Summit Public Schools
Summit, New Jersey
Grade Level / Content Area: 11-12 / Mathematics
Length of Course: 1 Year

AP Calculus BC

Course Description:

AP Calculus BC is a college-level calculus course that addresses the concepts, mechanics, and applications of limits, derivatives, integrals, and power series. The calculus that students will be exposed to in this course will address functions that are represented algebraically, numerically, graphically, and verbally. The instructor will encourage the connections between these representations throughout the course. Students will make presentations, work cooperatively and communicate mathematically orally and in written form. Technology will be used extensively throughout the course. Students will be expected to use the calculator as a tool to answer conceptual problems that will often involve complicated functions. The standards that follow are from the College Board's course description. These standards are accepted by universities worldwide for college credit.

NOTE:

- The items listed below in "Course Pacing" represent textbook chapters, while the individual sections listed under "Instructional Focus" for each standard provide alignment to the sections in the required text by Finney, et. al.

- Relevant past AP questions are listed at the end of each standard's "Sample Assessments". These questions are selected from past exams that the College Board has released to the public. These exams can be found at http://apcentral.collegeboard.com/.

- As there are no NJ state standards for calculus, the sections in the College Board's official course description are instead listed for each unit.

Course Pacing:

1.	Limits and Continuity (Chapter 2) *	5 days
2.	Derivatives (Chapter 3) *	10 days
3.	Applications of Derivatives (Chapter 4)	20 days
4.	The Definite Integral (Chapter 5)	15 days
5.	Differential Equations and Mathematical Modeling (Chapter 6)	15 days
6.	Applications of Definite Integrals (Chapter 7)	15 days
7.	Applications of Integration (Chapter 8)	15 days
8.	Infinite Series (Chapter 9)	25 days
9.	Parametric, Vector, and Polar Functions (Chapter 10)	20 days
10.	Review for AP Test	15 days
11.	Vectors and Analytic Geometry (Chapter 11) **	20-25 days

*This topic was covered in the students' previous course, so it should be treated as review.
**This is not a required AP topic; other topics may be chosen at the teacher's discretion.

Unit 1: Limits and Continuity

AP Calculus BC Standards 1.1 – 1.16	
Big Ideas: All students will understand and apply the ideas of limit and continuity to functions; these concepts are what fundamentally distinguish calculus from algebra. Students will gain familiarity with the idea of an infinitesimal quantity, and will apply this idea to particular functions presented both in graphical, tabular, and algebraic form. Technology, such as graphing calculators and computer programs, will be used to aid in the exploration of these concepts.	
Essential Questions What provocative questions will foster inquiry, understanding, and transfer of learning?	**Enduring Understandings** What will students understand about the big ideas?
	Students will understand that…
• What is a limit, and why is it important?	• Limits are values that a function approaches as the function input approaches a particular value.
• How can limits be evaluated?	• Limits can be evaluated numerically, graphically, or by using a variety of algebraic techniques.
• What can we learn about the behavior of functions by considering limits?	• We can consider, in a meaningful way, the limits of functions that may seem "infinite".
• What is continuity, and why is it important?	• We can define continuity by connecting the limits of functions at particular values of the input with the functions themselves.
	• Continuity establishes consistent behavior of functions through a point, and is an important criterion for almost all of the foundational theorems in this course.
• How do limits connect to the algebraic idea of slope/rate of change?	• We can extend our idea of slope to functions that are non-linear by using limits.
Areas of Focus: Proficiencies (Cumulative Progress Indicators)	**Examples, Outcomes, Assessments**
Students will:	Instructional Focus:
be able to evaluate the limit of a function at a particular value when the function is presented in a graph, a table, or as an equation.	1. Rates of Change and Limits (2.1) 2. Using the Limit Definition (Appendix A3)

be able to determine whether a function is continuous at a particular point and on an interval by appropriate algebraic methods.

be able to use technology to investigate the behavior of functions in terms of limits and continuity, and to justify their conclusions.

be able to identify "infinite" limits (asymptotic behavior) of functions that exhibit such behavior.

be able to use the Intermediate Value Theorem to determine the existence of solutions of problems.

be able to determine slopes of tangent lines to curves by appropriate algebra.

be able to use the formal limit definition to prove the existence of limits for particular functions.

3. Limits involving Infinity (2.2)
4. Continuity (2.3)
5. Rates of Change and Tangent Lines (2.4)

Sample Assessments:

- Find the limit:
  $$\lim_{x \to 0} \frac{\frac{1}{2+x} - \frac{1}{2}}{x}$$

- Find the limit:
  $\lim_{x \to \infty} e^{-x} \cos(x)$.

  Interpret this limit in writing.

- Find a $\delta>0$ such that if for all x such that $0 < |x-3| < \delta$, then $\left|2\sqrt{x+1} - 4\right| < 0.2$.

- From the behavior shown below, find the limit of f as x → 3:

x	f(x)
2.9	4.1
2.99	4.01
2.999	4.001
3.001	3.999
3.01	3.99
3.1	3.9

- A function has $\lim_{x \to 4} f(x) = 3$ and $f(4) = 3$. Can you say anything about the continuity of this function at x = 2? Why or why not?

- Can a number be equal to one more than its cube? Why or why not?

- Previous AP Free Response Questions: 2010 BC (B) #5, 2009 BC #6, 2008 BC #2

- Previous AP Multiple Choice Questions: 2003 BC #13, #76, #81; 2008 BC #3

	Instructional Strategies: Interdisciplinary Connections • Many concepts in physics arise from the connection between limits and functions, such as the ideas of instantaneous velocity. Similarly, several important biological models involve the idea of an "equilibrium" population, or a "carrying capacity" in ecology; these are also concepts directly related to limits at infinity. These and other connections will be presented in context. Technology Integration • Students will learn how to use the TI-83+ calculator to investigate limit behavior for functions that cannot be graphed easily by hand. They will also be using it to investigate limits at infinity, by looking for trends in the graphs they display. Students will also be able to draw upon other programs, such as the Desmos calculator (http://www.desmos.com/calculator) and Wolfram|Alpha (http://www.wolframalpha.com) to investigate fundamental concepts of limits and continuity. Media Literacy Integration • Students may be asked to search for examples in news media for articles on "limits", as the word is commonly used, and to compare and contrast the common usage of limit with its mathematical usage. Global Perspectives • How does the idea of an "infinitesimal quantity" connect to concepts in philosophy? How do other cultures grasp the idea of "the infinite"?

Unit 2: Derivatives and their Applications

AP Calculus BC Standards 2.1-2.10, 3.1-3.6, 4.1-4.7, 5.1-5.12	
Big Ideas: All students will understand and apply the concept of the derivative of a function, one of the two major fundamental ideas in calculus. They will be able to estimate the derivative of functions presented in graphical and tabular (data) format, and will be able to perform appropriate algebraic calculations to determine the derivative of a function. They will also be able to connect the derivative of a function to the idea of a tangent line to a function at a particular value. They will also connect the derivative concept to applications, such as the velocity and acceleration of an object, finding extreme values of functions, optimization and related rates.	
Essential Questions What provocative questions will foster inquiry, understanding, and transfer of learning?	Enduring Understandings What will students understand about the big ideas?
	Students will understand that…
• What is a derivative, and what does it tell you about a function?	• The derivative arises from the fundamental idea of a limit.
• Under what conditions will a function have a derivative?	• If a function has a derivative, that means it can be approximated locally by a line.
• What do derivatives represent? • How can one calculate a derivative algebraically?	• Functions representing particular physical quantities (such as position as a function of time) may have meaningful derivatives (such as velocity as a function of time).
• What is the relationship between differentiability and continuity?	• Functions that have a derivative are continuous. The converse need not be true.
• How does continuity determine the existence of extrema?	• Continuous functions on a closed interval must have extreme values. This fact is useful when trying to optimize the value of a particular quantity.
• What does the graph of a function's derivative tell us about the function?	• Graphs of functions can be connected to the graphs of their derivatives, and vice versa.
• To what situations may the derivative be applied?	• Quantities that change with time can be analyzed in terms of their rates of change with time (i.e., their derivatives).

<table>
<tr><td></td><td></td></tr>
<tr><th>Areas of Focus: Proficiencies
(Cumulative Progress Indicators)</th><th>Examples, Outcomes, Assessments</th></tr>
<tr><td>Students will:</td><td rowspan="12">Instructional Focus:

1. The Derivative of a Function (3.1)
2. Differentiability (3.2)
3. Rules for Differentiation (3.3; 3.5-3.9)
4. Velocity and Rates of Change (3.4)
5. Extreme Values of Functions (4.1)
6. The Mean Value Theorem (4.2)
7. Connecting Graphs and their Derivatives (4.3)
8. Modeling, Optimization, Related Rates, and Linear Approximations (4.4-4.6)
9. L'Hopital's Rule (8.1)

Sample Assessments:
• Find the derivatives:
1. $y = \frac{2x+1}{2x-1}$
2. $y = \ln\left(\sqrt{x}\right)$
3. $xy^3 + 2x + 3y = 1$

• Determine where the following functions are differentiable:
1. $y = \sqrt{x^2 - 2x}$
2. $y = \tan(4x)$

• For what values of the constant m is the function $f(x) = \begin{cases} \sin(2x), & x < 0 \\ mx, & x \geq 0 \end{cases}$ continuous and differentiable?

• The spread of measles in a certain school is given by $P(t) = \frac{200}{1+e^{5-t}}$, where t is the number of days since the measles first appeared, and P is the total number of students who have caught measles to date. When will the rate of the spread of measles be the greatest?</td></tr>
<tr><td>Understand and apply the definitions of the dervatives.</td></tr>
<tr><td>Understand the relationship between differentiability and continuity.</td></tr>
<tr><td>be able to define and calculate the derivative of algebraic functions by appropriate methods.</td></tr>
<tr><td>be able to determine whether or not a function will have a derivative.</td></tr>
<tr><td>be able to connect the derivative of a function to its rate of change.</td></tr>
<tr><td>Understand and apply the Mean Value Theorem</td></tr>
<tr><td>Locate local extrema using both the First and Second Derivative Tests</td></tr>
<tr><td>Completely analyze functions by finding local extrema, inflection points, intervals that increase/decrease, and intervals that are concave up/down.</td></tr>
<tr><td>be able to locate extreme values of functions on closed intervals using the derivative.</td></tr>
<tr><td>be able to use the derivative to calculate limits that are not amenable to typical algebraic methods.</td></tr>
<tr><td>be able to apply the derivative to a variety of contexts, including graphs, optimization, linear approximation, and related rates of change.</td></tr>
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	• For the function $y = x^{\frac{4}{5}}(2 - x)$, find the intervals on which the function is increasing, decreasing, concave up, and concave doing. Then find any local extrema and points of inflection. • Find the height and radius of the largest right circular cylinder that can be contained completely inside a sphere of radius 2. • Sand falls from a conveyor belt at the rate of 10 m^3/min onto the top of a conical pile. The height of the pile is always 3/8 of the base diameter. How fast are the height and radius of the base changing when the pile is 4 m high? • Previous AP Free Response Questions: 2007 BC #4, 2008 BC #4, #5 • Previous AP Multiple Choice Questions: 2003 BC #1, #4; 2008 BC #10, #15 Instructional Strategies: Interdisciplinary Connections • The idea of taking derivatives of quantities that vary with time is fundamental to the branch of physics known as kinematics. We will explore the connection between a function, its first derivative, and its second derivative in the context of position, velocity, and acceleration with time. We will also look at other quantities that vary, including some from economics (the idea of "marginal cost") and from chemistry. Technology Integration

	• The TI-83+ calculator is extremely helpful in this chapter, as it allows many of the more tedious calculations of derivatives to be performed – and visualized – easily. The connections between a function and its derivative, in terms of its graphs, can be quickly observed using this tool. The skill of estimating derivatives and plotting curves using this calculator is required of the students for the AP exam in May. In addition, Desmos.com can be used to quickly determine functions’ rates of change. Global Perspectives • Within the context of the problems we investigate, it will be useful to look for real-world examples of changing quantities and making predictions based on them, such as the world population and rate of growth. Questions such as “what will the maximum population of the Earth be, based on data?”, naturally arise from investigations of the derivative.

Unit 3: Integration and its Applications

AP Calculus BC Standards 6.1-6.14, 7.1-7.9, 8.1-8.13	
Big Ideas: All students will understand and apply the idea of the integral of a function, the other fundamental idea in calculus. They will distinguish two seemingly-unrelated ideas, that of an antiderivative (indefinite integral) and of a definite integral. They then will be able to demonstrate the connection between the two, made through the Fundamental Theorem of Calculus. They will be able to estimate definite integrals in a variety of ways, and learn a variety of techniques for calculating antiderivatives exactly. They will apply their knowledge of integrals to a variety of situations.	
Essential Questions What provocative questions will foster inquiry, understanding, and transfer of learning?	Enduring Understandings What will students understand about the big ideas?
• What is a definite integral of a function, and how does it connect to the idea of area? • How can a definite integral be estimated? • What is the fundamental relationship between antiderivatives and definite integrals? • How can antiderivatives be determined? • What are differential equations, what are their solutions, and how can they be solved?	Students will understand that… • Definite integrals represent "accumulated quantities". This can be area under a curve, but could mean many other things. • Definite integrals can be estimated numerically using a calculator or computer, or graphically by estimating areas of rectangles (Riemann Sums) or trapezoids built between the x-axis and the function. • Finding the exact value of a definite integral requires finding the antiderivative of the function being integrated (the Fundamental Theorem of Calculus). This is not always possible; in such cases estimation is required. • Antiderivatives can be found by applying differentiation rules backwards, through algebraic manipulation, through application of algebraic identities, or through various integration techniques including u-substitution, integration by parts, partial fraction decomposition, or trig substitution. • Antiderivatives themselves can be used to solve differential equations using separation of variables. Other

• What is the meaning of a definite integral if one of its limits is infinite?	options include Euler's Method or slope fields. • Definite integrals with infinite or undefined bounds are defined to be limits of definite integrals with real-valued bounds.
Areas of Focus: Proficiencies (Cumulative Progress Indicators)	**Examples, Outcomes, Assessments**

Students will:	Instructional Focus:
	
	1. Estimating with Finite Sums (5.1)
	2. Definite Integrals (5.2)
	3. Definite Integrals and Antiderivatives (5.3)
	4. The Fundamental Theorem of Calculus (5.4)
	5. The Trapezoidal Rule (5.5)
	6. Antiderivatives and Slope Fields (6.1)
	7. Integration and Estimation Techniques (6.2, 6.3, 6.6, 8.4)
	8. Applications of Integration:
	9. Mathematical Modeling of Exponential Growth (6.4)
	10. Population Growth (6.5)
	11. Net Change (7.1)
	12. Areas, Volumes, and Lengths of Curves (7.2-7.4)
	13. Work and Pressure (7.5)
	14. Relative Rates of Growth and Improper Integrals (8.2, 8.3)
	
	Sample Assessments:
	• Estimate the area of the region of the first quadrant bounded by the curve y = cos x. Partition the region into ten equal widths, and estimate this area using:
	  o left-hand rectangles
	  o trapezoids
	
	• Find the total area between the curve y = 4 – x, bounded by the x-axis and the vertical lines x = 0 and x = 6.
	
	• Evaluate the integral:

be able to define a definite integral of a function, and estimate it using techniques from geometry.	
be able to define the antiderivative of a function, and find it for a wide variety of functions using various techniques.	
be able to connect the definite integral of a function with the appropriate antiderivative through the Fundamental Theorem of Calculus.	
be able to use antiderivatives to solve initial value problems.	
use integration techniques including u-substitution, integration by parts, partial fraction decomposition, and trig substitution to solve complicated integrals.	
be able to use slope fields and Euler's method to approximate solutions to initial value problems.	
be able to use definite integrals to solve problems involving physics, area, volume, length, and surface area.	
be able to evaluate definite integrals having either infinite limits or taking on infinite values within their limits of integration.	

	$\int_{-1}^{1} 2x\sin(1-x^2)\,dx$. • Show that $0 \le \int_0^1 \sqrt{1+\sin^2 x}\;dx \le \sqrt{2}$ • Find dy/dx, if $y = \int_x^1 \frac{6}{3+t^4}\,dt$. • Skydivers A and B are in a helicopter hovering at 6400 feet. Skydiver A jumps and descends for 4 seconds before opening her parachute. The helicopter then climbs to 7000 feet and hovers there. 45 seconds after A leaves the helicopter, B jumps and descends for 13 seconds before opening her parachute. Both skydivers fall at 16 ft/sec with their parachutes open, and both fall freely with an acceleration of -32 ft/sec^2 before their parachutes open. Which skydiver lands first? • Solve the initial value problem analytically: $\frac{dy}{dx} = \left(x + \frac{1}{x}\right)^2$, $y(1) = 1$. • A deep-dish apple pie, whose internal temperature was 220°F before being removed from the oven, was set out on a breezy 40°F porch to cool. Fifteen minutes later, the pie's internal temperature was 180°F. How long will it take for the pie to cool to 70°F? • The number of billboards per mile along a 100-mile stretch of an interstate highway approaching a city is modeled by the function $B(x) = 21 - e^{0.03x}$, where x is the distance from the city in miles.

	About how many billboards are along that stretch of highway? • Find the volume of the solid generated by revolving the region enclosed by the parabola $y^2 = 4x$ and the line y = x around the line y = 4. • If a force of 80N is required to hold a spring 0.3 m beyond its unstressed length, how much work does it take to stretch the spring this far? How much work would be needed to stretch the spring a meter further? • Evaluate the following integrals or state that they diverge: ○ $\int_0^4 \frac{dr}{\sqrt{4-r}}$ ○ $\int_{-1}^4 dx/\sqrt{\|x\|}$ ○ $\int_0^\infty \frac{dx}{(1+x)\sqrt{x}}$ • Previous AP Free Response Questions: 2006 BC #4, 2010 BC #3, 2009 BC #2 • Previous AP Multiple Choice Questions: 2003 BC #6, 2008 BC #2, #11, #19 Instructional Strategies: Interdisciplinary Connections • Integrals are fundamental for the evaluation of any quantity that is calculated by "accumulation". The area under a curve, finding the total cost of a project (using marginal costs), or finding the net force on a glass fish tank's wall all require the use of integrals. In addition, many scientific, economic, and engineering problems require solving initial value problems – equivalent to finding antiderivatives of given functions.

	Technology Integration • Students will use the TI-83+ calculator to estimate definite integrals, including those for which an exact value cannot be found. They will also use the TI-83+ to explore the geometric approaches to estimating such integrals, by summing areas of rectangles or trapezoids. They can also generate slope fields, or graphical representations of initial value problems, using this technology, which will allow them to explore the solutions more fully. These are required skills for the students to have for the AP examination. In addition, websites like Wolfram|Alpha can explain, step-by-step, how certain integral problems can be done, and students should feel free to use that resource. Global Perspectives • Many of the problems that are solved by integration can be thought of in the context of global events; again, population growth can be modeled in this way. Students should look for situations in the news in which a rate of growth is quoted uncritically, and determine what it implies: is this growth constant with time? Changing? What would that mean in terms of the underlying quantity?

Unit 4: Infinite Series

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<tr><th colspan="2">AP Calculus BC Standards 10.1-10.15</th></tr>
<tr><td colspan="2">Big Ideas: All students will understand the fundamental concept of a power series, a way to represent a function in terms of infinite polynomials. They will connect their previously-learned knowledge of infinite geometric series, convergence and divergence, and infinite sums to that of power series. They will use Taylor's theorem to learn to represent any function in terms of an infinite power series, and will use the Remainder Theorem to be able to estimate errors in approximations derived from the use of such series. They will also investigate the convergence and divergence of series in general through the use of appropriate tests.</td></tr>
<tr><th>Essential Questions
What provocative questions will foster inquiry, understanding, and transfer of learning?</th><th>Enduring Understandings
What will students understand about the big ideas?</th></tr>
<tr><td>What is a power series, and how is it used?
How can power series be used to model functions?
What are the limitations of and advantages gained by using a Taylor Polynomial to represent functions?
What issues arise when determining the convergence of series, and how can this convergence be found?</td><td>Students will understand that…
Power series are polynomials with infinite numbers of terms.
A Taylor Series, with correctly-chosen coefficients, can represent any algebraic function over a particular interval.
The number degree of the polynomial and the distance from the center will both determine the margin of error in using a Taylor Polynomial to represent a differentiable function.
A variety of tests exist that will help to determine if an infinite series either converges or diverges. In certain circumstances, the value that a series converges to can be determined.</td></tr>
<tr><th>Areas of Focus: Proficiencies
(Cumulative Progress Indicators)</th><th>Examples, Outcomes, Assessments</th></tr>
<tr><td>Students will:</td><td rowspan="4">Instructional Focus:
Power Series (9.1)
Taylor Series (9.2)
Taylor's Theorem (9.3)
The Radius of Convergence (9.4)
Testing Convergence at Endpoints (9.5)
Sample Assessments:</td></tr>
<tr><td>Understand what it means for an infinite series to converge or diverge.</td></tr>
<tr><td>Find intervals on which geometric power series converge</td></tr>
<tr><td>Understand that if the correct set of coefficients are chosen any differentiable function can be represented by a power series, called a Taylor Series</td></tr>
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<td>be able to identify particular, commonly-occurring Taylor Series as representing particular functions. (i.e., $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$)</td>
<td rowspan="7">
a. Determine the sum of the series: 3 – 0.3 + 0.03 – 0.003 + 0.0003 - ….

b. Find the values of x for which the following geometric series converges and the function of x it represents:

$$\sum_{n=0}^{\infty} \sin^n(x)$$
c. Find the Taylor polynomial of order 3 generated by $f(x) = \cos(x)$ at x = π/4.

d. Find a power series to represent $f(x) = \frac{\sin(x)}{x}$ at x = 0.

e. Find the Taylor polynomial of order 4 at x = 0 for $f(x) = \ln(1 + x^2)$, and use it to approximate f (0.2).

f. The approximation $e^x \approx 1 + x + \frac{x^2}{2} + \frac{x^3}{3!}$ is used on small intervals about the origin. Estimate the magnitude of the approximation error for $|x| \leq 0.1$.

g. Determine the convergence or divergence of the series $\sum_{n=1}^{\infty} n!\, e^{-n}$ and state the test or tests you used.

h. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n}$.

i. Find the interval of convergence of the series
$\sum_{n=0}^{\infty} \frac{x^n \cdot \sqrt{n}}{3^n}$
For what values of x does the series converge (a) absolutely, or (b) conditionally?

j. Determine whether the series converges absolutely, converges conditionally, or diverges, and justify your answer:
$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1+n}{n^2}$
</td>
</tr>
<tr><td>be able to estimate errors in using Taylor Polynomials to represent functions through the use of the Remainder Estimation Theorem.</td></tr>
<tr><td>be able to exactly specify intervals on which Taylor Polynomials of a certain degree converge.</td></tr>
<tr><td>be able to determine if particular series converge or diverge using tests such as the n-th-term test, the Ratio test, the Limit Comparison test, the Direct Comparison Test, the Integral test, or p-series.</td></tr>
<tr><td>Apply the Alternating Series Test to determine the convergence of, and error in, alternating series.</td></tr>
<tr><td>be able to distinguish between absolute and conditional convergence, and the circumstances under which each apply.</td></tr>
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	k. Previous AP Free Response Questions: 2008 BC #3, 2010 BC #6, 2009 BC #4 l. Previous AP Multiple Choice Questions: 2003 BC #10, #11, #20; 2008 BC #4, #12, #16 Instructional Strategies: Interdisciplinary Connections • The use of series techniques to solve problems is one that occurs in physics, as well as in music. For example, different musical instruments generate a different infinite series of frequencies when playing the same note, which gives the unique sound, or timbre, of that instrument. Manipulation by synthesizers or other computer programs of these infinite series are what allow for the wide range of electronic musical instruments to be produced. Technology Integration • The TI-83+ calculator is used extensively to determine power series for different functions, and graphical representations of the partial sums of these series can demonstrate the fundamental concepts of convergence and interval of convergence. More powerful standalone programs, such as Mathematica (http://www.wolfram.com/mathematica/), are used to demonstrate more sophisticated concepts, such as absolute and conditional convergence, to the entire class. Global Perspectives • Geometric series can be used to demonstrate fundamental concepts of the accumulation of interest, for example, or of population growth. Atmospheric research uses functions that can be modeled more easily with power series.

Unit 5: Parametric, Vector, and Polar Functions

AP Calculus BC Standards: 9.1-9.9

Big Ideas: All students will understand how parametric and vector equations can be used to easily model situations in which objects are moving, and the connections that can be made to calculus. They will understand how to use the polar coordinate system to more readily understand functions that are dependent only on distance and angle, and how the earlier ideas from calculus can be used to determine areas and lengths of figures represented in this way.

Essential Questions What provocative questions will foster inquiry, understanding, and transfer of learning?	Enduring Understandings What will students understand about the big ideas?
	Students will understand that…
• What are parametric functions, and how are they used?	• Curves in the plane can be represented by pairs of parametric equations x(t), y(t).
• What ideas from calculus, such as arc length and velocity, can be applied to parametric equations?	• Knowing the parametric equations for a curve also can lead to understanding how an object traverses the curve (i.e., in which direction).
• What are vector-valued functions, and how are they used?	• Curves can also be represented by vector-valued functions, the components of which are parametric equations for that curve.
• How can one use vector-valued functions to model motion of an object in the plane, including details about its direction and speed of travel?	• Vector-valued functions can be differentiated or integrated, just as single-valued functions can. • Vector-valued functions provide a physically meaningful context for interpreting laws of physics, such as Newton's Laws.
• What are polar coordinates, and how are they used?	• Polar coordinates are a different way of locating points in the plane, identifying them in terms of their distance from the origin and a particular angle.
• How can polar coordinates be used along with calculus to determine areas of regions in the plane that are complex?	• Curves represented in terms of polar coordinates can be investigated using similar calculus procedures to those of curves given in Cartesian (e.g., y = f(x)) coordinates. In particular,

	questions of length, area, and tangency can be addressed.

Areas of Focus: Proficiencies (Cumulative Progress Indicators)	Examples, Outcomes, Assessments
Students will: be able to perform calculations of tangency, arc length, and area with calculus for functions represented parametrically. be able to use vector-valued functions to model physical situations, and to apply their calculus concepts to such situations. be able to analyze points and curves that are represented using polar coordinates instead of Cartesian coordinates, and to be able to translate between the two. be able to do typical calculus computations, such as derivatives and integrals, on functions represented in polar coordinates, and to be able to interpret their results in a meaningful context. Find the length of polar curves Find the area bounded between polar curves	Instructional Focus: 1. Parametric Functions (10.1) 2. Vectors In The Plane (10.2) 3. Vector-Valued Functions (10.3) 4. Modeling Projectile Motion (10.4) 5. Polar Coordinates and Polar Graphs (10.5) 6. Calculus of Polar Curves (10.6) Sample Assessments: • Find the equation for the tangent line to the curve $x = \frac{1}{2}\tan t,\ y = \frac{1}{2}\ sec(t)$ at t = π/3, and find the value of $\frac{d^2y}{dx^2}$ at that point. • Find the points at which the tangent to the curve $x = -\cos t,\ y = cos^2(t)$ is horizontal and the points at which the tangent is vertical. • A particle's position vector in the plane is given by $r(t) = (2\cos t)i + (3\ sin\ t)j$. Sketch the path of the particle, find the velocity and acceleration vectors, and find the particle's speed and direction of motion at the time t = π/2. • Solve the initial value problem for r as a function of t: $\frac{dr}{dt} = \frac{3}{2}(t+1)^{\frac{1}{2}}i + e^{-t}j, r(0) = 0$ • The position of a kite is given by $r(t) = \frac{t}{8}i - \frac{3}{64}t(t-160)j$, where t ≥ 0 is measured in seconds, and distance is measured in meters. How long is the kite above the ground, and at what time does the kite begin losing altitude?

	• Describe the symmetries of the polar curve $r = 2 + 3\sin\theta$. • Find the slope of the tangent line to the curve $r = 3(1 - \cos\theta)$ at θ = 3π/2. • Find equations for all horizontal and vertical tangent lines to the curve $r = 1 + 2\sin\theta$. • Find the area inside the six-leaved rose $r^2 = 2\sin 3\theta$. • Find the area inside the circle $r = 3\,a\cos\theta$ and outside the cardioid $r = a(1 + \cos\theta)$. • Find the length of the spiral $r = e^{\theta}/\sqrt{2}$ from 0 ≤ θ ≤ π. • Previous AP Free Response Questions: 2006 BC #3, 2004 BC #3, 2010 BC #3, 2009 BC #3, 2007 BC #3, 2005 BC #2 • Previous AP Multiple Choice Questions: 2003 BC #4, #84, 1998 BC #19, 2008 BC #1, #26 Instructional Strategies: Interdisciplinary Connections • Parametric equations and vectors are fundamental in physics, particularly in the analysis of motion (kinematics). Many of the examples that will be discussed will be drawn from there, such as projectile motion. Polar coordinates are relevant to astronomy, as that is the coordinate system used to track the motion of comets, planets, and other objects in the solar system. Technology Integration

	• Aside from the use of the TI-83+ graphing calculator, we will be using online calculators such as Desmos (http://www.desmos.com/calculator) to explore properties of curves in polar and parametric form. This calculator is freely available to students. The graphing program GeoGebra (http://www.geogebra.org) is also excellent for exploring curves represented in this way Global Perspectives • Physics and astronomy are, by their nature, globally-applicable subjects, and an understanding of navigation is important as well. All of these areas can be better understood through the use of different coordinate systems (such as polar coordinates) and different ways of representing functions (such as parametrically).

Required Texts and Resources:

Textbook: "Calculus: Graphical, Numerical, and Algebraic", by Ross L. Finney, Franklin D. Demana, Bert K. Waits, and Daniel Kennedy (Menlo Park: Scott Foresman Addison Wesley, 1999).

Electronic resource: Texas Instruments' TI-83+ graphing calculator.

Summit Public Schools
Summit, New Jersey

Curricular Addendum

Career-Ready Practices **CRP1**: Act as a responsible and contributing citizen and employee. **CRP2**: Apply appropriate academic and technical skills. **CRP3**: Attend to personal health and financial well-being. **CRP4**: Communicate clearly and effectively and with reason. **CRP5**: Consider the environmental, social and economic impacts of decisions. **CRP6**: Demonstrate creativity and innovation. **CRP7**: Employ valid and reliable research strategies. **CRP8**: Utilize critical thinking to make sense of problems and persevere in solving them. **CRP9**: Model integrity, ethical leadership and effective management. **CRP10**: Plan education and career paths aligned to personal goals. **CRP11**:. Use technology to enhance productivity. **CRP12**: Work productively in teams while using cultural global competence.	**Interdisciplinary Connections** ● Close Reading of works of art, music lyrics, videos, and advertisements ● Use Standards for Mathematical Practice and Cross-Cutting Concepts in science to support debate/inquiry across thinking processes **Technology Integration** Ongoing: ● Listen to books on CDs, Playaways, videos, or podcasts if available. ● Use document camera or overhead projector for shared reading of texts. Other: ● Use Microsoft Word, Inspiration, or SmartBoard Notebook software to write the words from their word sorts. ● Use available technology to create concept maps of unit learning.

Instructional Strategies:
Supports for English Language Learners:

Sensory Supports	Graphic Supports	Interactive Supports
Real-life objects (realia)	Charts	In pairs or partners
Manipulatives	Graphic organizers	In triads or small groups
Pictures & photographs	Tables	In a whole group
Illustrations, diagrams, & drawings	Graphs	Using cooperative group structures
Magazines & newspapers	Timelines	With the Internet (websites) or software programs
Physical activities	Number lines	In the home language
Videos & films		With mentors
Broadcasts		
Models & figures		

from https://wida.wisc.edu

Media Literacy Integration

- Use multiple forms of print media (including books, illustrations/photographs/artwork, video clips, commercials, podcasts, audiobooks, Playaways, newspapers, magazines) to practice reading and comprehension skills.

Global Perspectives

- The Global Learning Resource Library

Differentiation Strategies:

Accommodations	Interventions	Modifications
Allow for verbal responses	Multi-sensory techniques	Modified tasks/ expectations
Repeat/confirm directions	Increase task structure (e.g., directions, checks for understanding, feedback)	Differentiated materials
Permit response provided via computer or electronic device	Increase opportunities to engage in active academic responding (e.g., writing, reading aloud, answering questions in class)	Individualized assessment tools based on student need
Audio Books	Utilize prereading strategies and activities: previews, anticipatory guides, and semantic mapping	Modified assessment grading