Summit Public Schools
Summit, New Jersey
Grade Level / Content Area: 11-12 / Mathematics
Length of Course: 1 Year

AP Calculus AB

Course Description:

AP Calculus AB is a college-level calculus course that addresses the concepts, mechanics, and applications of limits, derivatives, and integrals. The calculus that students will be exposed to in this course will address functions that are represented algebraically, numerically, graphically, and verbally. The instructor will encourage the connections between these representations throughout the course. Students will make presentations, work cooperatively and communicate mathematically orally and in written form. Technology will be used extensively throughout the course. Students will be expected to use the calculator as a tool to answer conceptual problems that will often involve complicated functions. The standards that follow are from the College Board's course description. These standards are accepted by universities worldwide for college credit.

NOTE:

- The items listed below in "Course Pacing" represent instructional units, while the individual sections listed under "Instructional Focus" for each standard provide alignment to the sections in the required text by Hughes-Hallett.

- Relevant past AP questions are listed at the end of each standard's "Sample Assessments". These questions are selected from past exams that the College Board has released to the public. These exams can be found at http://apcentral.collegeboard.com/.

- As there are no NJ state standards for calculus, the sections in the College Board's official course description are instead listed for each unit.

Course Pacing:

1.	Limits and Derivatives	11 days
2.	Rules for Differentiation	7 days
3.	Applications of Differentiation	15 days
4.	Optimization	15 days
5.	The Definite Integral	18 days
6.	Antiderivatives and the Fundamental Theorem	14 days
7.	Area and Volume	20 days
8.	Differential Equations	25 days
9.	AP Exam Review	19 days
10.	Advanced Integration Techniques or Optional Study	26 days

Unit 1: Limits and Derivatives

AP Calculus AB Standards: 1.1-1.16, 2.1-2.4	
Big Ideas: This unit exposes students to the concept of a limit, which is the basis for the definitions of derivatives and definite integrals in calculus. Students will become familiar with the concept of an infinitesimal quantity. Limits of functions that are represented in algebraic and graphic form will be emphasized. This unit also introduces the definition of the derivative and various interpretations of this quantity. Technology will aide in the discovery of limit existence in interesting functions.	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
	Students will understand that…
- What are limits, and why are they important to the study of calculus?	- a limit is the value that a function approaches as the input approaches a particular value.
- How can limit notation be used to better describe infinite behavior of functions?	- limits can be used to describe rates of growth of functions, asymptotic behavior of functions, and the behavior of functions at points of discontinuity.
- What does it mean for a function to be continuous?	- for a function to be continuous at a point, its limit as the input approaches that point must equal the function value at that point.
- How does limit existence affect continuity of a function?	- continuity of a function at a point requires the limit of the function at that point to exist.
- What is the difference between an average rate of change and an instantaneous rate of change?	- An instantaneous rate of change takes the time interval to zero.
- What mathematical challenge is encountered when computing an instantaneous rate of change? How can this challenge be resolved?	- To compute an instantaneous rate of change, division by zero is required. Derivatives are defined to be the limit of the average rates of change as the time interval *approaches* zero.
- What is the relationship between differentiability and continuity?	- If a function is differentiable at a point, then it must be continuous at

	that point. The converse of this is not necessarily true.

Areas of Focus: Proficiencies (Cumulative Progress Indicators)	**Examples, Outcomes, Assessments**
Students will:	Instructional Focus: (Corresponding Textbook Sections) 1. Limits, Conceptually (1.8) 2. Limits, Algebraically (1.8) 3. Infinite Limits (1.8) 4. Continuity and the Intermediate Value Theorem (1.7 and 1.8) 5. Average and Instantaneous Rates of Change (2.1) 6. Definition of the Derivative (2.2) 7. The Derivative Function (2.3) 8. Interpreting Derivatives (2.4) 9. Differentiability (2.6) Sample Assessments: • For $f(x) = x\sin\left(\frac{1}{x}\right) + 2$, estimate the value of $\lim_{x \to 0} f(x)$ using your TI graphing calculator. Examine both a table of values and a graph of f(x). • Evaluate $\lim_{x \to 5} \frac{x^2 + 3x - 40}{x - 5}$ • Find the value of k such that f(x) is continuous. Let $f(x) = \begin{cases} -0.4x + 2, x \le 1 \\ 0.3x + k, x > 1 \end{cases}$ • A searchlight shines on a wall. The perpendicular distance from the light to the wall is 100 ft. How close to 90 degrees must the angle be in order for the length of the beam to be at least 1000 feet? Write your answer in terms of the definition of an infinite
Analyze graphs with the aid of technology.	
Use analytic and geometric information to predict and explain the local and global behavior of a function.	
Have an intuitive understanding of the limiting process, including one-sided limits.	
Calculate limits using algebra.	
Estimate limits from graphs or tables of data.	
Understand asymptotes in terms of graphical behavior	
Describe asymptotic behavior in terms of limits involving infinity	
Compare relative magnitudes of functions and their rates of change	
Have an intuitive understanding of continuity	
Use the Intermediate Value Theorem and the Extreme Value Theorem to gain a geometric understanding of graphs of continuous functions	
Understand continuity in terms of limits	
Understand that a derivative can be presented graphically, numerically, and analytically	
Understand that the derivative can be interpreted as an instantaneous rate of change.	
Understand that the derivative is defined as the limit of the difference quotient.	
Explain the relationship between differentiability and continuity.	
Interpret the derivative at a point as the slope of a curve at that point.	
Identify points where derivatives do not exist, such as at vertical tangent lines, cusps, and corners.	
Compute the tangent line to a curve at a point and local linear approximations.	

<table>
<tr><td>Define the instantaneous rate of change to be the limit of the average rate of change.</td><td rowspan="4">limit. (Hint, use the limit as the angle approaches 90 degrees.

• Given the graph of a polynomial, examine the rate of change of the function at particular points. Describe the rate of change as fast or slow and increasing or decreasing.

• Given a table of values, estimate the instantaneous rate of change both at given points and points that are not given.

• Given an algebraic function, estimate the instantaneous rate of change by creating a table in which several average rates of change are computed, narrowing the time interval at each computation.

• Given $f(x)=x^2+6x-2, c=-4$, use the definition of the derivative to evaluate $f'(c)$.

• Relevant AP Questions:
i. Open Ended: 2011 AB 6, 2011 AB 2 (Form B)
ii. Multiple Choice: 2003 AB 3, 6, 79

Sample Project:

Students will be asked to create a display showing a function with either a jump, removable, or infinite discontinuity. The display must contain algebraic, tabular, and graphic analysis of the function’s limit and the function’s continuity at the point of discontinuity. The display should be made using technology. The use of Geogebra software (www.geogebra.org) will be encouraged.</td></tr>
<tr><td>Approximate rates of change from graphs and tables of values.</td></tr>
<tr><td>Identify corresponding characteristics of the graphs of f, f', and f''.</td></tr>
<tr><td></td></tr>
</table>

	Instructional Strategies: Interdisciplinary Connections - In this early part of the course, students will be constantly exposed to functions that model real-world phenomena. Some examples include trigonometric models for biological processes that are periodic, exponential models for economic and sociological phenomena, and polynomial models for functions representing average cost. Students will be expected to use limits and appropriate limit notation to effectively describe the behavior or such models. Technology Integration - Students will be made aware that although many examples of evaluating limits provided in textbooks are solvable using pencil and paper, most models used in real life are not as convenient. Students will be encouraged to use technology as a means of analyzing these complex models. - The TI-83 and/or TI-89 calculators will aide students in evaluating the limit of a function by quickly creating tables of values and by quickly graphing functions at points of interest. It is important to discourage students from using the "trace" feature other than for very informal (or very obvious) investigations. - http://www.geogebra.org/ The above link provides free software that students can use to graph functions, solve equations, and

	evaluate limits with. The software can be downloaded or used on the web. - http://www.desmos.com The above link is a free graphing calculator that includes explorative activities to guide students through visualizing limits and derivatives. Media Literacy Integration - Students will be asked to find instances in the media (print, web, television) of derivatives in the social and physical sciences. Through this, students will become aware of how widespread the notion of a rate of change is in the physical and social sciences. This will also promote the discussion of an average rate of change versus and instantaneous rate of change. - Students will be asked to find examples of long-term projections in the news. An in-class discussion can contrast these models with the models studied in class, as well as the relationship between long-term projections and limits at infinity. Global Perspectives - Students can investigate the development of the limit as a more "modern" way to solidify concepts in calculus that pre-date the limit. The concept of a limit can be found in many cultures' mathematical history, and is a very interesting part of the history of calculus.

Unit 2: Rules for Differentiation

<table>
<tr><th colspan="2">AP Calculus AB Standards: 2.4-2.10, 3.1-3.2, 3.5-3.6</th></tr>
<tr><td colspan="2">Big Ideas: Students should understand the meaning of the derivative in terms of an instantaneous rate of change of a function at a point. Students should understand the derivative function as a function that measures this rate of change given any input. The derivative should be understood for functions represented graphically, numerically, analytically, or verbally. Students should be able to quickly and accurately find the derivative of both explicitly and implicitly defined functions. Both algebra and technology will be used to answer questions about the behavior of functions.</td></tr>
<tr><th>Essential Questions
What provocative questions will foster inquiry, understanding, and transfer of learning?</th><th>Enduring Understandings
What will students understand about the big ideas?</th></tr>
<tr><td>- Do patterns exist that make the computation of derivatives simpler?

- What is the relationship between polynomial, rational, exponential, and trigonometric functions and their derivatives?

- In what ways are f, f', and f'' related to each other?

- How can the differentiation rules for explicitly defined functions be applied to implicitly defined functions?</td><td>Students will understand that…

- Shortcuts such as the power, product, quotient, and chain rules exist that make the algebraic computation of derivative functions faster.

- Formulas can be derived, using the definition of the derivative, that generate the derivative function of all such functions as well as combinations of such functions

- Both algebraic and geometric patterns exist between a function and its first and second derivative.

- Implicit differentiation allows differentiation to become an operator that's valid across any equal sign.</td></tr>
<tr><th>Areas of Focus: Proficiencies (Cumulative Progress Indicators)</th><th>Examples, Outcomes, Assessments</th></tr>
<tr><td>Students will:</td><td rowspan="5">Instructional Focus: (Corresponding Textbook Sections)

1. Derivatives of Polynomials (3.1)
2. Derivatives of Exponential Functions (3.2)
3. Product and Quotient Rules (3.3)
4. The Chain Rule (3.4)
5. Derivatives of the Trig Functions (3.5)
6. Implicit Differentiation (3.7)</td></tr>
<tr><td>Determine, by definition, general rules for differentiation of functions, including sums, differences, constants, and coefficients.</td></tr>
<tr><td>Determine the power rule by definition.</td></tr>
<tr><td>Determine the derivative of $y = e^x$, and then $y = a^x$, by definition.</td></tr>
<tr><td>Determine the product rule by definition.</td></tr>
</table>

Understand and apply the chain rule. Determine the quotient rule by applying the product and chain rule. Determine the derivatives of $y = \sin(x)$ and $y = cos(x)$ by definition. Determine the derivatives of the other trigonometric functions by using the quotient rule. Compute derivatives using the chain rule and implicit differentiation.	Sample Assessments: • Prove the power rule, $\frac{d}{dx}x^n = nx^{n-1}$. • Compute the derivative of the following: ▪ $y = \frac{5}{2}x^3 - \frac{1}{3x^2}$ ▪ $y = 2\cos(2x)$ ▪ $f(x) = -9\sin^3(3x-2)$ ▪ $g(w) = w^2 \sin^2 w$ ▪ $x(t) = \frac{12}{7t^2}$ ▪ $y = \frac{3x-7}{6x+5}$ ▪ $g(t) = -0.4e^{.18t}$ • Prove, using the quotient rule, $\frac{d}{dx}\sec x = \sec x \tan x$ • Use implicit differentiation to find $\frac{dy}{dx}$ if $x^2 - xy + y^2 = 1$. Then, determine the coordinates at which the curve has vertical tangent lines. • Relevant AP Questions: ▪ Open Ended: 2010 AB 2a, 2007 AB 3d, 2010 AB 2a,c (Form B) ▪ Multiple Choice: 2003 AB 1, 4, 7, 9, 13, 14, 16, 24; 1997 AB 76, 79, 80, 86, Sample Project: - Students will create their own derivative matching game. Each student individually will create 10 "interesting" graphs. Then, students will pair up and attempt to draw the graph of the derivative function for each "interesting" graph. Once both students agree that both sets of graphs and derivatives are drawn correctly, a final draft of the 10 "interesting" graphs and corresponding derivatives will be drawn on index cards for a matching game.

	Instructional Strategies: Interdisciplinary Connections - Students will be analyzing the derivatives of functions that are mathematical models for scenarios in the social sciences, physics, biology, and economics. Technology Integration - TI graphing calculators will be used extensively to create graphs and tables, as well as assist in numerically evaluating the derivative of a function at a point. This skill is critical for success on the AP Calculus AB exam. • http://www.geogebra.org • http://www.desmos.com The above websites allow students to graph functions, and then easily graph the derivative of the function or numerically evaluate the derivative of the function at a point. Media Literacy Global Perspectives - Students can examine the growth rates of different nations. Students will be asked to draw conclusions about how and why different societies' populations have varied growth rates, and what implications these rates have on nations' economies. http://www.un.org/esa/population/publications/longrange2/WorldPop2300final.pdf The above link provides an example of such a study. Students should be able to identify the population function and growth rate functions' graphs in the document.

Unit 3: Applications of Derivatives

AP Calculus AB Standards: 3.3-3.4, 4.1-4.6, 5.1	
Big Ideas: Students will continue to apply what they learned in the last unit about implicit differentiation as a way to determine the derivatives of famous "inverse functions". Word problems that involve implicitly defined equations relating quantities that are functions of time can be solved using implicit differentiation. Other applications of derivatives include linear and quadratic approximations, differentials, and the Mean Value Theorem.	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
	Students will understand that…
- What are inverse functions, and how can implicit differentiation be used to determine derivatives of any inverse of a function?	- Functions can easily be rewritten as their inverses. Derivatives can then be easily computed using implicit differentiation.
- What "famous" inverse functions are commonly used, and what are their derivatives?	- Derivative formulas for logarithms and inverse trig functions can be computed using implicit differentiation.
- How can problems that relate changing quantities that are functions of time be solved using calculus?	- Related rates problems can be solved by using implicit differentiation.
- How can complicated functions be modeled with simple linear or quadratic representations?	- Any differentiable function can be locally approximated by a polynomial.
- What is the Mean Value Theorem and how does it appear in our world?	- The Mean Value Theorem is what relates the average rate of change on an interval to instantaneous rates of change. Aside from obvious connections to motion and travel, this essential theorem connects both branches of calculus.

<table>
<tr><th>Areas of Focus: Proficiencies
(Cumulative Progress Indicators)</th><th>Examples, Outcomes, Assessments</th></tr>
<tr><td>Students will:</td><td rowspan="11">Instructional Focus: (Corresponding Textbook Sections)

1. Derivatives of Logarithms (3.6)
2. Derivatives of other Inverse Functions (3.6)
3. Related Rates (4.6)
4. Linearizations and Differentials (3.9)
5. The Mean Value Theorem (3.10)

Sample Assessments:

• Find a general formula for $\frac{d}{dx}f^{-1}(x)$.

• Use implicit differentiation to show: $$\frac{d}{dx}\arctan(x)=\frac{1}{1+x^2}$$

• Find $\frac{dy}{dx}$ for each:
a. $y=\ln(x^2)$
b. $y=\frac{\arcsin(x)}{x}$
c. $y=\arctan(xy)$

• If a balloon is a perfect sphere and you want the radius to increase at 2cm/sec, how fast must you be blowing air into the balloon when the radius is 3 cm?

• Given $f(x)=x^2$, find the value of x=c on [0, 2] that satisfies the conclusions of the mean value theorem.

• Create your own function that does not satisfy the hypotheses of Rolle's Theorem.

• Find the linearization of $f(x)=\sqrt{x}$ at $x=4$. Use the linearization to estimate $\sqrt{5}$. Is this approximation an over or under estimate? How do you know?</td></tr>
<tr><td>Determine, by definition, the derivatives of $y=ln(x)$ and then $y=\log_a(x)$.</td></tr>
<tr><td>Determine a general rule for the derivative of any inverse function.</td></tr>
<tr><td>Determine, by definition, the derivatives of all of the inverse trig functions.</td></tr>
<tr><td>Solve problems involving related rates.</td></tr>
<tr><td>Compute linear and quadratic approximations to functions at a point.</td></tr>
<tr><td>Correctly interpret the approximations found as being valid on larger intervals for higher degree polynomials.</td></tr>
<tr><td>Compute and interpret the value of a differential given a point and a distance from that point.</td></tr>
<tr><td>Understand and explain why the conditions of existence theorems are "sufficient but not necessary".</td></tr>
<tr><td>Understand and apply the Mean Value Theorem to real-life scenarios as well as mathematical exercises.</td></tr>
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	• Relevant AP Questions: a. Open Ended: 2010 AB 2a, 2007 AB 3d, 2010 AB 2a,c (Form B) b. Multiple Choice: 2003 AB 1, 4, 7, 9, 13, 14, 16, 24; 1997 AB 76, 79, 80, 86, Sample Project: • Students will find both a linear and quadratic model for a function. They will create a graph with all three representations on a reasonable scale. The project will ask students to compare the two models based on their accuracy, error, and intervals on which these approximations are reasonable. Students will then research Taylor Series and learn about how higher-degree polynomials can provide better local approximations to functions. Instructional Strategies: Interdisciplinary Connections • Students will be analyzing the derivatives of functions that are mathematical models for scenarios in the social sciences, physics, biology, and economics. Technology Integration • TI graphing calculators will be used extensively to create graphs and tables, as well as assist in numerically evaluating the derivative of a function at a point. This skill is critical for success on the AP Calculus AB exam. • http://www.geogebra.org • http://www.desmos.com The above websites allow students to graph functions, and then easily graph the derivative of the function or numerically evaluate the derivative of the function at a point.

	Media Literacy Global Perspectives

Unit 4: Optimization

AP Calculus AB Standards: 4.7, 5.2-5.12	
Big Ideas: Students will learn how to completely analyze functions using derivatives. Features such as extreme values, intervals that increase and decrease, intervals that are concave up and concave down, and inflection points will be included in this analysis. Students will then use these skills to solve real-world problems that involve optimization of resources such as packing materials, volumes, and time to complete a series of tasks. L'Hopital's Rule will be investigated as a final application of differentiation.	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
	Students will understand that…
- How does the second derivative help describe concavity?	- Sign changes across the second derivative of a function indicate changes in concavity of said function.
- Are extrema of a function guaranteed?	- The Extreme Value Theorem's conditions must be met in order to guarantee extrema.
- What methods are available for determining extreme values of functions?	- The First Derivative Test, the Second Derivative Test, and the Candidate Test can all be used to determine extrema of a function.
- What applied problems can be solved using the tests in this unit?	- Any problem asking to find a maximum or minimum value of a quantity can be solved using calculus if there exists a continuous function modeling such quantity.
- What is L'Hopital's Rule?	- L'Hopital's Rule is a method of resolving certain indeterminate forms by using derivatives.

<table>
<tr><th>Areas of Focus: Proficiencies
(Cumulative Progress Indicators)</th><th>Examples, Outcomes, Assessments</th></tr>
<tr><td>Students will:</td><td rowspan="13">Instructional Focus: (Corresponding Textbook Sections)

1. Concavity (2.5)
2. The First and Second Derivative Tests (4.1)
3. Extreme Value Theorem (4.2)
4. Optimization (4.3)
5. L"Hopital's Rule (4.7)

Sample Assessments:

• Given $f(x) = x^{5/3} + 5x^{2/3}$, identify all critical points, inflection points, intervals increasing/decreasing, and intervals concave up/down.

• Classify the critical points in the example above as local maxima, local minima, or neither. Justify using both the First and Second Derivative Tests.

• Draw a function on a closed interval [a,b] that does not have an absolute maximum. Explain what condition(s) of the EVT are not satisfied.

• Use the Candidate Test to determine the absolute minimum value of $f(x) = 3x^3 - 2x$ on [-1, 1].

• A rectangular box with a square base and no top is to be made from a total of 120 square centimeters of cardboard. What are the dimensions of the box with the maximum volume?

• John is on a boat 150 yards from the shore. His house is on the shoreline, 400 yards from the point on the shore perpendicular to the boat. If he travels on the water 4 feet per second but runs 5 feet per second on land, how far down the shoreline should he dock his boat so he makes it home as fast as possible?</td></tr>
<tr><td>Determine inflection points by using the second derivative of a function.</td></tr>
<tr><td>Determine where a function is concave up and concave down by analyzing the second derivative.</td></tr>
<tr><td>Understand the conditions and be able to apply the First Derivative Test</td></tr>
<tr><td>Understand the conditions and be able to apply the Second Derivative Test</td></tr>
<tr><td>Provide examples of extrema where the First or Second Derivative test is inconclusive.</td></tr>
<tr><td>Understand the sufficient but necessary conditions of the Extreme Value Theorem</td></tr>
<tr><td>Provide examples of functions where the EVT can and cannot be applied</td></tr>
<tr><td>Use the "Candidate Test" as a method for determining absolute maximum and minimum values under the conditions of the EVT</td></tr>
<tr><td>Write single-variable functions for quantities that need to be optimized in real-world geometry problems ("Optimization problems")</td></tr>
<tr><td>Use the appropriate test to solve optimization problems</td></tr>
<tr><td>Analyze graphs of f ' (x) to determine features of f (x) such as critical points, inflection points, intervals that are increasing/decreasing, intervals that are concave up/down, and extreme values.</td></tr>
<tr><td></td></tr>
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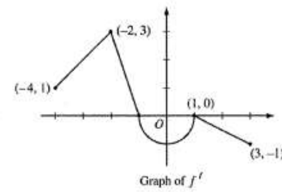
	•  Given the graph above, for f (x) determine the location of: local maxima, local minima, inflection points, intervals increasing, intervals decreasing, intervals concave up, intervals concave down • Relevant AP Questions: a. Open Ended Questions: 2010 AB 5b, c, 2006 AB 2 (Form B), 2011 AB 5, 2008 AB 5, 2010 AB 6, 2005 AB 2 (Form B), 2009 AB 1, 2007 AB 4, 2011 AB 3, 2009 AB 4 b. Multiple Choice Questions: 1997 AB 5, 8, 9, 16, 20, 22, 23; 2003 AB 76, 78, 81, 82, 83, 84, 86, 87, 88, 91, 92 Sample Project: • Students will analyze a product in a cylindrical container from a store. Holding the volume constant, students will determine the dimensions of the cylinder that minimizes surface area. Students will then analyze their results and consider additional constraints such as material, packing, transportation, and other logistical possibilities. Instructional Strategies: Interdisciplinary Connections • Students will be analyzing the derivatives of functions that are mathematical models for scenarios in the social sciences, physics, biology, and economics. Technology Integration • TI graphing calculators will be used extensively to create graphs and tables, as well as assist in numerically evaluating the derivative of a function

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Unit 5: The Definite Integral

AP Calculus AB Standards: 6.2-6.3, 6.6-6.7, 8.1	
Big Ideas: Students will understand a definite integral to be a limit of a Riemann Sum, or alternately as the net accumulation of change. Students will discover the inverse relationship between the derivative and the definite integral in their study of the Fundamental Theorem of Calculus. Algebraic, geometric, and technological strategies will be used to evaluate definite integrals. Students will understand an apply properties of and relationships between definite integrals, including the average value of a function.	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
	Students will understand that…
- What is the difference between "total area" and "net area"?	- In calculus, area below the x-axis is represented as negative, due to the negative value of the function bounding the area.
- How can displacement be retrieved from a function that measures velocity?	- The product of the width of the time interval and the height of the bounded function gives the displacement. This simple computation is only practical in a constant function, resulting in the area of a rectangle.
- How can displacement be geometrically interpreted on a graph of time vs. velocity? How is this different from total distance traveled?	- Displacement can be geometrically interpreted as the net area between the velocity curve and the x-axis. The total distance traveled is simply the *total* area between the velocity curve and the x-axis.
- How can irregular areas be approximated?	- Irregular areas can be approximated by building either many rectangles or many trapezoids with bases on the x-axis and heights determined by the function values. When using rectangles, this is called a Riemann Sum.
- How can this approximation be improved?	- The approximation is improved by taking the limit of the rectangles in the Riemann Sum to zero.
- How are definite integrals and derivatives related to each other?	- The Fundamental Theorem of Calculus both relates derivatives to definite integrals as well as provides an algebraic method for evaluating definite integrals.

- How can definite integrals be exactly evaluated?	- An antiderivative evaluated from the lower to upper bound of the interval provides the exact value of a definite integral.
- Does a definite integral always represent area?	- The value of definite integral can be interpreted and displayed as area on a rate vs. time graph, but need not represent area.
- How can we find the average of an infinite number of values along a function?	- How can we find the average of an infinite number of values along a function?

Areas of Focus: Proficiencies (Cumulative Progress Indicators)	**Examples, Outcomes, Assessments**
Students will:	Instructional Focus: 1. Distance Traveled (5.1) 2. The Definite Integral (5.2) 3. Definite Integrals Numerically (7.5) 4. Formal Definition of Antiderivative and Indefinite Integrals (5.4) 5. The Fundamental Theorem of Calculus (5.3) 6. Theorems about Definite Integrals (5.4) 7. Average Value of a Function (5.4) Sample Assessments: • Let $v(t) = 100(1-0.9^t)$ be the velocity of a sports car. Plot the graph, then estimate the area between the graph and the x-axis on the domain [0, 10]. Interpret the value of this area in the context of the problem. • Given a table of data, estimate the value of the definite integral using rectangular and trapezoidal approximations. • Compute the antiderviative of $f'(x) = (3x-1)^2$
Define definite integrals as the limit of Riemann Sums.	
Interpret the definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval.	
Apply basic properties of definite integrals including additivity and linearity.	
Use the Fundamental Theorem of Calculus to evaluate definite integrals.	
Use the Fundamental Theorem of Calculus to represent a particular antiderviative, and the analytical and graphical analysis of functions so defined.	
Compute antiderivatives following directly from derivatives of basic functions.	
Compute antiderivatives by substitution of variables, including change of limits for definite integrals.	
Use Riemann Sums, using left, right, and midpoint evaluation points, to numerically approximate definite integrals of functions represented algebraically, graphically, and by tables of values.	
Use Trapezoidal Sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values.	

Use definite integrals to compute, and correctly interpret, the average value of a function over an interval.	• Estimate $\int_1^2 2^x dx$ using a left Riemann Sum with 4 equal partitions. Is this answer an over or under estimate for $\int_1^2 2^x dx$? Explain. Use www.geogebra.org to compute Riemann Sums as $n \to \infty$. Use these results to predict the exact value of $\int_1^2 2^x dx$. • If $g'(x) = f(x)$, prove $\int_a^b f(x)dx = g(b) - g(a)$ • Evaluate $\int_{-1}^{1} \sqrt{4x+5}\, dx$ using the Fundamental Theorem of Calculus. • If the force to stretch a spring x inches is given by F=0.6x, use a definite integral to compute the amount of work (in inch-lbs) to stretch the spring from 0 to 9 inches. • Find the average value of $f(x) = x^3$ on the interval [0, 2]. Explain why the average value is not 4, the midpoint of the y coordinates. • Relevant AP Questions: a. Open Ended: 2010 AB 2c, 5a, 2009 AB 6b (Form B), 2007 AB 2a, 2b, 3c, 2009 AB 2, 5b b. Multiple Choice: 2003 AB 2, 5, 8, 11, 23, 77, 85, 92 Sample Project: • In pairs, students will be asked to go on a “road trip” for 15 minutes. One student will drive, and the other will simply record data. At predetermined intervals, the recorder will write down the car’s speed as well as the odometer

	readings at the beginning and end of the trip. With this discrete data, students will create a table of values from which they will estimate the total distance traveled during their trip. Students will use rectangular and trapezoidal estimates. Then, students will use their TI graphing calculator to create a regression equation for their velocity during the trip. The definite integral of this equation on the appropriate time interval will give a different estimate for the total distance traveled. Finally, students will be asked to estimate their acceleration at various times. Instructional Strategies: Interdisciplinary Connections • Students will be analyzing the derivatives and integrals of functions that are mathematical models for scenarios in the social sciences, physics, biology, and economics. The relationships between the derivatives and integrals of the different functions will be discussed in the context of the model. Technology Integration • TI graphing calculators will be used extensively to create graphs and tables, as well as assist in numerically evaluating the definite integral of a function on an interval. This skill is critical for success on the AP Calculus AB exam. • http://www.geogebra.org, http://www.wolframalpha.com/ The above websites allow students to graph functions and then easily graph a Riemann or trapezoidal sum with n partitions, or even compute the exact value of a definite integral.

	Media Literacy • Students will find examples of articles or news stories where graphs that provide a rate generate information about an amount. [For example, the area under a graph that has time on the x-axis and cases per day of a disease on the y-axis measures the total number of cases] Global Perspectives • Students will research and learn about Zeno's Dichotomy Paradox and examine how it relates to infinitesimal quantities, infinite sums, and definite integrals.

Unit 6: Antiderviatives and The Fundamental Theorem Part 2

AP Calculus AB Standards: 6.1, 6.4-6.5, 6.8-6.10, 6.14, 8.2
Big Ideas: Students will learn the second part of the Fundamental Theorem of Calculus, which fully connects the derivative and the integral as inverse mathematical operations. In doing so, students will study accumulation functions and observe their structure and properties. Students will also learn elementary and advanced methods for working out antiderivatives as a means of applying the Fundamental Theroem of Calculus.

Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
	Students will understand that…
• What are accumulation functions?	- Accumulation functions are functions whose input is a variable or function in the upper bound of a definite integral.
- How are definite integrals and derivatives related to each other?	- The derivative and definite integral are inverse operations: $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$
- What are some simple techniques for working out antiderivatives?	- Antiderivatives can be found by using known differentiation rules backwards as well as by applying algebraic techniques to the integrand first.
- How can the Chain Rule be worked backwards?	- The method of "u-substitution" can be used to work the Chain Rule backwards.
- How can calculus help solve physics problems involving projectile motion?	- Antiderivatives of acceleration functions can generate both velocity and position functions.

Areas of Focus: Proficiencies (Cumulative Progress Indicators)	**Examples, Outcomes, Assessments**
Students will:	Instructional Focus: 1. Graphs of Antiderivatives (6.1) 2. Algebraic Antidervatives (6.2) 3. Projectile Motion (6.3) 4. Accumulation Functions (6.4) 5. Fundamental Theorem of Calculus Part 2 (6.4)
Graph and interpret the behavior of accumulation functions	
Determine minima, maximia, inflection points, and intervals of change and concavity of accumulation functions	

<table>
<tr><td>Understand that the Fundamental Theorem Part 2 establishes derivatives and integrals of as inverse operations.</td><td rowspan="7">6. U-Substitution (7.1)

Sample Assessments:

• Compute the antiderviative of $f'(x) = (3x-1)^2$

• Evaluate the indefinite integral:
a. $\int 9x^{-3}\,dx$
b. $\int \sin r\,dr$
c. $\int 3.4e^{-2x}\,dx$
d. $\int \frac{\ln^2 x}{x}dx$

• Compute the derivatives:
a. $\int_1^x t^2\,dt$
b. $\int_{\sin(x)}^0 \arcsin(t)\,dt$

• Use antiderivatives to determine how long a ball it will take for a ball to hit the ground if it is launched with an initial velocity of 20 m/s with an acceleration due to gravity of -9.8 m/s^2.

• Relevant AP Questions:
a. Open Ended: 2010 AB 2c, 5a, 2009 AB 6b (Form B), 2007 AB 2a, 2b, 3c, 2009 AB 2, 5b
b. Multiple Choice: 2003 AB 2, 5, 8, 11, 23, 77, 85, 92

Sample Project:

• Students will draw a graph comprised of line segments called $g(t)$. From there, they will determine the following for $f(x) = \int_0^x g(t)\,dt$
1. Local extreme values
2. Points of inflection
3. Intervals that are increasing and decreasing
4. Intervals that are concave up and concave down
5. The absolute extreme values</td></tr>
<tr><td>Find derivatives of accumulation functions using the Fundamental Theorem of Calculus Part 2, applying the chain rule when appropriate.</td></tr>
<tr><td>Use the Fundamental Theorem of Calculus to represent a particular antiderviative, and the analytical and graphical analysis of functions so defined.</td></tr>
<tr><td>Compute antiderivatives following directly from derivatives of basic functions.</td></tr>
<tr><td>Compute antiderivatives by substitution of variables, including change of limits for definite integrals.</td></tr>
<tr><td>Apply antiderivatives to physics problems involving projectile motion.</td></tr>
<tr><td>Compute antiderivatives by first applying algebraic manipulations such as expanding polynomials, canceling factors, and applying trigonometric identities.</td></tr>
</table>

	Instructional Strategies: Interdisciplinary Connections • Students will be able to show that commonly used formulas in physics can be generated using calculus. Technology Integration • TI graphing calculators will be used extensively to create graphs and tables, as well as assist in numerically evaluating the definite integral of a function on an interval. This skill is critical for success on the AP Calculus AB exam. • http://www.geogebra.org, http://www.wolframalpha.com/ The above websites allow students to graph and analyze accumulation functions. Teacher-generated applets and animations will be provided. Media Literacy - Students will be asked to find examples of accumulation functions (functions defined by integrals) that are used as models in the physical sciences. Students will be asked to research academic sources to explain the significance of the function in the particular field of study. Global Perspectives

Unit 7: Area and Volume

AP Calculus AB Standards: 8.4-8.12	
Big Ideas: Students will study geometric problems involving finding area between curves, volumes of objects with geometric cross-sections, and volumes of objects with rotational symmetry. Students will also use calculus to generate and justify famous formulas from geometry.	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
	Students will understand that…
- How can areas between curves be found? How are these areas different than definite integrals? How are definite integrals used to solve such problems?	- Integrals that involve differences in curves bounding regions will determine the area of the bounded region.
- How can calculus be used to determine the volume of objects using cross-sections?	- Integrals of the generalized area of an object's cross-sections give the volume of the object.
- What is special about objects with rotational symmetry as it pertains to finding their volume?	- Objects with rotational symmetry always have circular cross-sections, allowing one to easily generalize their area and then compute the object's volume.
- What methods can be used to find the volume of solids of revolution?	- The "washer" method uses circular cross-sections while the "shell" method uses thin cylinders to break down the solid before integrating.
- Where do commonly used volume formulas come from (such as volume of cylinder, sphere, cone)?	- Integral calculus can be used to find the volume of any object with geometrically similar cross-sections. These objects have circular cross-sections, making the procedure particularly simple.

Areas of Focus: Proficiencies (Cumulative Progress Indicators)	Examples, Outcomes, Assessments
Students will:	Instructional Focus: 1. Area Between Curves (8.1) 2. Volume by Slicing (8.1) 3. Volume with Known Cross-Sections (8.1) 4. Solids of Revolution: Disks (8.2) 5. Solids of Revolution: Washers (8.2) 6. Solids of Cylindrical Shells (not in text) Sample Assessments: • Set up an integral that represents the area of the region bounded by $y = 2e^{0.2x}$ and $y = \cos x$ between 0 and 5. • The region bounded by $y = 4 - x^2$, $x = 0$, and $y = 0$ is rotated about the line x=3. Find the resulting volume using both the method of washers and cylindrical shells. • Find a region bounded by a system of equations than when rotated about an axis generates a cone with a height of 10 and a base radius of 4. Then, find the volume using either shells or washers. Show that your answer matches the expected result using the formula for the volume of a cone learned in geometry. • Relevant AP Questions: a. Open Ended: 2015 AB #2a, 2009 AB Form B #4c, 2010 AB #4b, 2011 AB #3c, 2013 AB #5b b. Multiple Choice: 2015 AB #21, 26, 81, 2017 AB #23, 2018 AB #29, 86 Sample Projects: • Students will analyze the packaging of a product. Given a fixed volume and the geometric shape of the packaging, students will compute the optimal dimensions of the
Find the exact area bounded by several curves.	
Find the volume of solid objects with known geometric cross-sections.	
Draw and visualize cross-sections of solids of revolution.	
Find the volume of solids of revolution using the method of disks.	
Find the volume of solids of revolution using the method of washers.	
Find the volume of solids of revolution using the method of cylindrical shells.	
Determine the easiest and fastest method for the finding the volume of a solid of revolution.	
Use integrals to find formulas for the volume of a sphere, cylinder, cone, and square pyramid.	

	packaging that would minimize surface area, and in turn, help to minimize packaging cost. These optimum dimensions will be compared to the actual dimensions of the packaging. Students will be asked to consider logistical restrictions in describing why the company chose to not use the optimal dimensions. • Students will create a 3-dimensional model for a volume with known cross-sections problem that was solve in class or on homework. A sheet of paper or cardboard should hold the 2-dimensional coordinate plane with the bounding functions drawn to scale. Several cross-sections should be attached to the coordinate plane and fastened so that they are perpendicular to the axis described in the problem. Instructional Strategies: Interdisciplinary Connections • Students will re-examine formulas from geometry and physics and validate them using calculus. Technology Integration • Geogebra (www.geogebra.org/) and Wolfram Alpha (www.wolframalpha.com/) can be used to graph the curves bounding a region that is then rotated to form a solid of revolution. Teacher-created Geogebra applets will also display the solid objects after rotation. Media Literacy Global Perspectives • Students will research how the ancient Egyptians and Babylonians found precise volumes without the formal use of calculus. Students will also make connections to how these cultures' contributions to the fields of geometry led us to the eventual discovery of calculus.

Unit 8: Differential Equations

<table>
<tr><th colspan="2">AP Calculus AB Standards: 7.1-7.4, 7.6-7.8</th></tr>
<tr><td colspan="2">Big Ideas: Students will apply the skills learned in units 1-6 in working with differential equations which model changes in various quantities with respect to time. Students will analyze differential equations through three types of solution methods: algebraic, graphic, and numeric. Advantages and shortcomings of each of these methods will be considered on a wide array of differential equations. Students will apply these skills to problems that occur in biology, chemistry, economics, and finance.</td></tr>
<tr><th>Essential Questions
What provocative questions will foster inquiry, understanding, and transfer of learning?</th><th>Enduring Understandings
What will students understand about the big ideas?</th></tr>
<tr><td>- How can functions be written that model common rates of change?

- What does the solution to a differential equation look like?

- How can differential equations be solved algebraically?

- How can differential equations be solved numerically?

- How can differential equations be graphed while considering all possible initial conditions?

- Why bother with Euler's Method or slope fields if they do not provide an exact solution to the differential equation?</td><td>Students will understand that…

- Differential equations are equations that use a derivative as a variable representing a rate of change. The rate of change of special growth patterns such as linear, exponential, and logistic growth can be easily modeled using a differential equation.

- Solutions to differential equations are either families of functions (in the absence of an initial condition) or a particular function given an initial condition.

- Differential equations that are "separable" can be solving using the method of separation of variables.

- All differential equations' solutions, given an initial condition, can be approximated using Euler's Method.

- A slope field is a visual representation of the family of solutions to a given differential equation.

- Numeric and graphic solution methods such as these allow</td></tr>
</table>

Areas of Focus: Proficiencies (Cumulative Progress Indicators)	Examples, Outcomes, Assessments
Students will:	Instructional Focus: 1. What is a Differential Equation (11.1) 2. Slope Fields (11.2) 3. Euler's Method (11.3) 4. Separation of Variables (11.4) 5. Growth and Decay (11.5) Sample Assessments: • The rate of growth of a species of bacteria is directly proportional to the amount of bacteria present. If there were initially 5 million bacteria and after 3 minutes there are 7 million bacteria, how many bacteria are present after 10 minutes? • Solve the differential equation: a. $\frac{dy}{dx} = xy$ b. $\frac{dx}{dt} = \frac{x}{t}$ c. $\frac{dW}{dt} = F - kW$ • Match 5 given differential equations with 5 given slope fields. • Draw a slope field for $\frac{dy}{dx} = x - 1$. • A savings account accrues 1% annual interest, compounded continuously. Write a differential equation that models the change in the balance in the account t years after an initial deposit of $5,000. Solve the differential equation to find a function that models the account balance t years after the initial deposit. How long will it take the account to double? Can you generalize this into a "doubling formula" for these types of savings accounts with interest rate r% and initial deposit $D.
Understand and apply the Mean Value Theorem, as well as the "Mean Value Theorem for Integrals"	
Completely analyze the features of a function, including notions of monotonicity and concavity.	
Optimize functions, identifying both absolute and relative extrema.	
Interpret the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.	
Model the geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations.	
Use integrals appropriately to model physical, biological, or economic situations.	
Solve problems involving integrals with specific applications including finding area of a region, the volume of a solid with a known cross section, the average value of a function, the distance traveled by a particle along a line, and the accumulated change from a rate of change.	
Find specific antiderivatives using initial conditions, including applications to motion along a line.	
Solve separable differential equations and use them in modeling.	

	• Relevant AP Questions: a. Free Response: 2002 AB Form B #5b, 2011 AB #5c, 2012 AB # 5c, 2014 #6c b. Multiple Choice: 2015 AB # 8, 18, 24, 2018 AB #22, 25, 30 Sample Projects: • Students will solve a separable differential equation (randomly assigned) in three different ways: separation of variables, Euler's Method, and by drawing slope fields. Then, write an analysis comparing and contrasting the three methods. Finally, each student should come up with his/her own examples of a differential equation that is ideally suited for each of the three methods. Instructional Strategies: Interdisciplinary Connections • Students will investigate how differential equations are commonly used to model the rate of change of populations within a certain ecosystem (including humans!). Constant, linear, exponential, and logistic growth rates will be examined. Students will be asked to discuss how limiting factors in the environment of the species may or may not inhibit unbounded population growth. Technology Integration • The TI-89 graphing calculator will be used to quickly sketch slope fields for a given differential equation. • Students will learn how to program a spreadsheet in Microsoft Excel to model

	many computations from Euler's Method. The resulting data points from Euler's Method can be graphed on Excel, creating an approximate solution to a given differential equation. • Geogebra (www.geogebra.org/) and Wolfram Alpha (www.wolframalpha.com/) can be used to graph the slope fields of complex differential equations, as well as find solutions to differential equations given an initial condition. Media Literacy • Students will be asked to research the spread of a recent epidemic and discuss whether or not the epidemic followed an exponential, logistic, or different growth rate. Then students will be asked to find a differential equation and particular solution that models the rate of change and the number infected, respectively. Global Perspectives • The population growth rates for different nations will be used to motivate the topic of differential equations.

Required Texts and Resources:

Hughes-Hallett, Deborah. Calculus: Single Variable, 6th ed. Wiley, 2012.

TI-83 Plus Graphing Calculator. Texas Instruments.

TI-89 Graphing Calculator. Texas Instruments.

College Board course resources: https://apstudents.collegeboard.org/courses/ap-calculus-ab

Summit Public Schools

Summit, New Jersey

Curricular Addendum

Career-Ready Practices	Interdisciplinary Connections
CRP1: Act as a responsible and contributing citizen and employee. **CRP2**: Apply appropriate academic and technical skills. **CRP3**: Attend to personal health and financial well-being. **CRP4**: Communicate clearly and effectively and with reason. **CRP5**: Consider the environmental, social and economic impacts of decisions. **CRP6**: Demonstrate creativity and innovation. **CRP7**: Employ valid and reliable research strategies. **CRP8**: Utilize critical thinking to make sense of problems and persevere in solving them. **CRP9**: Model integrity, ethical leadership and effective management. **CRP10**: Plan education and career paths aligned to personal goals. **CRP11**:. Use technology to enhance productivity. **CRP12**: Work productively in teams while using cultural global competence.	• Close Reading of works of art, music lyrics, videos, and advertisements • Use Standards for Mathematical Practice and Cross-Cutting Concepts in science to support debate/inquiry across thinking processes **Technology Integration** Ongoing: • Listen to books on CDs, Playaways, videos, or podcasts if available. • Use document camera or overhead projector for shared reading of texts. Other: • Use Microsoft Word, Inspiration, or SmartBoard Notebook software to write the words from their word sorts. • Use available technology to create concept maps of unit learning.

Instructional Strategies:
Supports for English Language Learners:

Sensory Supports	Graphic Supports	Interactive Supports
Real-life objects (realia)	Charts	In pairs or partners
Manipulatives	Graphic organizers	In triads or small groups
Pictures & photographs	Tables	In a whole group
Illustrations, diagrams, & drawings	Graphs	Using cooperative group structures
Magazines & newspapers	Timelines	With the Internet (websites) or software programs
Physical activities	Number lines	In the home language
Videos & films		With mentors
Broadcasts		
Models & figures		

from https://wida.wisc.edu

Media Literacy Integration

- Use multiple forms of print media (including books, illustrations/photographs/artwork, video clips, commercials, podcasts, audiobooks, Playaways, newspapers, magazines) to practice reading and comprehension skills.

Global Perspectives

- The Global Learning Resource Library

Differentiation Strategies:

Accommodations	Interventions	Modifications
Allow for verbal responses	Multi-sensory techniques	Modified tasks/ expectations
Repeat/confirm directions	Increase task structure (e.g., directions, checks for understanding, feedback)	Differentiated materials
Permit response provided via computer or electronic device	Increase opportunities to engage in active academic responding (e.g., writing, reading aloud, answering questions in class)	Individualized assessment tools based on student need
Audio Books	Utilize prereading strategies and activities: previews, anticipatory guides, and semantic mapping	Modified assessment grading