}$	$\frac{1125}{16}$

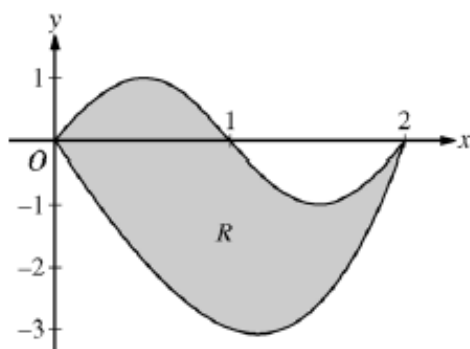
3. Let  $h$  be a function having derivatives of all orders for  $x > 0$ . Selected values of  $h$  and its first four derivatives are indicated in the table above. The function  $h$  and these four derivatives are increasing on the interval  $1 \leq x \leq 3$ .
- (a) Write the first-degree Taylor polynomial for  $h$  about  $x = 2$  and use it to approximate  $h(1.9)$ . Is this approximation greater than or less than  $h(1.9)$  ? Explain your reasoning.
  - (b) Write the third-degree Taylor polynomial for  $h$  about  $x = 2$  and use it to approximate  $h(1.9)$ .
  - (c) Use the Lagrange error bound to show that the third-degree Taylor polynomial for  $h$  about  $x = 2$  approximates  $h(1.9)$  with error less than  $3 \times 10^{-4}$ .

## Example of an Area/Volume Problem (w/calc)

### 2008 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS

CALCULUS BC  
SECTION II, Part A  
Time—45 minutes  
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

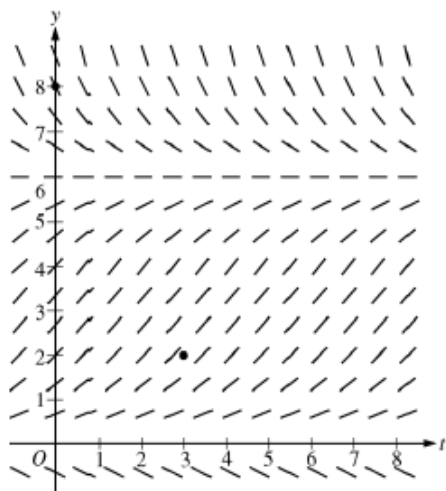


1. Let  $R$  be the region bounded by the graphs of  $y = \sin(\pi x)$  and  $y = x^3 - 4x$ , as shown in the figure above.
    - (a) Find the area of  $R$ .
    - (b) The horizontal line  $y = -2$  splits the region  $R$  into two parts. Write, but do not evaluate, an integral expression for the area of the part of  $R$  that is below this horizontal line.
    - (c) The region  $R$  is the base of a solid. For this solid, each cross section perpendicular to the  $x$ -axis is a square. Find the volume of this solid.
    - (d) The region  $R$  models the surface of a small pond. At all points in  $R$  at a distance  $x$  from the  $y$ -axis, the depth of the water is given by  $h(x) = 3 - x$ . Find the volume of water in the pond.
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## Example of a Slope Field/Differential Equation/Euler's Method Problem (w/o calculator)

### 2008 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS

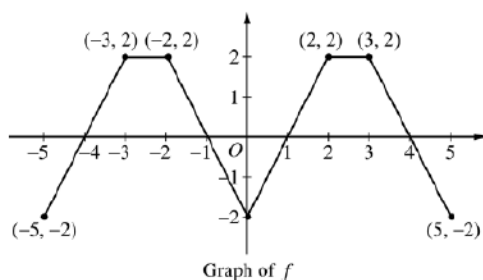
6. Consider the logistic differential equation  $\frac{dy}{dt} = \frac{y}{8}(6 - y)$ . Let  $y = f(t)$  be the particular solution to the differential equation with  $f(0) = 8$ .
- (a) A slope field for this differential equation is given below. Sketch possible solution curves through the points  $(3, 2)$  and  $(0, 8)$ .
- (Note: Use the axes provided in the exam booklet.)



- (b) Use Euler's method, starting at  $t = 0$  with two steps of equal size, to approximate  $f(1)$ .
- (c) Write the second-degree Taylor polynomial for  $f$  about  $t = 0$ , and use it to approximate  $f(1)$ .
- (d) What is the range of  $f$  for  $t \geq 0$ ?

## Example of a Function Defined as an Integral Problem (w/Calculator)

### 2006 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS



3. The graph of the function  $f$  shown above consists of six line segments. Let  $g$  be the function given by  $g(x) = \int_0^x f(t) dt$ .
- (a) Find  $g(4)$ ,  $g'(4)$ , and  $g''(4)$ .
- (b) Does  $g$  have a relative minimum, a relative maximum, or neither at  $x = 1$ ? Justify your answer.
- (c) Suppose that  $f$  is defined for all real numbers  $x$  and is periodic with a period of length 5. The graph above shows two periods of  $f$ . Given that  $g(5) = 2$ , find  $g(10)$  and write an equation for the line tangent to the graph of  $g$  at  $x = 108$ .

## Example of a Data Problem (w/calculator)

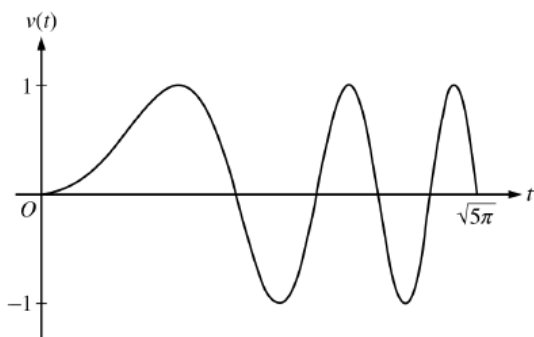
### 2008 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS

$t$ (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

2. Concert tickets went on sale at noon ( $t = 0$ ) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time  $t$  is modeled by a twice-differentiable function  $L$  for  $0 \leq t \leq 9$ . Values of  $L(t)$  at various times  $t$  are shown in the table above.
- Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. ( $t = 5.5$ ). Show the computations that lead to your answer. Indicate units of measure.
  - Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.
  - For  $0 \leq t \leq 9$ , what is the fewest number of times at which  $L'(t)$  must equal 0? Give a reason for your answer.
  - The rate at which tickets were sold for  $0 \leq t \leq 9$  is modeled by  $r(t) = 550te^{-t/2}$  tickets per hour. Based on the model, how many tickets were sold by 3 P.M. ( $t = 3$ ), to the nearest whole number?
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## Example of a Position, Velocity, Acceleration Problem (w/Calculator)

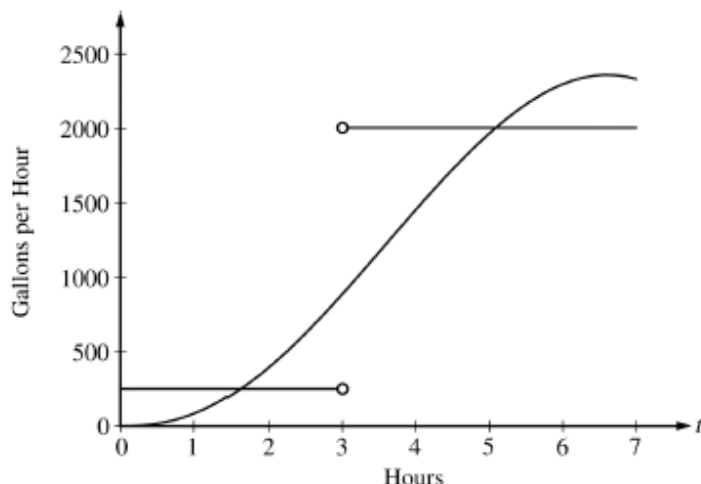
### 2007 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)



2. A particle moves along the  $x$ -axis so that its velocity  $v$  at time  $t \geq 0$  is given by  $v(t) = \sin(t^2)$ . The graph of  $v$  is shown above for  $0 \leq t \leq \sqrt{5\pi}$ . The position of the particle at time  $t$  is  $x(t)$  and its position at time  $t = 0$  is  $x(0) = 5$ .
- Find the acceleration of the particle at time  $t = 3$ .
  - Find the total distance traveled by the particle from time  $t = 0$  to  $t = 3$ .
  - Find the position of the particle at time  $t = 3$ .
  - For  $0 \leq t \leq \sqrt{5\pi}$ , find the time  $t$  at which the particle is farthest to the right. Explain your answer.

## Example of an Integral Application Problem (w/Calculator)

### 2007 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS



2. The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval  $0 \leq t \leq 7$ , where  $t$  is measured in hours. In this model, rates are given as follows:

- (i) The rate at which water enters the tank is  $f(t) = 100t^2 \sin(\sqrt{t})$  gallons per hour for  $0 \leq t \leq 7$ .
- (ii) The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \leq t < 3 \\ 2000 & \text{for } 3 < t \leq 7 \end{cases} \text{ gallons per hour.}$$

The graphs of  $f$  and  $g$ , which intersect at  $t = 1.617$  and  $t = 5.076$ , are shown in the figure above. At time  $t = 0$ , the amount of water in the tank is 5000 gallons.

- (a) How many gallons of water enter the tank during the time interval  $0 \leq t \leq 7$ ? Round your answer to the nearest gallon.
- (b) For  $0 \leq t \leq 7$ , find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.
- (c) For  $0 \leq t \leq 7$ , at what time  $t$  is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

***Example of a Logistic Growth Problem (w/o calculator)***  
**2004 AP® CALCULUS BC FREE-RESPONSE QUESTIONS**

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5. A population is modeled by a function  $P$  that satisfies the logistic differential equation

$$\frac{dP}{dt} = \frac{P}{5} \left( 1 - \frac{P}{12} \right).$$

- (a) If  $P(0) = 3$ , what is  $\lim_{t \rightarrow \infty} P(t)$ ?

If  $P(0) = 20$ , what is  $\lim_{t \rightarrow \infty} P(t)$ ?

- (b) If  $P(0) = 3$ , for what value of  $P$  is the population growing the fastest?

- (c) A different population is modeled by a function  $Y$  that satisfies the separable differential equation

$$\frac{dY}{dt} = \frac{Y}{5} \left( 1 - \frac{t}{12} \right).$$

Find  $Y(t)$  if  $Y(0) = 3$ .

- (d) For the function  $Y$  found in part (c), what is  $\lim_{t \rightarrow \infty} Y(t)$ ?
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***Example of a Misc. Problem (w/Calculator)***

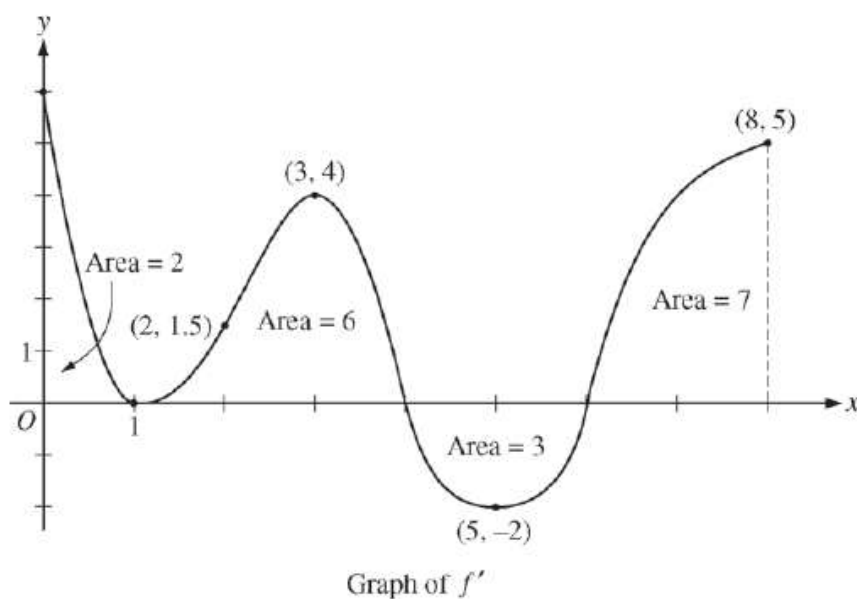
**2007 AP® CALCULUS AB FREE-RESPONSE QUESTIONS**

$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

3. The functions  $f$  and  $g$  are differentiable for all real numbers, and  $g$  is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of  $x$ . The function  $h$  is given by  $h(x) = f(g(x)) - 6$ .
- (a) Explain why there must be a value  $r$  for  $1 < r < 3$  such that  $h(r) = -5$ .
- (b) Explain why there must be a value  $c$  for  $1 < c < 3$  such that  $h'(c) = -5$ .
- (c) Let  $w$  be the function given by  $w(x) = \int_1^{g(x)} f(t) dt$ . Find the value of  $w'(3)$ .
- (d) If  $g^{-1}$  is the inverse function of  $g$ , write an equation for the line tangent to the graph of  $y = g^{-1}(x)$  at  $x = 2$ .
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## Example of a Min/Max/Inflection Point Problem (w/o Calculator)

### 2013 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS



4. The figure above shows the graph of  $f'$ , the derivative of a twice-differentiable function  $f$ , on the closed interval  $0 \leq x \leq 8$ . The graph of  $f'$  has horizontal tangent lines at  $x = 1$ ,  $x = 3$ , and  $x = 5$ . The areas of the regions between the graph of  $f'$  and the  $x$ -axis are labeled in the figure. The function  $f$  is defined for all real numbers and satisfies  $f(8) = 4$ .
- (a) Find all values of  $x$  on the open interval  $0 < x < 8$  for which the function  $f$  has a local minimum. Justify your answer.
  - (b) Determine the absolute minimum value of  $f$  on the closed interval  $0 \leq x \leq 8$ . Justify your answer.
  - (c) On what open intervals contained in  $0 < x < 8$  is the graph of  $f$  both concave down and increasing? Explain your reasoning.
  - (d) The function  $g$  is defined by  $g(x) = (f(x))^3$ . If  $f(3) = -\frac{5}{2}$ , find the slope of the line tangent to the graph of  $g$  at  $x = 3$ .
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