AP Calculus Blizzard Bag #3 Name___________________________
Complete the following showing all work Date/Block______________________

Solve each of the following problems, using available space for work. After examining the form of the choices, decide which is the best of the choices given. Calculators may **NOT** be used on this part of the exam. All work shown must be in a neat and organized manner.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

15. If $\int_1^7 \ln x\,dx$ is approximated by 3 circumscribed rectangles of equal width on the $x-axis$, then the approximation is

(A) $\frac{1}{2}(\ln 3+\ln 5+\ln 7)$

(B) $\frac{1}{2}(\ln 1+\ln 3+\ln 5)$

(C) $2(\ln 3+\ln 5+\ln 7)$

(D) $2(\ln 3+\ln 5)$

(E) $\ln 1+2\ln 3+2\ln 5+\ln 7$

16. The volume of an expanding sphere is increasing at a rate of 12 cubic feet per second. When the volume of the sphere is 36π cubic feet, how fast, in square feet per second, is the surface area increasing?

Note: ($V=\frac{4\pi r^3}{3}$ *and* $S=4\pi r^2$)

(A) 8

(B) 6

(C) 8π

(D) $\frac{8\pi}{3}$

(E) 10

17. If $f(x) = 15 - g(x)$ for $-2 \le x \le 2$, then $\int_{-2}^{2} [f(x) - g(x)]\, dx =$

(A) 60

(B) $2\int_{-2}^{2} g(x)\, dx - 60$

(C) $2\int_{-2}^{2} g(x)\, dx + 60$

(D) $60 - 4\int_{0}^{2} g(x)\, dx$

(E) $60 - 2\int_{-2}^{2} g(x)\, dx$

18. $\int_{2}^{4} \left[\frac{d}{dt}\left(3t^2 + 2t - 1\right) \right] dt =$

(A) 12

(B) 40

(C) 46

(D) 55

(E) 66

19. The velocity of a particle moving along the $x-axis$ is given by a third-degree polynomial $P(t)$. The roots of $P(t)$ are all in the open interval $0<t<a$. Which of the following statements must be true?

I. The velocity of the particle will be zero at least once and at most three times for $0<t<a$.
II. In the interval $0<t<a$, the particle moves both left and right.
III. The total distance traveled by the particle from $t=0$ *to* $t=a$ is given by $\int_0^a P(t)\,dt$.

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I, II, and III

20. Let f be a function that is differentiable on the open interval (a,b). If f has a relative minimum at $(c, f(c))$ and $a<c<b$, which of the following must be true?

I. $f'(c)=0$
II. $f''(c)$ must exist
III. If $f''(c)$ exists, then $f''(c)>0$

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I and III only

21. A solid has a circular base of radius 3. If every plane cross section perpendicular to the $x - axis$ is an equilateral triangle, then its volume is

(A) 36

(B) $12\sqrt{3}$

(C) $18\sqrt{3}$

(D) $24\sqrt{3}$

(E) $36\sqrt{3}$

22. If the substitution $u = 25 - x^2$ is made, the integral $\int_0^3 x\sqrt{25 - x^2}\, dx =$

(A) $\frac{1}{2}\int_0^3 \sqrt{u}\, du$

(B) $\frac{1}{2}\int_{25}^{16} \sqrt{u}\, du$

(C) $-\frac{1}{2}\int_0^3 \sqrt{u}\, du$

(D) $\frac{1}{2}\int_{16}^{25} \sqrt{u}\, du$

(E) $2\int_{16}^{25} \sqrt{u}\, du$

23. If $y = \arcsin\left(\frac{3x}{4}\right)$, then $\frac{dy}{dx} =$

(A) $\frac{-3}{\sqrt{16-9x^2}}$

(B) $\frac{12}{16+9x^2}$

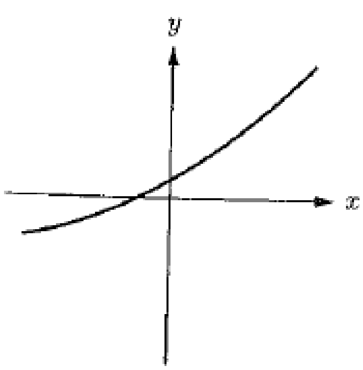
(C) $\frac{4}{\sqrt{16-9x^2}}$

(D) $\frac{12}{\sqrt{16-9x^2}}$

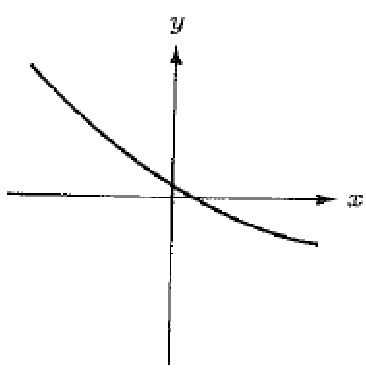
(E) $\frac{3}{\sqrt{16-9x^2}}$

24. Which of the functions sketched below is increasing at a decreasing rate?

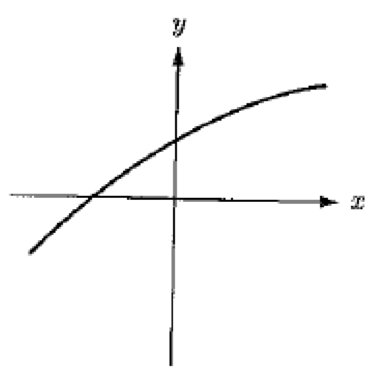
(A)



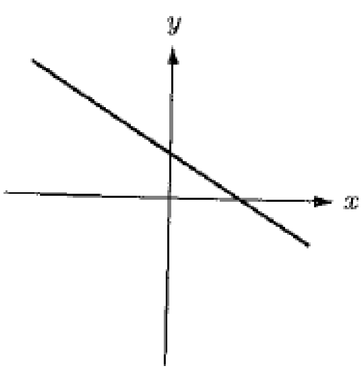
(B)



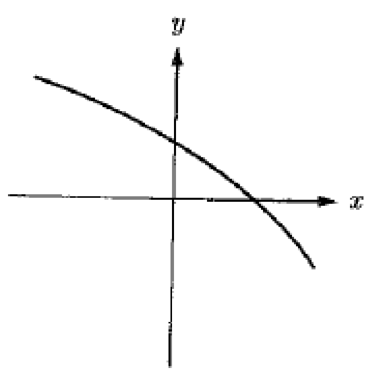
(C)



(D)



(E)



25. If at each point (x, y) on a certain curve, the slope of the curve is $\frac{2y}{x}$, then the curve is a(n)

(A) straight line

(B) parabola

(C) circle

(D) ellipse

(E) hyperbola

26. $\lim_{h \to 0} \frac{\tan(2(x+h)) - \tan(2x)}{h}$ is

(A) 0

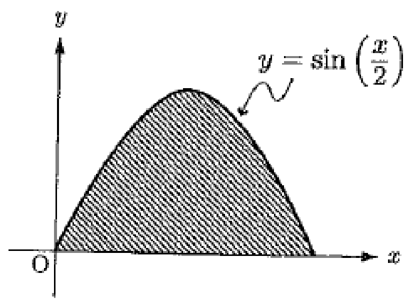
(B) $2\cot(2x)$

(C) $\sec^2(2x)$

(D) $2\sec^2(2x)$

(E) nonexistent

27. What is the area of the shaded region in the figure below?



(A) 2
(B) π
(C) 4
(D) $2\pi - 1$
(E) 2π

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28. Let f be a continuous function whose derivative is given by

$$f'(x) = \begin{cases} x^2, & x \le 2 \\ \frac{1}{2}x + 3, & x > 2 \end{cases}$$

For what values of x does the graph of f have a point of inflection?

(A) 0 only
(B) 2 only
(C) 4 only
(D) 0 and 2 only
(E) 0, 2, and 4