Curriculum Management System

MONROE TOWNSHIP SCHOOLS

Course Name: Advanced Placement calculus AB/BC
Grade: 11-12

For adoption by all regular education programs as specified and for adoption or adaptation by all Special Education Programs in accordance with Board of Education Policy # 2220.

Board Approved: <Type Date Here>

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Monroe Township Schools Administration and Board of Education Members

ADMINISTRATION

Dr. Kenneth R. Hamilton, Superintendent
Dr. Jeff C. Gorman, Assistant Superintendent

BOARD OF EDUCATION

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Mr. Ira Tessler, Board Vice President
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Jamesburg Representative
Mr. Robert Czarneski

WRITERS NAME

Mrs. Susan Okulewicz

CURRICULUM SUPERVISOR

Mrs. Susan Gasko

Mission, Vision, Beliefs, and Goals

Mission Statement

The Monroe Public Schools in collaboration with the members of the community shall ensure that all children receive an exemplary education by well-trained committed staff in a safe and orderly environment.

Vision Statement

The Monroe Township Board of Education commits itself to all children by preparing them to reach their full potential and to function in a global society through a preeminent education.

Beliefs

1. All decisions are made on the premise that children must come first.
2. All district decisions are made to ensure that practices and policies are developed to be inclusive, sensitive and meaningful to our diverse population.
3. We believe there is a sense of urgency about improving rigor and student achievement.
4. All members of our community are responsible for building capacity to reach excellence.
5. We are committed to a process for continuous improvement based on collecting, analyzing, and reflecting on data to guide our decisions.
6. We believe that collaboration maximizes the potential for improved outcomes.
7. We act with integrity, respect, and honesty with recognition that the schools serve as the social core of the community.
8. We believe that resources must be committed to address the population expansion in the community.
9. We believe that there are no disposable students in our community and every child means every child.

Board of Education Goals

1. Raise achievement for all students paying particular attention to disparities between subgroups.
2. Systematically collect, analyze, and evaluate available data to inform all decisions.
3. Improve business efficiencies where possible to reduce overall operating costs.
4. Provide support programs for students across the continuum of academic achievement with an emphasis on those who are in the middle.
5. Provide early interventions for all students who are at risk of not reaching their full potential.
6. To Create a 21st Century Environment of Learning that Promotes Inspiration, Motivation, Exploration, and Innovation.

Common Core State Standards (CSSS)

The Common Core State Standards provide a consistent, clear understanding of what students are expected to learn, so teachers and parents know what they need to do to help them. The standards are designed to be robust and relevant to the real world, reflecting the knowledge and skills that our young people need for success in college and careers. With American students fully prepared for the future, our communities will be best positioned to compete successfully in the global economy.

Links:

1. CCSS Home Page: http://www.corestandards.org
2. CCSS FAQ: http://www.corestandards.org/frequently-asked-questions
3. CCSS The Standards: http://www.corestandards.org/the-standards
4. NJDOE Link to CCSS: http://www.state.nj.us/education/sca
5. Partnership for Assessment of Readiness for College and Careers (PARCC): http://parcconline.org

Scope and Sequence
Advanced Placement Calculus AB/BC
Quarter 1

Unit Topics(s)

I. Limits and Rates of Change
   a. Average and Instantaneous Rates of Change
   b. Limits
   c. Continuity
   d. Tangent Lines and the Normal to a Curve
   e. Slope of a Curve at a Given Point

II. Derivatives
   a. Definition of a f'
   b. Relationship Between the Graphs of f and f'
   c. Differentiability
   d. Rules for Differentiation
   e. Velocity and Rate of Change
   f. Chain Rule and Implicit Differentiation
   g. Derivatives of Trigonometric Functions and Inverse Trigonometric, Exponential and Logarithmic Functions

III. Applications of Derivatives
   a. Extreme Values of functions
   b. Mean Value Theorem for Derivatives
   c. Analysis of Graphs using First and Second Derivatives
   d. Modeling and Optimization
   e. Linearization and Newton's Method
   f. Differentials and Change
   g. Related Rates

Scope and Sequence
Advanced Placement Calculus AB/BC
Quarter 2

Unit Topic(s)

I. Integrals
   a. Approximating Areas
   b. Definite Integrals and Antiderivatives
   c. Average Value of a Function
   d. Mean Value Theorem for Definite Integrals
   e. Area and Volume of a Function
   f. Connecting Differential and Integral Calculus
   g. The Fundamental Theorem of Calculus

II. Differential Equations
   a. Slope Fields
   b. Euler's Method
   c. Antiderivatives and the Indefinite Integral
   d. Techniques of Integration
   e. Separable Differential Equations
   f. Exponential Growth and Decay
   g. Logistic Growth

Scope and Sequence
Advanced Placement Calculus AB/BC
Quarter 3

Unit Topic(s)

I. Applications of Definite Integrals
   a. Net Change
   b. Motion on a Line
   c. Consumption Over Time
   d. Area in the Plane
   e. Volume Using Integrals
   f. Surface Area of a Solid of Revolution
   g. Lengths of Curves
   h. Work and Fluid Force

II. L'Hopital's Rule and Improper Integrals
   a. Sequences
   b. Indeterminate Forms and L'Hopital's Rule
   c. Relative Rates of Growth
   d. Improper Integrals

III. Infinite Series
   a. Geometric Series
   b. Power Series
   c. Taylor Series, Maclaurin Series and Taylor's Theorem
   d. Lagrange Form of the Remainder
   e. Euler's Formula
   f. Test for Convergence and Divergence
   g. Harmonic Series
   h. Test for Convergence at Endpoints
   i. Alternating Series (Leibniz's Theorem)
   j. Testing a Power Series for Convergence

IV. Parametric, Vector, and Polar Functions
   a. Parametric Functions
   b. Slope and Concavity
   c. Arc Length
   d. Vectors in the Plane
   e. Properties of Vector Operations
   f. Using Vectors to Model Motion in a Plane
   g. Velocity, Acceleration, Speed, Displacement, and Distance Traveled
   h. Polar functions
   i. Rectangular and parametric representations of polar curves
   j. Area Between Polar Curves

V. Formal Review of Advanced Applications of Derivatives and Integrals
   a. Spiral Applications and Practice Problems
   b. College Board Practice Problems
   c. College Board Practice tests
   d. Practice Problems from Other Sources

Scope and Sequence Advanced Placement Calculus AB/BC Quarter 4	
Unit Topic(s)	
I. Formal Review of Advanced Applications of Derivatives and Integrals a. Spiral Applications and Practice Problems b. College Board Practice Problems c. College Board Practice Tests d. Practice Problems from Other Sources	II. Real-Life Application Project a. Apply Derivatives and Integrals to Solve a Real-World Problem b. Use of Current Technology for Presentations

Unit 1-Limits and Rates of Change
Advanced Placement Calculus AB/BC
Stage 1 Desired Results

<table>
<tr>
<td rowspan="6">
ESTABLISHED GOALS
The student will be able to understand and apply the concepts of limits and rates of change.

<u>Common Core State Standards for Mathematics</u>

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums symmetries; end behavior; and periodicity
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
8. Write a function defined by an expression in different but equivalent forms to reveal and
</td>
<td colspan="2">Transfer</td>
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<tr>
<td colspan="2">Students will be able to independently use their learning to…
analyze functions and solve real world applications involving geometry, physics, business and economics.</td>
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<tr>
<td colspan="2">Meaning</td>
</tr>
<tr>
<td>UNDERSTANDINGS
Students will understand that…

Functions can behave differently at different points in their domain.
Limits can be used to analyze functions numerically, graphically, and analytically.
The concept of continuity plays an integral role in real life application problems.
</td>
<td>ESSENTIAL QUESTIONS

What is a limit?
How can we evaluate limits numerically, graphically, and analytically?
How are limits used to analyze functions?
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<tr>
<td colspan="2">Acquisition</td>
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<tr>
<td>Students will know…

Average and instantaneous speed
Definition of a limit
Properties of limits
One-sided and two-sided limits
Sandwich Theorem
Finite limits approaching infinity
Infinite limits approaching a constant
Definitions of vertical and horizontal asymptotes
End behavior models
Continuity at a point
Removable, jump, oscillating and infinite discontinuities
Continuous functions on an interval
Composites of continuous functions
Intermediate Value Theorem for
</td>
<td>Students will be skilled at…

Evaluating limits
Calculating average and instantaneous speed
Finding function values
Applying the properties of limits
Finding limits indirectly using the Sandwich Theorem
Calculating and verifying end behavior models for functions
Evaluating limits to identify vertical and horizontal asymptotes
Calculating the slope of a curve
Writing the equation of a tangent line to a curve at a given point
Writing the equation of a normal line to a curve at a given point
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</table>

explain different properties of the function. 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **Build new functions from existing functions** 3. Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x), f(kx),* and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. **Construct and compare linear, quadratic, and exponential models and solve problems** 1. Distinguish between situations that can be modeled with linear functions and with exponential functions. **Interpret expressions for functions in terms of the situation they model** 5. Interpret the parameters in a linear or exponential function in terms of a context. **Model periodic phenomena with trigonometric functions** 5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. **Interpret linear models** 9. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	Continuous Functions • Average rates of change • Slope of a curve • Tangent to a curve • Normal to a curve	• Calculating the average rate of change of a function.

Unit 1-Limits and Rates of Change
Advanced Placement Calculus AB/BC
Stage 2 - Evidence

Evaluative Criteria	Assessment Evidence
The following rubric will be applied to each problem on any summative assessment. 6 Complete and correct solution including documentation of all concepts. Appropriate units will be included as needed. 5 Solution contains one minor arithmetic or algebraic error but demonstrates complete understanding of necessary concepts. OR Complete and correct solution without units. 4 Solution contains more than one minor arithmetic or algebraic error but demonstrates complete understanding of necessary concepts. OR Complete and correct solution without units. 3 Solution contains one conceptual error or one or more major arithmetic or algebraic errors while demonstrating partial understanding of necessary concepts. 2 Limited understanding of necessary concepts while demonstrating proper mathematical skills. 1 Incorrect answer with no understanding of necessary concepts. 0 No answer is given.	PERFORMANCE TASK(S): Summative assessment questions should be open-ended and model the format established by The College Board. • Students will demonstrate ability to evaluate limits numerically, analytically and graphically, to use limits to determine continuity of a function at a given point, to find vertical and horizontal asymptotes, and to identify end behavior models. Students will solve real life problems using limits. • Students will demonstrate mastery of concepts on previous assessments in addition to writing equations of tangent and normal lines to a curve at a given point, sketching the graph of a function given conditions involving limits, and using limits to solve application problems.

The following is a student rubric to assess individual understanding during class activities. 4 - I understand completely and I can teach it to a classmate. 3 - I understand the concept but I do not think I can explain it to a classmate. 2 - I can complete the task with assistance. 1 - I need help!	OTHER EVIDENCE: Throughout each lesson student understanding will be assessed through informal questioning, formative assessments, and student's self-assessment. **Pre / Post-assessments -**Using limits to analyze a function Formative assessments should include but not be limited to the following concepts: • Given a function use limits to identify any points of discontinuity and all vertical and horizontal asymptotes. Use the information gathered to sketch a graph of the function. • Given a function find the slopes of the secant line through different points. Examine the change in slopes as the interval decreases and the secant approaches a tangent at a given point. Make a conclusion about the slope of the tangent to the curve at the given point.

Unit 1-Limits and Rates of Change
Advanced Placement Calculus AB/BC
Stage 3 – Learning Plan

Summary of Key Learning Events and Instruction

The teacher and students will use class discussion and small group cooperation to accomplish the following tasks:

- **Exploration:** Explore properties of limits of two functions by evaluating each one separately as $x \to c$ and comparing it to the limit of their product as $x \to c$
- Analyze a piecewise function by drawing its graph and using one-sided limits to determine whether a limit exists at a given point.
- **Activity**: Using geometric applications and one-sided limits to reinforce the Sandwich Theorem, students will evaluate the $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$.
- **Technology- Computer and/or iPad:** Use Calculus In Motion with Geometer's Sketchpad to explore limits at a constant and limits at infinity.
- Evaluate limits using various methods including graphical, analytical and numerical.
- Sketch the graph of a function that satisfies given conditions involving limits, including asymptotes.
- Evaluate limits using end-behavior models and evaluating continuity at $x = c$.
- **Exploration:** Explore the use of factoring to remove a discontinuity.
- **Technology- Computer and/or iPad:** Use Calculus In Motion with Geometer's Sketchpad to explore The Intermediate Value Theorem.
- Given a salary guide, transportation or parking fees defined using a greatest integer or piecewise function, graph the function and determine the intervals of continuity. Discuss the financial consequences of different time intervals.
- **Activity:** Given a function find the slopes of the secant line through different points. Examine the change in slopes as the interval decreases and the secant approaches a tangent at a given point. Make a conclusion about the slope of the tangent to the curve at the given point.
- Given the equation for the free fall of an object, find the speed of the object at a given time.
- Students will be encouraged to use alternative resources to prepare for the Advanced Placement Exam.
- **Technology- Computer and/or iPad:** Students will be encouraged to view the following videos to reinforce concepts discussed in class using the **VideoCalculus** application.
    - 2.1.2 Finding Limits Graphically
    - 2.2.2 Limits and Indeterminate Forms
    - 3.1.2 Finding Instantaneous Velocity

<u>Resources:</u>

Technology

Software:

Calculus In Motion™ software, Animations of calculus for the Geometer's Sketchpad v4, Audrey Weeks

The following websites offer video lessons on calculus topics:

http://online.math.uh.edu/HoustonACT/videocalculus/index.html

http://www.khanacademy.org
http://www.wolframalpha.com
http://m.socrative.com/student/#joinRoom

The following websites offer practice tests and test preparation models:
http://apcentral.collegeboard.com/apc/Controller.jpf
http://sat.collegeboard.org/practice/sat-question-of-the-day

i-Pad Applications
The following apps are available on student i-Pads.

- WolframAlpha
- VideoCalculus
- SketchExplorer

Activities
Activities referenced in this document have been created by Susan Gasko and Susan Okulewicz and can be found on the Mathematics shared teachers folder.

Explorations
Explorations referenced in this document can be found in the following text:
Calculus: Graphical, Numerical, Algebraic
Finney, Ross L.; Demana. Franklin D.; Waits, Bert K.; Kennedy, Daniel;
Pearson Education, Inc., 2012.
Teachers Resources;

- Teacher's Guide with Answers
- Teacher's AP Correlations and Preparation Guide
- Student Practice Workbook

Additional Sources for Review
Preparing for the Calculus AP Exam with Calculus: Graphical, Numerical, Algebraic, Barton, Brunsting, Diehl, Hill, Tyler, Wilson, Pearson Education, Inc., 2007.
Multiple-Choice & Free-Response Questions in Preparation for the AP Calculus (BC) Examination, 8th Edition, Lederman, David, D & S Marketing Systems, Inc., 2011.
Barron's AP Calculus, 11th Edition, Barron's Educational Series, Inc., 2012.
Barron's AP Calculus Flash Cards, Barron's Educational Series, Inc., 2008.
5 Steps to a 5 AP Calculus AB & BC, 2012-2013 edition, William Ma, McGraw Hill Companies, 2011.
Teaching AP Calculus, Lin McMulllin, D & S Marketing Systems, Inc., 2005.

Unit 2- Derivatives
Advanced Placement Calculus AB/BC
Stage 1 Desired Results

ESTABLISHED GOALS
The student will be able to find the derivative of a function and establish the connection between the derivative and the graph of the function.

<u>Common Core State Standards for Mathematics</u>

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums symmetries; end behavior; and periodicity*
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
8. Write a function defined by an expression in

Transfer

Students will be able to independently use their learning to…
Solve real world problems involving instantaneous rates of change, simple harmonic motion, motion along a line, sensitivity to change, growth and decay, and economics.

Meaning

UNDERSTANDINGS
Students will understand that…
- The derivative plays an essential role when modeling instantaneous change mathematically.
- The connection between a functions and its derivative can be explored using graphs, tables, and analytical methods.

ESSENTIAL QUESTIONS
- What is a derivative?
- How are derivatives used to model real life phenomena?

Acquisition

Students will know…
- The formal definition of a derivative
- The alternate definition of a derivative
- Derivative notation
- Relationships between the graph of a function and its derivative
- One-sided derivatives
- How a derivative might fail to exist at a point
- Differentiability implies local linearity
- The symmetric difference quotient
- Differentiability implies continuity
- The Intermediate Value Theorem for Derivatives
- Rules for differentiation
- Instantaneous rate of change
- Displacement
- Average velocity
- Instantaneous velocity

Students will be skilled at…
- Calculating derivatives using the definition of a derivative
- Calculating the slope of a curve at a point
- Graphing the derivative of a function given the graph of the original function
- Graphing the function given the graph of its derivative
- Graphing the derivative of a function given numerical data
- Identifying differentiability of a function at a point
- Identifying corners, cusps, discontinuities, vertical and horizontal tangents, and justifying their existence using limits
- Analyzing differentiable functions using a graphing calculator to explore local

different but equivalent forms to reveal and explain different properties of the function. 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **Build new functions from existing functions** 3. Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x), f(kx),* and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. **Construct and compare linear, quadratic, and exponential models and solve problems** 1. Distinguish between situations that can be modeled with linear functions and with exponential functions. **Interpret expressions for functions in terms of the situation they model** 5. Interpret the parameters in a linear or exponential function in terms of a context. **Model periodic phenomena with trigonometric functions** 5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. **Interpret linear models** 9. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	• Speed • Acceleration • Jerk • Marginal cost • Marginal revenue • Derivatives of trigonometric functions and their inverses • Chain rule • Slope of a parametric function • Implicit differentiation • Derivatives of exponential and logarithmic functions	linearity • Identifying various difference quotients that can be used to evaluate the derivative of a function • Approximating derivatives numerically and graphically • Applying The Intermediate Value Theorem • Finding the derivative of a constant function • Applying the power rule • Applying the constant multiple rule • Applying the sum and difference rule • Applying the product and quotient rules • Finding higher order derivatives • Analyzing particle motion along a line • Differentiating trigonometric functions and their inverses • Differentiating composite functions using the chain rule • Using implicit differentiation • Differentiating exponential and logarithmic functions

Unit 2- Derivatives Advanced Placement Calculus AB/BC Stage 2 - Evidence	
Evaluative Criteria	**Assessment Evidence**
The following rubric will be applied to each problem on any summative assessment. 6 Complete and correct solution including documentation of all concepts. Appropriate units will be included as needed. 5 Solution contains one minor arithmetic or algebraic error but demonstrates complete understanding of necessary concepts. OR Complete and correct solution without units. 4 Solution contains more than one minor arithmetic or algebraic error but demonstrates complete understanding of necessary concepts. OR Complete and correct solution without units. 3 Solution contains one conceptual error or one or more major arithmetic or algebraic errors while demonstrating partial understanding of necessary concepts. 2 Limited understanding of necessary concepts while demonstrating proper mathematical skills. 1 Incorrect answer with no understanding of necessary concepts. 0 No answer is given.	PERFORMANCE TASK(S): Summative assessment questions should be open-ended and model the format established by the College Board. • Students will demonstrate the ability to apply the definition of a derivative and to demonstrate understanding of differentiability. • Students will demonstrate the ability to apply the basic rules for differentiation, calculate derivatives of trigonometric functions, and apply derivatives to applications involving particle motion, sensitivity to change and economics. • Students will demonstrate mastery of concepts on previous assessments in addition to ability to apply the chain rule to these concepts. • Students will demonstrate mastery of all rules of differentiation including the use of chain rule and implicit differentiation; apply rules for differentiating inverse trigonometric functions, exponential functions and logarithmic functions.
The following is a student rubric to assess individual understanding during class activities. 4 - I understand completely and I can teach it to a classmate. 3 - I understand the concept but I do not think I can explain it to a classmate. 2 - I can complete the task with assistance. 1 - I need help!	OTHER EVIDENCE: Throughout each lesson student understanding will be assessed through informal questioning, formative assessments, and student's self-assessment. **Pre/Post-assessments -** Evaluating and graphing rates of change of a function Formative assessments should include but not be limited to the following concepts:

- Given the position function for a particle moving along a line the student will
  a. Find the displacement during a given time interval
  b. Find the average velocity on the same interval
  c. Find the instantaneous velocity at a given time
  d. Find the acceleration at the same time
  e. Determine the direction of the particle at any time on the interval
  f. Determine the location of the particle when the position function is at a minimum or maximum.

- Given a position function of a body moving in simple harmonic motion:
  a. Find the body's velocity, speed, acceleration and jerk at time t
  b. Describe the motion of the body

- Suppose the functions f and g and their first derivatives have the following values

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	0	-1	2	1
0	-1	-3	-2	4

Find the first derivative of the following combinations at the given value of x.

$a)\ \ 3f(x)-g(x), \qquad x=-1$

$b)\ \ f^2(x)g^3(x), \qquad x=0$

$c)\ \ g(f(x)), \qquad x=-1$

$d)\ \ f(g(x)), \qquad x=-1$

$e)\ \ \dfrac{f(x)}{g(x)+2}, \qquad x=0$

$f)\ \ g(x+f(x)), \qquad x=0$

Unit 2- Derivatives
Advanced Placement Calculus AB/BC
Stage 3 – Learning Plan

Summary of Key Learning Events and Instruction

The teacher and students will use class discussion and small group cooperation to accomplish the following tasks:

- **Activity:** Students will establish the relationship between the graph of the function and the graph of its derivative by calculating the derivative of the function at given points using the definition of a derivative.
- **Technology- Computer and/or iPad:** Use Calculus In Motion with Geometer's Sketchpad to explore the definition of derivative.
- Use one-sided derivatives to determine if a piecewise function has a derivative at a given point.
- **Activity:** FLASH CARDS **-** Given the graph of a function, students will find the graph of its derivative.
- **Exploration:** Using the graphing calculator to explore differentiability at a given point.
- Sketch the graph of a continuous function whose derivative is a piecewise function stated as the combination of two constant functions.
- **Exploration:** Comparing the definition of a derivative and the symmetric difference quotient.
- Compare left-hand and right-hand derivatives with limits to show that a function is not differentiable at a given point.
- Using the rules for differentiation, determine whether or not a function has a horizontal tangent.
- Apply the rules for differentiation given values of two functions and their derivatives where no function is given.
- Apply the rules of differentiation to write the equations of lines that are perpendicular, parallel, or tangent to a curve.
- Apply the rules of differentiation to find the point on a curve where the tangent is parallel or perpendicular to a given function.
- Given a formula for a real world situation, find the indicated rate of change.
- **Exploration:** Explore Rates of Change using the rate of change of the area of a cross section of a tree with respect to its radius.
- **Exploration / Technology:** Using the graphing calculator, students will model horizontal motion using parametric equations.
- **Exploration / Technology:** Use the graphing calculator to compare the graph of a function and the graph of its derivative.
- **Activity:** Students will explore the relationship between velocity, speed and acceleration through graphs.
- Describe the motion of a particle given the graph of its velocity or position.
- Calculate marginal cost or marginal revenue and discuss the meaning of the results in terms of production.
- Describe simple harmonic motion using velocity, speed, acceleration and jerk.
- Find the equation of the line tangent to a given parametric function at a point defined by a given value of t.
- Apply implicit differentiation to find the tangent line and normal line to an ellipse at a given point.
- Given a function in exponential or logarithmic form representing a real world example of growth or decay, apply rules of differentiation to explain growth rates and when maximum or minimum growth will occur.
- **Technology- Computer and/or iPad:** Students will be encouraged to view the following videos to reinforce concepts discussed in class using the **VideoCalculus** application.
    - 3.1.2 Finding Instantaneous Velocity
    - 3.1.3 The Derivative
    - 4.1.1 A Shortcut for Finding Derivatives
    - 4.2.1 The Product Rule

- 15.1.2 Differentiating Logarithmic functions

- Students will be encouraged to use alternative resources to prepare for the Advanced Placement Exam.

Resources:

Technology
Software:
Calculus In Motion™ software, Animations of calculus for the Geometer's Sketchpad v4, Audrey Weeks

The following websites offer video lessons on calculus topics:
http://online.math.uh.edu/HoustonACT/videocalculus/index.html
http://www.khanacademy.org
http://www.wolframalpha.com
http://m.socrative.com/student/#joinRoom

The following websites offer practice tests and test preparation models:
http://apcentral.collegeboard.com/apc/Controller.jpf
http://sat.collegeboard.org/practice/sat-question-of-the-day

i-Pad Applications
The following apps are available on student i-Pads.
- WolframAlpha
- VideoCalculus
- Sketch Explorer

Activities
Activities referenced in this document have been created by Susan Gasko and Susan Okulewicz and can be found on the Mathematics shared teachers folder.

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Pearson Education, Inc., 2012.
Teachers Resources;
- Teacher's Guide with Answers

- Teacher's AP Correlations and Preparation Guide
- Student Practice Workbook

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5 Steps to a 5 AP Calculus AB & BC, 2012-2013 edition, William Ma, McGraw Hill Companies, 2011.
Teaching AP Calculus, Lin McMulllin, D & S Marketing Systems, Inc., 2005.

Unit 3- Applications of Derivatives
Advanced Placement Calculus AB/BC
Stage 1 Desired Results

ESTABLISHED GOALS
The student will be able to apply properties of derivatives using functions, graphs and tables to model real-life situations.

<u>Common Core State Standards for Mathematics</u>

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums symmetries; end behavior; and periodicity*
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
8. Write a function defined by an expression in

Transfer

Students will be able to independently use their learning to…
Model discrete phenomena to solve real world problems involving optimization, approximation and related rates.

Meaning

UNDERSTANDINGS
Students will understand that…
- Rates of change and extreme values play a critical role in solving real world applications.
- Approximation plays an important role in the engineering and science fields.

ESSENTIAL QUESTIONS
- How are derivatives used to solve real world applications?
- How can derivatives be used to estimate change in real world situations?

Acquisition

Students will know…
- Absolute and local extreme values
- The Extreme Value Theorem
- The Local Extreme Value Theorem
- Definition of critical point
- The Mean Value Theorem for Derivatives
- Increasing and decreasing functions
- Definition of antiderivative
- First Derivative Test for Local Extrema
- Definition of concavity
- Concavity test
- Definition of point of inflection
- Second Derivative Test for Local Extrema
- Strategy for solving maximum and minimum problems
- Definition of linearization
- Newton's Method
- Definition of Differentials

Students will be skilled at…
- Finding extreme values
- Applying the Extreme Value Theorem
- Identifying critical points and endpoints of a function within a given interval
- Applying the Mean Value Theorem for Derivatives
- Finding the intervals on which a function is increasing or decreasing
- Applying The First Derivative Test For Local Extrema to find local maximum and minimum values of a function
- Determining the concavity of a function
- Identifying points of inflection
- Applying the Second Derivative Test for Local Extrema
- Graphing a function given information about its first and second derivatives
- Modeling geometric and economic applications of optimization
- Applying linearization to approximate

different but equivalent forms to reveal and explain different properties of the function. 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **Build new functions from existing functions** 3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. **Construct and compare linear, quadratic, and exponential models and solve problems** 1. Distinguish between situations that can be modeled with linear functions and with exponential functions. **Interpret expressions for functions in terms of the situation they model** 5. Interpret the parameters in a linear or exponential function in terms of a context. **Model periodic phenomena with trigonometric functions** 5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. **Interpret linear models** 9. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	• Differential estimate of change • Absolute, relative and percentage change • Solution strategy for solving related rate problems	the values of a function near a given point • Applying Newton's Method to approximate real solutions of an equation • Applying differentials to estimate change • Applying differentials to estimate maximum error • Analyzing the accuracy of estimates • Solving related rate application problems

Unit 3- Applications of Derivatives
Advanced Placement Calculus AB/BC
Stage 2 - Evidence

<table>
<tr><th>Evaluative Criteria</th><th>Assessment Evidence</th></tr>
<tr><td>The following rubric will be applied to each problem on any summative assessment.

6 Complete and correct solution including documentation of all concepts. Appropriate units will be included as needed.
5 Solution contains one minor arithmetic or algebraic error but demonstrates complete understanding of necessary concepts. OR Complete and correct solution without units.
4 Solution contains more than one minor arithmetic or algebraic error but demonstrates complete understanding of necessary concepts. OR Complete and correct solution without units.
3 Solution contains one conceptual error or one or more major arithmetic or algebraic errors while demonstrating partial understanding of necessary concepts.
2 Limited understanding of necessary concepts while demonstrating proper mathematical skills.
1 Incorrect answer with no understanding of necessary concepts.
0 No answer is given.</td><td>PERFORMANCE TASK(S):
Summative assessment questions should be open-ended and model the format established by the College Board.
Students will demonstrate ability to use analytical methods to find local extreme values, intervals on which the function is increasing or decreasing, inflection points, intervals on which the function is concave up or concave down; make connections between a function and its first and second derivatives both graphically and analytically, and to apply The Mean Value Theorem for Derivatives.
Students will demonstrate ability to solve application problems involving optimization, approximation, and related rates.
Students will demonstrate mastery of all concepts on both previous assessments using numerical, graphical and analytical approaches.
Optimization Project: Given specific dimensions for poster board, students will calculate the dimensions of a box with a lid having the maximum volume possible. Students will then design and construct the box with the calculated dimensions.</td></tr>
<tr><td>The following is a student rubric to assess individual understanding during class activities.

4 - I understand completely and I can teach it to a classmate.
3 - I understand the concept but I do not think I can explain it to a classmate.
2 - I can complete the task with assistance.
1 - I need help!</td><td>OTHER EVIDENCE:

Throughout each lesson student understanding will be assessed through informal questioning, formative assessments, and student's self-assessment.

Pre / Post-assessments - Applying rates of change to model real-life situations

Formative assessments should include but not be limited to the following concepts:</td></tr>
</table>

- Given a function find each of the following:
  a) local extrema
  b) intervals on which the function is increasing or decreasing
  c) inflection points
  d) intervals on which the function is concave up or concave down
- Example: Sketch a possible smooth curve $y = f(x)$ through the origin with the following properties:

  $f'(x) < 0 \text{ for } x \le 2$

  $f'(x) > 0 \text{ for } x \ge 2$

  $f''(x) < 0 \text{ for } x < -1$

  $f''(x) > 0 \text{ for } x > -1$

Unit 3- Applications of Derivatives
Advanced Placement Calculus AB/BC
Stage 3 – Learning Plan

Summary of Key Learning Events and Instruction

The teacher and students will use class discussion and small group cooperation to accomplish the following tasks:

- **Technology- Computer and/or iPad:** Use Calculus In Motion with Geometer's Sketchpad to explore The Mean Value Theorem.
- **Exploration:** Sketching a possible graph of a function given information about its first and second derivative.
- Apply the First and Second Derivative Tests to find local extrema of a given function. Discuss the appropriate use of each method to obtain the same results.
- Apply the strategy for solving optimization problems to solve problems involving area, volume, and motion.
- Apply the strategy for solving optimization problems that address maximum profit and minimum cost. Students will discuss the economic implications of their results.
- **Exploration / Technology:** Using a graphing calculator to explore the local linearity of a function.
- Apply differentials to approximate the change in surface area or volume of a solid given a small change one of its dimensions.
- Discuss the various aspects of error analysis in the context of approximation.
- **Exploration / Technology:** Using a graphing calculator to apply Newton's Method to estimate real solutions of a given equation.
- **Activity:** Students will complete a tiered activity involving related rates for a sliding ladder. Students will find the rate of change in position of the ladder, the rate of change of the angle formed by the ladder with the ground, or the rate of change of the area enclosed by the ladder the ground and the building.
- **Technology- Computer and/or iPad:** Use Calculus In Motion with Geometer's Sketchpad to explore a variety of related rate problems.
- Apply the strategy for solving related rate applications to solve problems involving area of geometric figures and solids.
- Apply the strategy for solving related rate applications to solve problems involving volumes of spheres, cones, cylindrical drums, and prisms.
- Apply the strategy for solving related rate applications to solve problems involving moving objects.
- **Technology- Computer and/or iPad:** Students will be encouraged to view the following videos to reinforce concepts discussed in class using the **VideoCalculus** application.
    - 7.3.4 The Can Problem
    - 7.4.1 The Pebble Problem
- Students will be encouraged to use alternative resources to prepare for the Advanced Placement Exam.

<u>Resources:</u>

Technology

Software:

Calculus In Motion™ software, Animations of calculus for the Geometer's Sketchpad v4, Audrey Weeks

The following websites offer video lessons on calculus topics:
http://online.math.uh.edu/HoustonACT/videocalculus/index.html
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http://www.wolframalpha.com
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http://sat.collegeboard.org/practice/sat-question-of-the-day

i-Pad Applications
The following apps are available on student i-Pads.

- WolframAlpha
- VideoCalculus
- SketchExplorer

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Pearson Education, Inc., 2012.
Teachers Resources;

- Teacher's Guide with Answers
- Teacher's AP Correlations and Preparation Guide
- Student Practice Workbook

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Multiple-Choice & Free-Response Questions in Preparation for the AP Calculus (BC) Examination, 8th Edition, Lederman, David, D & S Marketing Systems, Inc., 2011.
Barron's AP Calculus, 11th Edition, Barron's Educational Series, Inc., 2012.
Barron's AP Calculus Flash Cards, Barron's Educational Series, Inc., 2008.
5 Steps to a 5 AP Calculus AB & BC, 2012-2013 edition, William Ma, McGraw Hill Companies, 2011.
Teaching AP Calculus, Lin McMulllin, D & S Marketing Systems, Inc., 2005.

Unit 4- Indefinite and Definite Integrals
Advanced Placement Calculus AB/BC
Stage 1 Desired Results

<table>
<tr><td rowspan="6">ESTABLISHED GOALS
The student will be able to understand and apply the concepts of indefinite and definite integrals and their connection to the Riemann Sum.

<u>Common Core State Standards for Mathematics</u>

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums symmetries; end behavior; and periodicity
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
8. Write a function defined by an expression in</td><td colspan="2">Transfer</td></tr>
<tr><td colspan="2">Students will be able to independently use their learning to…
Apply approximation methods to solve real world problems where an analytical function is not available.
Apply the Fundamental Theorem of Calculus to calculate quantities given rates of change in the context of Geometry, Physics and Economics.</td></tr>
<tr><td colspan="2">Meaning</td></tr>
<tr><td>UNDERSTANDINGS
Students will understand that…
There is a relationship between the derivative and definite integral as expressed in both parts of The Fundamental Theorem of Calculus.</td><td>ESSENTIAL QUESTIONS
What is a definite integral and how is it different from the area under a curve?
When is an approximation method appropriate to solve real world application problems?
What is The Fundamental Theorem of Calculus and why is it so important?</td></tr>
<tr><td colspan="2">Acquisition</td></tr>
<tr><td>Students will know…
Area under a curve
Distance traveled
Rectangular Approximation Method (RAM) (LRAM)(RRAM)(MRAM)
Volume of a sphere
Riemann Sum
Definite integral as a limit of Riemann Sums
Integration notation
Area under a curve as a definite integral
Integrals on a calculator (numerical integral)
Discontinuous integrable functions
Properties of definite integrals
Average value of a function</td><td>Students will be skilled at…
Approximating the area under a curve of a nonnegative continuous function using RAM
Calculating a particle's position given a function for its velocity or a table of values
Applying RAM to approximate volumes of solids
Expressing a limit of Riemann Sums as a definite integral
Identifying intervals on which a function is above or below the x-axis
Evaluating definite integrals using area formulas
Expressing a quantity as a definite</td></tr>
</table>

different but equivalent forms to reveal and explain different properties of the function. 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **Build new functions from existing functions** 3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. **Construct and compare linear, quadratic, and exponential models and solve problems** 1. Distinguish between situations that can be modeled with linear functions and with exponential functions. **Interpret expressions for functions in terms of the situation they model** 5. Interpret the parameters in a linear or exponential function in terms of a context. **Model periodic phenomena with trigonometric functions** 5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. **Interpret linear models** 9. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	• Mean Value Theorem for Definite Integrals • The Fundamental Theorem of Calculus (Part 1) • Definition of antiderivative • The Fundamental Theorem of Calculus (Part 2) • Trapezoidal Rule • Simpson's Rule	integral of a constant rate function • Calculating the numerical integral using a graphing calculator • Identifying points of discontinuity when calculating definite integrals • Applying the properties of definite integrals • Calculating the average value of a function over a given interval • Applying the Mean Value Theorem for Definite Integrals • Applying The Fundamental Theorem of Calculus Part 1 • Identifying all possible functions with a given derivative (+constant) • Applying The Fundamental Theorem of Calculus Part 2 • Calculating total area of a function over a closed interval analytically • Calculating total area of a function over a closed interval numerically, using a graphing calculator • Analyzing antiderivatives graphically • Approximating the value of a definite integral using the Trapezoidal Rule • Approximating the value of a definite integral using Simpson's Rule • Determining of the approximation of a definite integral is an overestimate or underestimate

Unit 4- Indefinite and Definite Integrals
Advanced Placement Calculus AB/BC
Stage 2 - Evidence

Evaluative Criteria	**Assessment Evidence**
The following rubric will be applied to each problem on any summative assessment. 6 Complete and correct solution including documentation of all concepts. Appropriate units will be included as needed. 5 Solution contains one minor arithmetic or algebraic error but demonstrates complete understanding of necessary concepts. OR Complete and correct solution without units. 4 Solution contains more than one minor arithmetic or algebraic error but demonstrates complete understanding of necessary concepts. OR Complete and correct solution without units. 3 Solution contains one conceptual error or one or more major arithmetic or algebraic errors while demonstrating partial understanding of necessary concepts. 2 Limited understanding of necessary concepts while demonstrating proper mathematical skills. 1 Incorrect answer with no understanding of necessary concepts. 0 No answer is given.	PERFORMANCE TASK(S): Summative assessment questions should be open-ended and model the format established by the College Board. • Students will demonstrate the ability to apply RAM to approximate areas and volumes, to express a Riemann Sum as a definite integral, to evaluate definite integrals using area, to apply the properties of definite integrals, to calculate the average value of a function, to evaluate definite integrals using antiderivatives. • Students will demonstrate mastery of concepts on previous assessments in addition to applying The Fundamental Theorem of Calculus and the Trapezoidal Rule.
The following is a student rubric to assess individual understanding during class activities. 4 - I understand completely and I can teach it to a classmate. 3 - I understand the concept but I do not think I can explain it to a classmate. 2 - I can complete the task with assistance. 1 - I need help!	OTHER EVIDENCE: Throughout each lesson student understanding will be assessed through informal questioning, formative assessments, and student's self-assessment. **Pre/Post-assessments -** Applying definite and indefinite integrals Formative assessments should include but not be limited to the following concepts:

	• Given the graph of $f(x)$ where position is $s = \int_0^t f(x)\,dx$ Find the following: a) The particle's velocity at a given time b) Whether the particle's acceleration is positive or negative at a given time c) The particle's position at a given time d) When the position has the largest value e) When the acceleration is zero f) When the particle is moving toward or away from the origin • Given a table of values for the rate of change of a function over a closed interval, approximate the value of a definite integral and the average value of the function using both MRAM and The Trapezoidal Rule. Discuss accuracy of your results in terms of overestimates or underestimates.

Unit 4- Indefinite and Definite Integrals
Advanced Placement Calculus AB/BC
Stage 3 – Learning Plan

Summary of Key Learning Events and Instruction

The teacher and students will use class discussion and small group cooperation to accomplish the following tasks:

- **Technology- Computer and/or iPad:** Use Calculus In Motion with Geometer's Sketchpad to explore Riemann Sums.
- **Activity:** Students will complete a tiered activity to explore the differences between finding area with RRAM, MRAM, and LRAM, or using a shape other than a rectangle.
- **Exploration:** Examine the relationship between RRAM, MRAM, and LRAM, and whether a function is increasing or decreasing.
- **Technology- Computer and/or iPad:** Use Calculus In Motion with Geometer's Sketchpad to explore using definite integrals to calculate area under a curve.
- **Exploration:** Finding the value of definite integrals using transformations of a given integral.
- Apply the integral of a constant function to express a quantity, given a rate of change.
- **Technology- Computer and/or iPad:** Use Calculus In Motion with Geometer's Sketchpad to explore The Mean Value Theorem for Integrals.
- Apply the Properties of Definite Integrals to perform operations on given definite integrals by substituting numerical values for definite integrals.
- Calculate the average value of a function on a given integral using area formulas.
- Apply The Fundamental Theorem of Calculus to find the derivative of a definite integral of a function over the given interval from a to x.
- Apply The Fundamental Theorem of Calculus Part 2 to evaluate a definite integral analytically.
- Calculate the total area of a region between a function and the x-axis using The Fundamental Theorem of Calculus Part 2.
- **Technology- Computer and/or iPad:** Students will be encouraged to view the following video to reinforce concepts discussed in class using the **VideoCalculus** application.
    - 1.1.2 Two Questions of Calculus
- Students will be encouraged to use alternative resources to prepare for the Advanced Placement Exam.

Resources:

Technology

Software:

Calculus In Motion™ software, Animations of calculus for the Geometer's Sketchpad v4, Audrey Weeks

The following websites offer video lessons on calculus topics:

http://online.math.uh.edu/HoustonACT/videocalculus/index.html
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http://www.wolframalpha.com

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Unit 5-Differential Equations
Advanced Placement Calculus AB/BC
Stage 1 Desired Results

ESTABLISHED GOALS

The student will be able to understand and apply various methods of integration to solve problems requiring differential equations and mathematical modeling.

Common Core State Standards for Mathematics

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums symmetries; end behavior; and periodicity*

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

Transfer

Students will be able to independently use their learning to…

Apply advanced methods of integration to solve real world applications in Economics and Science.

Meaning

UNDERSTANDINGS

Students will understand that…

- There are various methods of integration not all applicable to every problem.
- The solution to a differential equation, given an initial condition, is a function.
- Calculus plays a key role in real-world applications of exponential growth and decay.

ESSENTIAL QUESTIONS

- What are the various methods of integration available to us and how do we choose the correct method for a given problem?
- How are differential equations used to solve real-world applications?

Acquisition

Students will know…

- Definition of differential equations
- Initial value problem
- Slope fields
- Euler's Method
- Definition of indefinite integral
- Properties of indefinite integrals
- Integration by Substitution
- Integration by Parts Formula
- Tabular Integration
- Properties of Logarithmic and Exponential Functions
- Definition of separable differential equations
- The Law of Exponential Change
- Compound interest formulas
- Definition of half-life

Students will be skilled at…

- Finding the general solution to a differential equation
- Finding the particular solution to differential equation given an initial value
- Constructing a slope field for a differential equation
- Sketching the graph of a particular solution to a differential equation on a slope field
- Apply Euler's Method to approximate the value of a function at a particular point
- Evaluating an indefinite integral
- Applying properties of indefinite integrals

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **Build new functions from existing functions** 3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. **Construct and compare linear, quadratic, and exponential models and solve problems** 1. Distinguish between situations that can be modeled with linear functions and with exponential functions. **Interpret expressions for functions in terms of the situation they model** 5. Interpret the parameters in a linear or exponential function in terms of a context. **Model periodic phenomena with trigonometric functions** 5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. **Interpret linear models** 9. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	• Newton's Law of Cooling • Partial Fraction Decomposition • The Logistic Differential Equation • Logistic growth models	• Applying integration by substitution to evaluate indefinite integrals • Restating a definite integral using a u-substitution • Evaluating a definite integral using integration by substitution • Applying Integration by Parts to evaluate integrals • Choosing the appropriate parts for use in the Integration by Parts Formula using LIPET. (natural **L**ogarithm, **I**nverse trigonometric, **P**olynomial, **E**xponential, and **T**rigonometric functions) • Solving for the unknown integral • Applying Tabular Integration to evaluate integrals • Evaluating integrals using Properties of Logarithmic and Exponential functions • Applying Separation of Variables to solve Differential Equations • Applying The Law of Exponential Change to solve real-world applications involving exponential growth and decay in Economics and Science. • Evaluating integrals using Partial Fraction Decomposition • Applying logistic differential equations to model population growth.

Unit 5-Differential Equations Advanced Placement Calculus AB/BC Stage 2 - Evidence	
Evaluative Criteria	**Assessment Evidence**
The following rubric will be applied to each problem on any summative assessment. 6 Complete and correct solution including documentation of all concepts. Appropriate units will be included as needed. 5 Solution contains one minor arithmetic or algebraic error but demonstrates complete understanding of necessary concepts. OR Complete and correct solution without units. 4 Solution contains more than one minor arithmetic or algebraic error but demonstrates complete understanding of necessary concepts. OR Complete and correct solution without units. 3 Solution contains one conceptual error or one or more major arithmetic or algebraic errors while demonstrating partial understanding of necessary concepts. 2 Limited understanding of necessary concepts while demonstrating proper mathematical skills. 1 Incorrect answer with no understanding of necessary concepts. 0 No answer is given.	PERFORMANCE TASK(S): Summative assessment questions should be open-ended and model the format established by the College Board. • Students will demonstrate the ability to evaluate definite and indefinite integrals analytically using properties of integrals and Integration by Substitution. • Students will demonstrate the ability to evaluate definite and indefinite integrals analytically using properties of integration, Integration by Substitution, and Integration by Parts; solve initial value problems and construct slope fields for differential equations. • Students will demonstrate ability to evaluate definite and indefinite integrals using partial fraction decomposition. • Students will demonstrate the ability to solve Logistic Growth Equations. • Students will demonstrate ability to apply definite and indefinite integrals to solve real world problems involving temperature change, population growth, radioactive decay, carbon dating, and compound interest.
The following is a student rubric to assess individual understanding during class activities. 4 - I understand completely and I can teach it to a classmate. 3 - I understand the concept but I do not think I can explain it to a classmate. 2 - I can complete the task with assistance. 1 - I need help!	OTHER EVIDENCE: Throughout each lesson student understanding will be assessed through informal questioning, formative assessments, and student's self-assessment. **Pre/Post–assessments -** Solving problems using differential equations and mathematical modeling Formative assessments should include but not be limited to the following concepts: • Choose the appropriate method of evaluating each integral. Explain your reasoning.

	a) $\int_0^1 \frac{dx}{3(1+x)^2}$ b) $\int_0^1 \frac{dx}{3x^2}$ c) $\int e^x \cos x \, dx$ d) $\int x^2 \sin 4x \, dx$ • Students will verify solutions to complex integration problems using *Wolfram* APP on their iPad. • Apply the Law of Exponential Change to solve differential equations resulting in specific formulas for compound interest, radioactive decay, half-life, Newton's Law of Cooling, and population growth**.** • Given a logistic equation representing the spread of a disease through a community, calculate the number of people infected when the disease is spreading the fastest.

Unit 5-Differential Equations
Advanced Placement Calculus AB/BC
Stage 3 – Learning Plan

Summary of Key Learning Events and Instruction

The teacher and students will use class discussion and small group cooperation to accomplish the following tasks:

- **Technology- Computer and/or iPad:** Use Calculus In Motion with Geometer's Sketchpad to explore Slope Fields and Euler's Method.
- **Activity:** Given a differential equation, students will create a slope field, sketch a particular solution to the differential equation for a given value on their slope field, solve the differential equation using Separation of Variables, and compare solutions from the two methods.
- **Activity:** Students will complete a tiered activity to apply Separation of Variables to solve initial value problems
- Apply the method of u-substitution for various types of functions including polynomial, trigonometric, exponential and logarithmic.
- **Exploration:** Given an integral, explore the choice of parts that leads to a successful solution using Integration by Parts.
- Use the Law of Exponential Change to solve differential equations resulting in specific formulas for compound interest, radioactive decay, half-life, Newton's Law of Cooling, and population growth.
- Apply the method of Partial Fraction Decomposition to evaluate indefinite integrals.
- Use Logistic Differential Equations to model population growth.
- **Technology- Computer and/or iPad:** Students will be encouraged to view the following video to reinforce concepts discussed in class using the **VideoCalculus** application.
  - 16.1.1 An Introduction to Integral Tables
  - 16.6.1 An Introduction to Integration by Parts
- Students will be encouraged to use alternative resources to prepare for the Advanced Placement Exam.

<u>Resources:</u>

Technology

Software:

Calculus In Motion™ software, Animations of calculus for the Geometer's Sketchpad v4, Audrey Weeks

The following websites offer video lessons on calculus topics:

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<u>5 Steps to a 5 AP Calculus AB & BC, 2012-2013 edition</u>, William Ma, McGraw Hill Companies, 2011.
<u>Teaching AP Calculus</u>, Lin McMulllin, D & S Marketing Systems, Inc., 2005.

Unit 6- Applications of Definite Integrals
Advanced Placement Calculus AB/BC
Stage 1 Desired Results

<table>
<tr><td rowspan="6">ESTABLISHED GOALS
The student will be able to understand and apply the concepts definite integrals as it relates to net change.

<u>Common Core State Standards for Mathematics</u>

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums symmetries; end behavior; and periodicity
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
8. Write a function defined by an expression in</td><td colspan="2">Transfer</td></tr>
<tr><td colspan="2">Students will be able to independently use their learning to…
Apply their knowledge of integral calculus to model problems involving rates of change.</td></tr>
<tr><td colspan="2">Meaning</td></tr>
<tr><td>UNDERSTANDINGS
Students will understand that…
The integral is an invaluable tool when calculating net change and total accumulation.
Applications involving area and volume play an important role in the fields of Engineering and Business.</td><td>ESSENTIAL QUESTIONS
How do we determine the appropriate method of integration for a given problem?
What is net change?
Explain the importance of calculus in the manufacturing industry and the relationship it fosters between engineering, manufacturing, and business.
What type of real world problems can be solved by applying integrals as accumulation functions?</td></tr>
<tr><td colspan="2">Acquisition</td></tr>
<tr><td>Students will know…
Position
Displacement
Total distance
Relationship between position, velocity and acceleration
Relationship between Net Change and Total Accumulation
Work
Hooke's Law
Definition of area between two curves
Definition of boundaries with changing functions</td><td>Students will be skilled at…
Describing the motion of a particle given its velocity
Calculating a particle's displacement and final position given its velocity on an interval
Calculating the total distance traveled by a particle given its velocity on an interval
Calculating total consumption given a rate of change analytically
Approximating total consumption given data representing the rate of change</td></tr>
</table>

different but equivalent forms to reveal and explain different properties of the function. 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **Build new functions from existing functions** 3. Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x), f(kx),* and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. **Construct and compare linear, quadratic, and exponential models and solve problems** 1. Distinguish between situations that can be modeled with linear functions and with exponential functions. **Interpret expressions for functions in terms of the situation they model** 5. Interpret the parameters in a linear or exponential function in terms of a context. **Model periodic phenomena with trigonometric functions** 5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. **Interpret linear models** 9. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	• Definition of volume of a solid • Strategy for calculating volume of a solid by slicing • Strategies for calculating the volume of a solid of revolution • Strategy for calculating the surface area of a solid of revolution • Definition of arc length • Strategies of calculating arc length including vertical tangents, corners and cusps • Work as a definite integral • Fluid force and fluid pressure	• Applying Hooke's Law to determine the amount of work done in stretching and compressing a spring • Calculating the area between two curves on a given interval analytically • Identifying boundaries when calculating area between two curves • Calculating area using subregions • Calculating the area of a region by integrating with respect to either axis • Applying the slicing method to calculate the volume of a solid with a given cross section • Calculating the volume of a solid by revolving a particular region about a given axis • Recognizing when a cross section is a disk or a washer when finding volumes for solids of revolution • Calculating the volume of a solid using the Cylindrical Shell Method • Visualizing solids that are formed when a cross section of a solid is perpendicular to the axis over a given interval • Visualizing solids formed by rotating a plane region about a given axis • Applying integrals to calculate the surface area of a solid of revolution • Applying integrals to calculate lengths of curves in a plane • Applying integrals to solve problems involving work, fluid force, and fluid pressure

Unit 6- Applications of Definite Integrals
Advanced Placement Calculus AB/BC
Stage 2 - Evidence

Evaluative Criteria	**Assessment Evidence**
The following rubric will be applied to each problem on any summative assessment. 6 Complete and correct solution including documentation of all concepts. Appropriate units will be included as needed. 5 Solution contains one minor arithmetic or algebraic error but demonstrates complete understanding of necessary concepts. OR Complete and correct solution without units. 4 Solution contains more than one minor arithmetic or algebraic error but demonstrates complete understanding of necessary concepts. OR Complete and correct solution without units. 3 Solution contains one conceptual error or one or more major arithmetic or algebraic errors while demonstrating partial understanding of necessary concepts. 2 Limited understanding of necessary concepts while demonstrating proper mathematical skills. 1 Incorrect answer with no understanding of necessary concepts. 0 No answer is given.	PERFORMANCE TASK(S): Summative assessment questions should be open-ended and model the format established by the College Board. • Students will demonstrate the ability to describe particle motion in the context of position, displacement and total distance given its velocity, understand the connection between net change and total accumulation, and calculate the area of a region. • Students will demonstrate mastery of all concepts on previous assessments in addition to calculating volumes of solids using slicing method, rotation, and cylindrical shells. Students will demonstrate ability to calculate the surface area of a solid of revolution. • **Project:** Solids of Revolution in our Diet. Students will choose a food item or container with a circular cross section and calculate its volume by creating functions to model the boundary and rotating the functions about an axis. Students will present their findings to the class in a creative display. • Students will demonstrate mastery of all concepts on previous assessments in addition to calculating the length of a smooth curve including curves that contain vertical tangents, corners and cusps. • Students will demonstrate the ability to apply definite integrals in science applications involving work, force, and fluid force.
The following is a student rubric to assess individual understanding during class activities. 4 - I understand completely and I can teach it to a classmate. 3 - I understand the concept but I do not think I can explain it to a classmate. 2 - I can complete the task with assistance. 1 - I need help!	OTHER EVIDENCE: Throughout each lesson student understanding will be assessed through informal questioning, formative assessments, and student's self-assessment. **Pre/Post-assessments -** Applying definitive integrals to solve applications involving area and volume Formative assessments should include but not be limited to the following concepts: • Given the function $v(t)$ in m/sec, of a particle moving along the x axis on a specified

	interval at a specific starting position a) Determine when the particle is moving to the right, left, and stopped. b) Find the particle's displacement for the given time interval and its final position. c) Find the total distance traveled by the particle. • Given a continuous function on the closed interval defined in the first quadrant, calculate the volume of the solid formed by rotating the function about the x-axis, the y-axis, the line x=4, and the line y=5. • Calculate the perimeter of the region bounded by the functions $f(x)=x^3+5\|x\|+2$, $g(x)=-2$, and the vertical lines $x=-2$ and $x=1$. • A particular frozen drink is served in a truncated conical container with height 7in. The diameter of the base is 2.5 in and the opening on the top has diameter 3.5 in. If the beverage weighs (4/9) oz/in^3 and the straw sticks up 1 in above the top, how much work will it take to drink the beverage? (ignoring friction, give answer in inch-ounces)

Unit 6- Applications of Definite Integrals
Advanced Placement Calculus AB/BC
Stage 3 – Learning Plan

Summary of Key Learning Events and Instruction

The teacher and students will use class discussion and small group cooperation to accomplish the following tasks.

- Apply definite integrals to describe net change in the context of particle motion and consumption over time.
- Apply definite integrals to solve problems involving area and perimeter of a region in the plane.
- Apply definite integrals to solve problems involving volume and surface area generated by revolving a region about a given axis.
- **Activity-**Students will work in pairs or small groups to determine the best method of calculating the volume of a given piece of candy. Each group will have a different item and will have to justify their choice of slicing, rotating, or cylindrical shells method including the reason why each of the alternative methods would be less effective. Students will take measurements and create the necessary function(s) to model the boundary, then apply the method of their choice to calculate its volume.
- **Technology- Computer and/or iPad:** The teacher can choose specific sketches from Calculus In Motion with Geometer's Sketchpad for guided class discussion on calculating the area between two curves, the volume of a solid by slicing with a given cross section, and the volume of a solid by revolution including washers and cylindrical shells.
- **Technology- Computer and/or iPad:** Students will be encouraged to view the following video to reinforce concepts discussed in class using the **VideoCalculus** application.
    - 10.1.2 Gravity and Vertical Motion
    - 13.1.2 Calculus in 20 Minutes
- **Technology- Computer and/or iPad**: Students will be encouraged to use the video clips listed below to help visualize concepts involving 3-dimensional solids. **http://online.math.uh.edu/HoustonACT/videocalculus/index.html**
    - Video #26 Area between two curves
    - Video #27 Volumes I: Slicing method
    - Video #28 Volumes II: Rotation method
    - Video #29 Volumes III: Cylindrical shell method
- Apply definite integrals to solve real world problems involving work, fluid force and fluid pressure.
- Students will be encouraged to use alternative resources to prepare for the Advanced Placement Exam.

Resources:

Technology

Software:

Calculus In Motion™ software, Animations of calculus for the Geometer's Sketchpad v4, Audrey Weeks

The following websites offer video lessons on calculus topics:
http://online.math.uh.edu/HoustonACT/videocalculus/index.html
http://www.khanacademy.org
http://www.wolframalpha.com
http://m.socrative.com/student/#joinRoom

The following websites offer practice tests and test preparation models:
http://apcentral.collegeboard.com/apc/Controller.jpf
http://sat.collegeboard.org/practice/sat-question-of-the-day

i-Pad Applications
The following apps are available on student i-Pads.
- WolframAlpha
- VideoCalculus
- SketchExplorer

Activities
Activities referenced in this document have been created by Susan Gasko and Susan Okulewicz and can be found on the Mathematics shared teachers folder.

Explorations
Explorations referenced in this document can be found in the following text:
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Finney, Ross L.; Demana. Franklin D.; Waits, Bert K.; Kennedy, Daniel;
Pearson Education, Inc., 2012.
Teachers Resources;
- Teacher's Guide with Answers
- Teacher's AP Correlations and Preparation Guide
- Student Practice Workbook

Additional Sources for Review
Preparing for the Calculus AP Exam with Calculus: Graphical, Numerical, Algebraic, Barton, Brunsting, Diehl, Hill, Tyler, Wilson, Pearson Education, Inc., 2007.
Multiple-Choice & Free-Response Questions in Preparation for the AP Calculus (BC) Examination, 8th Edition, Lederman, David, D & S Marketing Systems, Inc., 2011.
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Unit 7- L'Hopital's Rule and Improper Integrals
Advanced Placement Calculus AB/BC
Stage 1 Desired Results

ESTABLISHED GOALS

The student will be able to understand and apply the limits involving infinity.

Common Core State Standards for Mathematics

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums symmetries; end behavior; and periodicity*

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

8. Write a function defined by an expression in different but equivalent forms to reveal and

Transfer

Students will be able to independently use their learning to…

Write and evaluate appropriate limits to draw conclusions about sequences, compare rates of growth, and solve problems requiring indefinite integrals.

Meaning

UNDERSTANDINGS

Students will understand that…

- Sequences are frequently used in mathematics and applied fields.
- Limits can be used to find real answers to applications involving infinity.
- There are proven analytical methods to evaluate limits that result in indeterminate forms.

ESSENTIAL QUESTIONS

- How can we use limits to compare rates of growth?
- How can we determine if an improper integral converges or diverges?

Acquisition

Students will know…

- Definition of Sequence
- nth term of an arithmetic sequence
- nth term of a geometric sequence
- Explicit vs recursive definitions of a sequence
- Limit of a sequence
- Properties of limits of sequences
- The Sandwich Theorem for sequences
- The Absolute Value Theorem
- Indeterminate forms
- L'Hopital's Rule
- Definitions of faster, slower, and same-rate growth as $x \to \infty$
- Transitivity of growing rates

Students will be skilled at…

- Recognizing a sequence as finite or infinite and arithmetic, geometric, or neither
- Writing the nth term of a given sequence in explicit form
- Using sequences to solve application problems
- Finding the limit of a sequence
- Understanding and applying the Absolute Value Theorem
- Evaluating limits of indeterminate form using L'Hopital's Rule
- Comparing rates of growth as $x \to \infty$ using limits

explain different properties of the function. 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **Build new functions from existing functions** 3. Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x), f(kx),* and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. **Construct and compare linear, quadratic, and exponential models and solve problems** 1. Distinguish between situations that can be modeled with linear functions and with exponential functions. **Interpret expressions for functions in terms of the situation they model** 5. Interpret the parameters in a linear or exponential function in terms of a context. **Model periodic phenomena with trigonometric functions** 5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. **Interpret linear models** 9. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	• Improper integrals • Improper integrals with infinite discontinuities • Comparison test for convergence and divergence	• Evaluating improper integrals • Evaluating improper integrals with discontinuities • Applying The Comparison Test for convergence or divergence of an improper integral

Unit 7- L'Hopital's Rule and Improper Integrals
Advanced Placement Calculus AB/BC
Stage 2 - Evidence

Evaluative Criteria	**Assessment Evidence**
The following rubric will be applied to each problem on any summative assessment. 6 Complete and correct solution including documentation of all concepts. Appropriate units will be included as needed. 5 Solution contains one minor arithmetic or algebraic error but demonstrates complete understanding of necessary concepts. OR Complete and correct solution without units. 4 Solution contains more than one minor arithmetic or algebraic error but demonstrates complete understanding of necessary concepts. OR Complete and correct solution without units. 3 Solution contains one conceptual error or one or more major arithmetic or algebraic errors while demonstrating partial understanding of necessary concepts. 2 Limited understanding of necessary concepts while demonstrating proper mathematical skills. 1 Incorrect answer with no understanding of necessary concepts. 0 No answer is given.	PERFORMANCE TASK(S): Summative assessment questions should be open-ended and model the format established by the College Board. • Students will demonstrate the ability to expand a sequence given in explicit or recursive form, write the explicit or recursive rule for the nth term of an arithmetic or geometric sequence and use the rule to find a particular term of the sequence. • Students will demonstrate the ability to find the limit of a given sequence and determine the convergence or divergence of a given sequence analytically or graphically. • Students will demonstrate the ability to identify indeterminate forms including $0/0$, ∞/∞, $\infty \cdot 0$, $\infty - \infty$, 1^∞, 0^0, ∞^0. • Students will demonstrate the ability to apply L'Hopital's rule to evaluate limits that lead to indeterminate forms analytically and graphically. • Students will demonstrate the ability to use L'Hopital's rule to compare growth rates. • Students will demonstrate the ability to evaluate an improper integral analytically and graphically, and use the Comparison Test to determine whether an integral converges.
The following is a student rubric to assess individual understanding during class activities. 4 - I understand completely and I can teach it to a classmate. 3 - I understand the concept but I do not think I can explain it to a classmate. 2 - I can complete the task with assistance. 1 - I need help!	OTHER EVIDENCE: Throughout each lesson student understanding will be assessed through informal questioning, formative assessments, and student's self-assessment. Formative assessments should include but not be limited to the following concepts: • Determine whether each sequence is arithmetic or geometric then find an explicit and

recursive rule for the nth term of each.

$$a)\ 2\,,\ \frac{1}{2}\,,-1\,,-\frac{5}{2}\,,\ldots$$

$$b)\ 81\,,-27\,,9\,,-3\,,\ldots$$

- Determine if each sequence converges or diverges. If it converges then find its limit.

$$a)\ a_n = n\sin\left(\frac{3\pi}{n}\right)$$

$$b)\ a_n = (-1)^n\left(\frac{3n-1}{n+2}\right)$$

- Explain how to apply L'Hopital's rule to each group of indeterminate forms below:
  - $0/0\,,\ \infty/\infty$
  - $\infty\cdot 0\,,\ \infty-\infty$
  - $1^\infty\,,\ 0^0\,,\ \infty^0$
- Use L'Hopital's rule to evaluate the limit.

$$\lim_{x\to 0}\left(e^x + x\right)^{\frac{1}{x}}$$

- Order the given functions from slowest growing to fastest growing as $x \to \infty$.

3^x , x^3 , $(\ln x)^x$ and e^x

- Find the point(s) of discontinuity, then evaluate the integral.

$$\int_{-2}^{0}\frac{dt}{(t+1)^{3/5}}$$

	• The infinite region in the first quadrant bounded by the coordinate axes and the curve $y=\frac{1}{x}-1$ is revolved about the y-axis to generate a solid. a) Write the volume of the solid as an improper integral. b) Express the integral as a limit of a definite integral. c) Find the volume of the solid.

Unit 7- L'Hopital's Rule and Improper Integrals
Advanced Placement Calculus AB/BC
Stage 3 – Learning Plan

Summary of Key Learning Events and Instruction

The teacher and students will use class discussion and small group cooperation to accomplish the following tasks.

- Apply the properties of limits to sequences.
- Apply the Sandwich Theorem to show convergence of a sequence.
- Apply The Absolute Value Theorem for convergence of a sequence.
- Identify different indeterminate forms of a limit and apply L'Hopital's rule to find the limits.
- **Exploration / Technology –** Using a graphing utility to explore L'Hopital's Rule graphically.
- **Exploration –** Comparing rates of growth as $x \to \infty$.
- Analyze rates of growth analytically and graphically.
- Evaluate improper integrals with infinite limits of integration.
- Evaluate improper integrals whose integrands have infinite discontinuities.
- **Exploration –** Investigation of convergence or divergence of $\int_0^1 \frac{dx}{x^p}$.
- Apply the Comparison Test for convergence.
- **Exploration –** Calculating the volume of the infinite solid called Gabriel's Horn**.**
- **Technology- Computer and/or iPad:** Students will be encouraged to view the following video to reinforce concepts discussed in class using the **VideoCalculus** application.
    - 14.1.2 An Introduction to L'Hopital's Rule

Resources:

Technology

Software:

Calculus In Motion™ software, Animations of calculus for the Geometer's Sketchpad v4, Audrey Weeks

The following websites offer video lessons on calculus topics:

http://online.math.uh.edu/HoustonACT/videocalculus/index.html
http://www.khanacademy.org
http://www.wolframalpha.com
http://m.socrative.com/student/#joinRoom

The following websites offer practice tests and test preparation models:

http://apcentral.collegeboard.com/apc/Controller.jpf
http://sat.collegeboard.org/practice/sat-question-of-the-day

i-Pad Applications
The following apps are available on student i-Pads.

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- VideoCalculus
- SketchExplorer

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Teachers Resources;

- Teacher's Guide with Answers
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Unit 8- Infinite Series
Advanced Placement Calculus AB/BC
Stage 1 Desired Results

ESTABLISHED GOALS	Transfer	
The student will be able to understand and apply strategies to identify infinite series and test for their convergence. **<u>Common Core State Standards for Mathematics</u>** **Interpret functions that arise in applications in terms of the context** 4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums symmetries; end behavior; and periodicity* 5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. 6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **Analyze functions using different representations** 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. 8. Write a function defined by an expression in different but equivalent forms to reveal and	*Students will be able to independently use their learning to…* apply tests for convergence of infinite series to represent functions that can impact the study of science, mathematics and engineering.	
	Meaning	
	UNDERSTANDINGS *Students will understand that…* • A strategy can be developed to determine which tests of convergence will ultimately determine if a series converges to a function on it's interval of convergence. • Power series play an important role in understanding the physical universe and can be used to represent functions.	ESSENTIAL QUESTIONS • How can an infinite series be identified as divergent or convergent? • Describe how differentiation and integration can be used to identify the connection between a function and a series. • Explain how to apply a strategy to test a series for convergence.
	Acquisition	
	Students will know… • Definition of a series • Sigma notation • nth partial sum of a series • Infinite Series • Geometric Series • Convergent and divergent series • Interval of convergence • Power series • Strategies for finding a power series by differentiation or integration • Strategies for constructing series • Taylor polynomials • Taylor and Maclaurin Series • Taylor's Theorem with remainder	*Students will be skilled at…* • Recognizing and writing geometric and infinite series • Determining whether a series converges or diverges • Finding a power series using differentiation or integration • Finding the interval of convergence • Finding the radius of convergence • Constructing Taylor Polynomials and Taylor Series generated by f at $x=0$ and f at $x=a$ • Using the Maclaurin Series to construct other series • Using Taylor's Theorem with remainder

explain different properties of the function. 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **Build new functions from existing functions** 3. Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x), f(kx),* and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. **Construct and compare linear, quadratic, and exponential models and solve problems** 1. Distinguish between situations that can be modeled with linear functions and with exponential functions. **Interpret expressions for functions in terms of the situation they model** 5. Interpret the parameters in a linear or exponential function in terms of a context. **Model periodic phenomena with trigonometric functions** 5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. **Interpret linear models** 9. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	• Remainder Estimation Theorem • Euler's Formula • Convergence Theorem for power series • The nth term test for divergence • Direct Comparison Test • Absolute Convergence • The Ratio Test • Endpoint Convergence • The Integral Test • P-series • Harmonic Series • The Limit Comparison Test (LCT) • The Alternating Series Test (Leibniz's Theorem) • Alternating Series Estimation Theorem • Strategies for rearrangement of convergent series • Strategies for testing a power series for convergence	to prove convergence of a Maclaurin Series • Applying the convergence theorem for a power series • Understanding and applying The Remainder Estimation Theorem • Understanding and applying Euler's Formula • Determining convergence using the nth-Term Test, Direct Comparison Test, and Ratio Test • Solving application problems involving Harmonic Series and P-series • Determining convergence using the Limit Comparison Test, Alternating Series Test, and Integral Test • Determining absolute and conditional convergence • Testing power series for intervals of convergence

Unit 8- Infinite Series
Advanced Placement Calculus AB/BC
Stage 2 - Evidence

Evaluative Criteria	**Assessment Evidence**
The following rubric will be applied to each problem on any summative assessment. 6 Complete and correct solution including documentation of all concepts. Appropriate units will be included as needed. 5 Solution contains one minor arithmetic or algebraic error but demonstrates complete understanding of necessary concepts. OR Complete and correct solution without units. 4 Solution contains more than one minor arithmetic or algebraic error but demonstrates complete understanding of necessary concepts. OR Complete and correct solution without units. 3 Solution contains one conceptual error or one or more major arithmetic or algebraic errors while demonstrating partial understanding of necessary concepts. 2 Limited understanding of necessary concepts while demonstrating proper mathematical skills. 1 Incorrect answer with no understanding of necessary concepts. 0 No answer is given.	PERFORMANCE TASK(S): Summative assessment questions should be open-ended and model the format established by the College Board. • Students will demonstrate the ability to identify or write a geometric series in expanded form or explicit form, to find the sum of a given geometric series if it converges, and to identify the interval of convergence of an infinite geometric series. • Students will demonstrate the ability to write or identify a Taylor Series given a particular Taylor polynomial, to write or identify a Maclaurin series for a given function. • Students will be able to write a Taylor polynomial for a given function and use it to approximate a value of the function. • Students will demonstrate the ability to write a power series for a function using a Maclaurin series, to write a Taylor polynomial of a specified order given appropriate values for derivatives of the function at x=0, and To use the Taylor polynomial to approximate a value of the function close to 0. • Students will demonstrate the ability to estimate the error of an approximation using the remainder theorem and the graphical method. • Students will demonstrate the ability to recognize all forms of the Maclaurin series and use an appropriate form to write a series for a new function. • Students will demonstrate the ability to find the radius of convergence for a power series and write the interval of convergence for the series. • Students will demonstrate the ability to use an appropriate test for convergence of a power series, and to distinguish between absolute and conditional convergence.
The following is a student rubric to assess individual understanding during class activities. 4 - I understand completely and I can teach it to a classmate. 3 - I understand the concept but I do not think I can explain it to a classmate. 2 - I can complete the task with assistance. 1 - I need help!	OTHER EVIDENCE: Throughout each lesson student understanding will be assessed through informal questioning, formative assessments, and student's self-assessment. Formative assessments should include but not be limited to the following concepts:

- Which of the following series converge? JUSTIFY YOUR ANSWERS!!!

$I. \sum_{n=1}^{\infty}\left(\frac{4}{3}\right)^n$ $\quad$ II. $\sum_{n=1}^{\infty}\frac{\cos(n\pi)}{n}$ $\quad$ $III. \sum_{n=1}^{\infty}\frac{1}{n}$

$IV. \sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$ $\quad$ $V. \sum_{n=1}^{\infty}\frac{n!}{n^n}$ $\quad$ $VI. \sum_{n=1}^{\infty}\frac{1}{n^3+1}$

- Consider the infinite series $\sum_{n=1}^{\infty}4\left(\frac{-1}{3}\right)^{n-1}$
  a) What is the sum of the series?
  b) How many terms are required in the partial sum to approximate the sum of the infinite series to within 0.001?
- Given the Maclaurin Series for e^x, find a series representation for xe^x.
- Let $P(x)=7-3(x-4)+5(x-4)^2-2(x-4)^3+6(x-4)^4$ be the fourth-degree Taylor polynomial for the function f about 4. Assume f has derivatives of all orders for all real numbers. What is the third-degree Taylor polynomial for $g(x)=\int_4^x f(t)dt$ about 4?
- What are the values of x for which the series $\sum_{n=1}^{\infty}\frac{(2x-1)^n}{n\cdot 4^n}$ converges?

Unit 8- Infinite Series
Advanced Placement Calculus AB/BC
Stage 3 – Learning Plan

Summary of Key Learning Events and Instruction

The teacher and students will use class discussion and small group cooperation to accomplish the following tasks.

- Given an infinite geometric series in expanded form, write an expression using sigma notation that will generate the series.
- Justify whether a series converges and state the sum if it converges.
- Find the interval of convergence for an infinite geometric series and write the function of x represented by the series.
- **Exploration-** Find Power Series for Other Functions given the function $1/(1-x)=1+x+x^2+...+x^n+...$ on the interval $(-1,1)$.
- Find a power series by differentiating or integrating a known power series.
- **Exploration –** Find a power series for $\tan^{-1} x$ and determine if it converges at $x=1$.
- **Exploration –** Derive a Taylor Polynomial by Constructing a polynomial $P(x)$ that matches the behaviors of a given function at $x=0$ through $P(0)$, $P'(0)$, $P''(0)$, $P'''(0)$ and $P^4(0)$.
- Construct the seventh-order Taylor polynomial and the Taylor series for $\sin x$ at $x=0$. a power series for using a Taylor polynomial.
- **Exploration –** Students will work in pairs to construct the sixth-order Taylor polynomial and the Taylor series for $\cos x$ at $x=0$. Groups can then compare methods to explore possible shortcuts.
- Use a Maclaurin Series to find power series to represent other functions centered at $x=0$and $x=2$.
- Students will work in pairs to find the Taylor polynomial of order four for a given function at $x=0$ and use it to approximate the value of the function at $x=0.2$. Through guided class discussion students will see the connection between Taylor polynomials and the linearization of f at $x=0$.
- **Explorations / Group Activity –** Students will reinforce each other's understanding of the Maclaurin series by

  a) Writing the Maclaurin series for $\sin^2 x$ using the identity $\sin^2 x = \frac{1}{2}(1-\cos 2x)$,

  b) Differentiating the series to obtain the Maclaurin series for $2\sin x\cos x$

  c) Verifying the series for $\sin 2x$.
- Use Taylor's Theorem with remainder to prove convergence of a Maclaurin series.
- Prove the convergence of a Maclaurin series with the Remainder Estimation Theorem.
- **Exploration –** Derive and apply Euler's formula.
- Calculate the interval of convergence for a power series.
- Apply the nth Term Test for Divergence.
- Prove the convergence of a series of nonnegative numbers using The Direct Comparison Test.
- Show that a given series converges absolutely.

- **Exploration –** Prove and apply the Ratio Test for convergence.
- Use the Ratio Test to examine series with a radius of convergence at a point, on an interval, and approaching endpoints.
- Identify the convergence or divergence of a given series. Justify an appropriate method.
- Find the sum of a telescoping series.
- Apply the integral test for convergence.
- **Exploration –** Use the Integral Test to prove the convergence or divergence of Harmonic Series and P-Series $\sum_{n=1}^{\infty}\left(1/n^p\right)$ by considering $p>1$, $p<1$, and $p=1$.
- Apply the Limit Comparison test (LCT) to determine convergence or divergence of P-Series.
- Recognize an Alternating Series and apply The Alternating Series Test (Leibniz's Theorem)
- Apply the Alternating Series Estimation Theorem.
- Prove the Alternating Harmonic Series is convergent but not absolutely convergent and find a bound for the truncation error after 99 terms.
- Examine the consequences of rearranging the terms of an absolutely convergent or conditionally convergent series. Rearrange the terms of the Alternating Harmonic Series to confirm the results.
- Apply the strategy to test a power series $\sum_{n=0}^{\infty} c_n(x-a)^n$ for convergence.
- Outline the procedure for determining convergence.
- **Group Activity –** Students will randomly be assigned an infinite series to test for convergence. After students have solved their assigned problem individually, groups of students with the same infinite series will be identified and will compare their results. When members of each group agree on a solution, it will be shared with the class.

Resources:

Technology
Software:
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Unit 9- Parametric , Vector, and Polar Functions
Advanced Placement Calculus AB/BC
Stage 1 Desired Results

ESTABLISHED GOALS
The student will be able to understand and apply properties of vectors, parametric equations, and polar equations to study nonlinear curves.

Common Core State Standards for Mathematics

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums symmetries; end behavior; and periodicity*
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
8. Write a function defined by an expression in

Transfer

Students will be able to independently use their learning to…
Apply parametric, vector, and polar equations to model real world phenomena.

Meaning

UNDERSTANDINGS
Students will understand that…
- Parametric, vector and polar equations can be used to describe curves that are difficult or impossible to define in the form $y = f(x)$.
- Vectors can be used to model motion in the coordinate plane using directed distance.
- Parametric and polar curves can be used to gain understanding of non-linear motion in the plane.

ESSENTIAL QUESTIONS
- Explain how parametric equations can be used with vectors to describe motion.
- What are some benefits of using polar equations to model areas?

Acquisition

Students will know…
- Parametric Equation
- Parametric Curve
- Parametric Differentiation Formulas
- Strategies to analyze slope and concavity of a parametric curve
- Arc Length of a parametric curve
- Cycloid
- Two-Dimensional vectors
- Vector operations
- Properties of vector operations
- Planar Motion

Students will be skilled at…
- Sketching a parametric curve in the plane
- Applying differentiation formulas for parametric equations
- Calculating the slope at a point on a parametric curve
- Calculating arc length of a parametric curve
- Finding the magnitude and direction of a vector
- Applying properties of vector operations

different but equivalent forms to reveal and explain different properties of the function. 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **Build new functions from existing functions** 3. Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x), f(kx),* and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. **Construct and compare linear, quadratic, and exponential models and solve problems** 1. Distinguish between situations that can be modeled with linear functions and with exponential functions. **Interpret expressions for functions in terms of the situation they model** 5. Interpret the parameters in a linear or exponential function in terms of a context. **Model periodic phenomena with trigonometric functions** 5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. **Interpret linear models** 9. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	• Displacement and distance traveled • Rectangular and polar coordinates • Polar curves • Polar-Rectangular conversion formulas • Parametric equations of polar curves • Slopes of polar curves • Area in polar coordinates • Area between polar curves	• Calculating speed, acceleration and direction of motion using properties of vectors • Calculating displacement and total distance traveled using vectors • Converting coordinates of a point from polar to rectangular form • Converting coordinates of a point from rectangular form to polar form • Graphing polar equations by hand and on a graphing utility • Converting polar equations to rectangular form • Calculating the area of a region bounded by polar curves

Unit 9- Parametric , Vector, and Polar Functions
Advanced Placement Calculus AB/BC
Stage 2 - Evidence

<table>
<tr><th>Evaluative Criteria</th><th>Assessment Evidence</th></tr>
<tr><td>The following rubric will be applied to each problem on any summative assessment.

6 Complete and correct solution including documentation of all concepts. Appropriate units will be included as needed.
5 Solution contains one minor arithmetic or algebraic error but demonstrates complete understanding of necessary concepts. OR Complete and correct solution without units.
4 Solution contains more than one minor arithmetic or algebraic error but demonstrates complete understanding of necessary concepts. OR Complete and correct solution without units.
3 Solution contains one conceptual error or one or more major arithmetic or algebraic errors while demonstrating partial understanding of necessary concepts.
2 Limited understanding of necessary concepts while demonstrating proper mathematical skills.
1 Incorrect answer with no understanding of necessary concepts.
0 No answer is given.</td><td>PERFORMANCE TASK(S):
Summative assessment questions should be open-ended and model the format established by the College Board.

• Students will demonstrate the ability to find first and second order derivatives of a parameterized curve in terms of t, find the point on a parameterized curve where a horizontal tangent occurs, and find the length of a parameterized curve.
• Students will demonstrate the ability to rewrite a parametric equation in rectangular form and determine if the resulting relation is a function of x; sketch the curve.
• Students will demonstrate the ability to find the magnitude and direction angle of a given vector.
• Students will demonstrate the ability to find the magnitude, velocity vector and speed of a particle at a particular value of t given a parametric equation describing its position, and calculate the total distance traveled by the particle for a specified interval.
• Students will demonstrate the ability to find the area of a region bounded by two polar equations.
• Students will demonstrate the ability to apply conversion formulas to replace a polar equation with an equivalent Cartesian equation.</td></tr>
<tr><td>The following is a student rubric to assess individual understanding during class activities.

4 - I understand completely and I can teach it to a classmate.
3 - I understand the concept but I do not think I can explain it to a classmate.
2 - I can complete the task with assistance.
1 - I need help!</td><td>OTHER EVIDENCE:
Throughout each lesson student understanding will be assessed through informal questioning, formative assessments, and student's self-assessment.

Formative assessments should include but not be limited to the following concepts:

• Find the area of the planar region inside the "figure-eight" $r = 1 + 2\cos\theta$ and outside the circle $r = 1$. Sketch the region.</td></tr>
</table>

	- Given position function $r(t)=\left\langle \sqrt{3}\sec t\ ,\ \sqrt{3}\tan t \right\rangle$ of a particle moving in the plane at time t a) Find the velocity and acceleration vectors b) Find the speed at a given value of $t=0$. - Replace the polar equation $r\cos\left(\theta+\pi/3\right)=2\sqrt{3}$ with an equivalent Cartesian equation.

Unit 9- Parametric , Vector, and Polar Functions
Advanced Placement Calculus AB/BC
Stage 3 – Learning Plan

Summary of Key Learning Events and Instruction

The teacher and students will use class discussion and small group cooperation to accomplish the following tasks.

- Analyze a parametric curve by sketching the graph for a given domain, finding the highest point on the curve analytically, and finding all points of inflection on the curve.
- Find the arc length of a parameterized curve.
- **Exploration –** Analyze a cycloid represented with parametric equations.
- Apply properties of vector operations in the plane.
- Use vectors to describe position, velocity, speed, acceleration and direction of motion of a particle given its position function in terms of differentiable functions $x(t)$, $y(t)$.
- Calculate displacement and distance traveled by a particle given its velocity $v(t)=(v_1(t)\ ,\ v_2(t))$.
- Sketch the graphs of polar curves by hand and using a graphing utility.
- **Exploration –** Graph Polar Curves Parametrically.
- Calculate the area between two polar curves.
- Apply conversion formulas to replace a polar equation with an equivalent Cartesian equation.

Resources:

Technology

Software:

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Unit 10- Formal Review
Advanced Applications of Derivatives and Integrals
Advanced Placement Calculus AB/BC
Stage 1 Desired Results

ESTABLISHED GOALS

The student will be able to apply derivatives and integrals as needed to solve application problems.

<u>Common Core State Standards for Mathematics</u>

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums symmetries; end behavior; and periodicity*

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

8. Write a function defined by an expression in

Transfer

Students will be able to independently use their learning to…

Apply differential and integral calculus as needed to solve real world application problems.

Apply problem-solving skills to unfamiliar situations involving derivatives and integrals.

Meaning

UNDERSTANDINGS

Students will understand that…

- Calculus plays in integral role in our everyday lives.

ESSENTIAL QUESTIONS

- How do we recognize which mathematical concepts are necessary to solve elaborate application problems?

Acquisition

Students will know…

- All definitions and properties associated with
  - Limits
  - Derivatives
  - Integrals
- All rules for differentiation
- All rules for integration

Students will be skilled at…

- Applying all definitions and properties to solve advanced calculus problems numerically, analytically, and graphically.
- Applying all definitions and properties to solve application problems in unfamiliar contexts, involving geometry, science, and economics.

different but equivalent forms to reveal and explain different properties of the function. 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **Build new functions from existing functions** 3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. **Construct and compare linear, quadratic, and exponential models and solve problems** 1. Distinguish between situations that can be modeled with linear functions and with exponential functions. **Interpret expressions for functions in terms of the situation they model** 5. Interpret the parameters in a linear or exponential function in terms of a context. **Model periodic phenomena with trigonometric functions** 5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. **Interpret linear models** 9. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.		

Unit 10- Formal Review
Advanced Applications of Derivatives and Integrals
Advanced Placement Calculus AB/BC
Stage 2 - Evidence

Evaluative Criteria	**Assessment Evidence**
The following rubric will be applied to each problem on any summative assessment. 6 Complete and correct solution including documentation of all concepts. Appropriate units will be included as needed. 5 Solution contains one minor arithmetic or algebraic error but demonstrates complete understanding of necessary concepts. OR Complete and correct solution without units. 4 Solution contains more than one minor arithmetic or algebraic error but demonstrates complete understanding of necessary concepts. OR Complete and correct solution without units. 3 Solution contains one conceptual error or one or more major arithmetic or algebraic errors while demonstrating partial understanding of necessary concepts. 2 Limited understanding of necessary concepts while demonstrating proper mathematical skills. 1 Incorrect answer with no understanding of necessary concepts. 0 No answer is given.	PERFORMANCE TASK(S): Summative assessment questions should be open-ended and model the format established by the College Board. • Students will demonstrate their ability to apply their knowledge of calculus to successfully complete multiple choice and free response problems at the Advanced Placement level. • Project: Given an economics situation, students will use differential and integral calculus to solve a task in a small business setting. Students will present their findings in written form using MathType, and in video form using iMovie. • Students will individually analyze the results of their project to make suggestions and improvements.
The following is a student rubric to assess individual understanding during class activities. 4 - I understand completely and I can teach it to a classmate. 3 - I understand the concept but I do not think I can explain it to a classmate. 2 - I can complete the task with assistance. 1 - I need help!	OTHER EVIDENCE: Throughout each lesson student understanding will be assessed through informal questioning, formative assessments, and student's self-assessment. Formative assessments should include but not be limited to the following: • College Board released exams • D&S Marketing Systems published practice exams

Unit 10- Formal Review
Advanced Applications of Derivatives and Integrals
Advanced Placement Calculus AB/BC
Stage 3 – Learning Plan

Summary of Key Learning Events and Instruction

- Students will complete the 2008 and 2012 released College Board practice exams in a classroom setting within the designated time frame.
- Students will analyze and make corrections in areas needing improvement.
- Blocks of time will be designated to implement strategies for successfully completing problems in the multiple-choice format.
- Blocks of time will be designated to implement strategies for successfully completing problems in the free response format.
- Free response questions will be discussed using the College Board released 9-point rubric.
- Students will organize their study materials to prepare for the Advanced Placement Test.

Resources:

Technology

Software:

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Benchmark Assessment Quarter 1

1. Students will demonstrate ability to evaluate limits numerically, analytically and graphically.
2. Students will use limits to determine continuity of a function at a given point, to find vertical and horizontal asymptotes, and to identify end behavior models.
3. Students will be able to write equations of tangent and normal lines to a curve at a given point.
4. Students will be able to sketch the graph of a function given conditions involving limits.
5. Students will successfully use limits to solve application problems.
6. Students will demonstrate ability to apply the definition of a derivative and to demonstrate understanding of differentiability.
7. Students will demonstrate ability to apply the basic rules for differentiation.
8. Students will demonstrate ability to apply the chain rule and implicit differentiation to calculate derivatives.
9. Students will demonstrate ability to differentiate trigonometric functions, inverse trigonometric functions, exponential functions and logarithmic functions.
10. Students will successfully apply derivatives to applications involving particle motion, sensitivity to change and economics.
11. Students will demonstrate the ability to use analytical methods to find local extreme values, intervals on which a function is increasing or decreasing, points of inflection, and intervals on which a function is concave up or concave down.
12. Students will be able to make connections between a function and its first and second derivatives both graphically and analytically.
13. Students will successfully apply The Mean Value Theorem for Derivatives.
14. Students will successfully solve application problems involving optimization, approximation, and related rates.

Benchmark Assessment Quarter 2

1. Students will demonstrate the ability to approximate area and volume using RAM.
2. Students will be able to express a Riemann Sum as a definite integral.
3. Students will be able to evaluate definite integrals using area formulas and antiderivatives.
4. Students will demonstrate the ability to apply the properties of definite integrals to evaluate integrals and to calculate the average value of a function.
5. Students will successfully apply The Fundamental Theorem of Calculus.
6. Students will successfully apply The Trapezoidal Rule
7. Students will be able to apply the properties of integrals to evaluate definite and indefinite integrals analytically.
8. Students will be able to evaluate definite and indefinite integrals analytically using Integration by Substitution.
9. Students will be able to evaluate definite and indefinite integrals analytically using Integration by Parts.
10. Students will successfully solve initial value problems and construct slope fields for differential equations.
11. Students will be able to evaluate definite and indefinite integrals analytically using Partial Fraction Decomposition.
12. Students will successfully solve application problems involving exponential growth and logistic growth.

Benchmark Assessment Quarter 3

1. Students will be able to describe particle motion in the context of position, displacement and total distance given its velocity.
2. Students will demonstrate an understanding of the connection between net change and total accumulation.
3. Students will be able to apply definite integrals to calculate the area of a region.
4. Students will be able to calculate volumes of solids using slicing and rotation methods, and cylindrical shells.
5. Students will be able to calculate surface area of a solid and length of a smooth curve.
6. Students will successfully solve application problems involving science and statistics.
7. Students will demonstrate an understanding of arithmetic and geometric sequences, explicitly and recursively.
8. Students will successfully apply L'Hopital's rule to evaluate limits of indeterminate form.
9. Students will successfully apply limits to compare growth rates.
10. Students will demonstrate the ability to evaluate improper integrals.
11. Students will demonstrate the ability to find a power series for a given function.
12. Students will successfully apply strategies to test an infinite series for convergence.
13. Students will successfully complete applications involving infinite series.
14. Students will successfully solve applications involving parametric, vector and polar equations.
15. Students will demonstrate the ability to apply their knowledge of differential and integral calculus to successfully complete multiple choice and free response problems at the Advanced Placement level.

Benchmark Assessment Quarter 4

1. Students will demonstrate the ability to apply their knowledge of differential and integral calculus to successfully complete multiple choice and free response problems at the Advanced Placement level.
2. Students will apply their knowledge of differential and integral calculus by completing and presenting an in-depth real-life application project.
3. Students will present their project to the class using MathType, Keynote, and iMovie.
4. Students will individually analyze the results of their project to make suggestions and improvements.