# Curriculum Management System

MONROE TOWNSHIP SCHOOLS

**Course Name:** **Advanced Placement Calculus AB**

**Grade:** **10-12**

***For adoption by all regular education programs as specified and for adoption or adaptation by all Special Education Programs in accordance with Board of Education Policy # 2220.***

***Board Approved: September 2012***

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# Monroe Township Schools Administration and Board of Education Members

## ADMINISTRATION

**Dr. Kenneth R. Hamilton, Superintendent**
**Dr. Jeff C. Gorman, Assistant Superintendent**

## BOARD OF EDUCATION

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**Mr. Louis C. Masters**
**Mr. Ira Tessler**
Jamesburg Representative
**Ms. Patrice Faraone**

## WRITERS NAME

**Ms. Susan Gasko**
**Ms. Susan Okulewicz**

## MATHEMATICS CURRICULUM INCHARGE (9-12)

**Dr. Manjit K. Sran**

# Mission, Vision, Beliefs, and Goals

## Mission Statement

**The Monroe Public Schools in collaboration with the members of the community shall ensure that all children receive an exemplary education by well-trained committed staff in a safe and orderly environment.**

## Vision Statement

**The Monroe Township Board of Education commits itself to all children by preparing them to reach their full potential and to function in a global society through a preeminent education.**

## Beliefs

**1. All decisions are made on the premise that children must come first.**
**2. All district decisions are made to ensure that practices and policies are developed to be inclusive, sensitive and meaningful to our diverse population.**
**3. We believe there is a sense of urgency about improving rigor and student achievement.**
**4. All members of our community are responsible for building capacity to reach excellence.**
**5. We are committed to a process for continuous improvement based on collecting, analyzing, and reflecting on data to guide our decisions.**
**6. We believe that collaboration maximizes the potential for improved outcomes.**
**7. We act with integrity, respect, and honesty with recognition that the schools serves as the social core of the community.**
**8. We believe that resources must be committed to address the population expansion in the community.**
**9. We believe that there are no disposable students in our community and every child means every child.**

## Board of Education Goals

**1. Raise achievement for all students paying particular attention to disparities between subgroups.**
**2. Systematically collect, analyze, and evaluate available data to inform all decisions.**
**3. Improve business efficiencies where possible to reduce overall operating costs.**
**4. Provide support programs for students across the continuum of academic achievement with an emphasis on those who are in the middle.**
**5. Provide early interventions for all students who are at risk of not reaching their full potential.**
**6. To Create a 21st Century Environment of Learning that Promotes Inspiration, Motivation, Exploration, and Innovation.**

# Philosophy

Monroe Township Schools are committed to providing all students with a quality education resulting in life-long learners who can succeed in a global society. The mathematics program, grades K - 12, is predicated on that belief and is guided by the following six principles as stated by the National Council of Teachers of Mathematics (NCTM) in the *Principles and Standards for School Mathematics, 2000.* First, a mathematics education requires equity. All students will be given worthwhile opportunities and strong support to meet high mathematical expectations. Second, a coherent mathematics curriculum will effectively organize, integrate, and articulate important mathematical ideas across the grades. Third, effective mathematics teaching requires the following: **a)** knowing and understanding mathematics, students as learners, and pedagogical strategies **b)** having a challenging and supportive classroom environment and **c)** continually reflecting on and refining instructional practice. Fourth, students must learn mathematics with understanding. A student's prior experiences and knowledge will actively build new knowledge. Fifth, assessment should support the learning of important mathematics and provide useful information to both teachers and students. Lastly, technology enhances mathematics learning, supports effective mathematics teaching, and influences what mathematics is taught.

As students begin their mathematics education in Monroe Township, classroom instruction will reflect the best thinking of the day. Children will engage in a wide variety of learning activities designed to develop their ability to reason and solve complex problems. Calculators, computers, manipulatives, technology, and the Internet will be used as tools to enhance learning and assist in problem solving. Group work, projects, literature, and interdisciplinary activities will make mathematics more meaningful and aid understanding. Classroom instruction will be designed to meet the learning needs of all children and will reflect a variety of learning styles.

In this changing world those who have a good understanding of mathematics will have many opportunities and doors open to them throughout their lives. Mathematics is not for the select few but rather is for everyone. Monroe Township Schools are committed to providing all students with the opportunity and the support necessary to learn significant mathematics with depth and understanding.

# Common Core State Standards (CSSS)

The Common Core State Standards provide a consistent, clear understanding of what students are expected to learn, so teachers and parents know what they need to do to help them. The standards are designed to be robust and relevant to the real world, reflecting the knowledge and skills that our young people need for success in college and careers. With American students fully prepared for the future, our communities will be best positioned to compete successfully in the global economy.

**Links:**

1. CCSS Home Page: http://wwws.corstandards.org
2. CCSS FAQ: http://www.corestandards.org/frequently-asked-questions
3. CCSS The Standards: http://www.corestandards.org/the-standards
4. NJDOE Link to CCSS: http://www.state.nj.us/education/sca
5. Partnership for Assessment of Readiness for College and Careers (PARCC): http://parcconline.org

| Scope and Sequence<br>Advanced Placement Calculus AB<br>Quarter 1 |
| --- |
| Unit Topics(s) |

I. Limits and Rates of Change
   a. Average and Instantaneous Rates of Change
   b. Limits
   c. Continuity
   d. Tangent Lines and the Normal to a Curve
   e. Slope of a Curve at a Given Point

II. Derivatives
   a. Definition of a $f'$
   b. Relationship Between the Graphs of $f$ and $f'$
   c. Differentiability
   d. Rules for Differentiation
   e. Velocity and Rate of Change
   f. Chain Rule and Implicit Differentiation
   g. Derivatives of Trigonometric Functions and Inverse Trigonometric, Inverse, Exponential and Logarithmic Functions

| Scope and Sequence<br>Advanced Placement Calculus AB<br>Quarter 2 |
| --- |
| Unit Topic(s) |

I. Applications of Derivatives
   a. Extreme Values of Functions
   b. Mean Value Theorem for Derivatives
   c. Analysis of Graphs using First and Second Derivatives
   d. Modeling and Optimization
   e. Linearization and Newton's Method
   f. Differentials and Change
   g. Related Rates

II. Integrals
   a. Approximating Areas
   b. Definite Integrals and Antiderivatives
   c. Average Value of a Function
   d. Mean Value Theorem for Definite Integrals
   e. Area and Volume of a Function
   f. Connecting Differential and Integral Calculus
   g. The Fundamental Theorem of Calculus

| Scope and Sequence<br>Advanced Placement Calculus AB<br>Quarter 3 |
| --- |
| Unit Topic(s) |

I. Differential Equations
   a. Slope Fields
   b. Euler's Method
   c. Antiderivatives and the Indefinite Integral
   d. Techniques of Integration
   e. Separable Differential Equations
   f. Exponential Growth and Decay

II. Applications of Definite Integrals
   a. Net Change
   b. Motion on a Line
   c. Consumption Over Time
   d. Area in the Plane
   e. Volume Using Integrals

| Scope and Sequence<br>Advanced Placement Calculus AB<br>Quarter 4 |
| --- |
| Unit Topic(s) |

I. Advanced Applications of Derivatives and Integrals
   a. Formal Review of Derivatives, Integrals and Applications
      i. Spiral Applications and Practice Problems
      ii. Practice Tests from The College Board
      iii. Practice Problems from Review Sources
   b. Real-Life Application Project encompassing all major concepts from the course ***(Culminating Project lasting approximately four weeks)***

# Unit 1- Limits and Rates of Change
## Stage 1 Desired Results

ESTABLISHED GOALS

Students will be able to understand and apply concepts of limits and rates of change.

**<u>Common Core State Standards for Mathematics</u>**

**Interpret functions that arise in applications in terms of the context**

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums symmetries; end behavior; and periodicity*
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

**Analyze functions using different representations**

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
8. Write a function defined by an expression in different but equivalent forms to reveal and

***Transfer***

*Students will be able to independently use their learning to…*

Analyze functions and solve real world applications involving geometry, physics, business, and economics.

***Meaning***

UNDERSTANDINGS

*Students will understand that…*

- Functions can behave differently at different points in their domain.
- Limits can be used to analyze functions numerically, graphically and analytically.

ESSENTIAL QUESTIONS

- What is a limit?
- How can we evaluate limits numerically, graphically and analytically?
- How can limits be used to analyze functions?

***Acquisition***

*Students will know…*

- Average and instantaneous speed
- Definition of a limit
- Properties of limits
- One-sided and two-sided limits
- Sandwich Theorem
- Finite limits approaching infinity
- Infinite limits approaching a constant
- Definitions of vertical and horizontal asymptotes
- End behavior models
- Continuity at a point
- Removable, jump, oscillating and infinite discontinuities
- Continuous functions on an interval
- Composites of continuous functions
- Intermediate Value Theorem for Continuous Functions
- Average rates of change

*Students will be skilled at…*

- Evaluating limits
- Calculating average and instantaneous speed
- Finding function values
- Applying the properties of limits
- Finding limits indirectly using the Sandwich Theorem
- Calculating and verifying end behavior models for functions
- Evaluating limits to identify vertical and horizontal asymptotes
- Calculating the slope of a curve
- Writing the equation of a tangent line to a curve at a given point
- Writing the equation of a normal line to a curve at a given point
- Calculating the average rate of change of a function.

| | | |
|---|---|---|
| explain different properties of the function.<br>9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).<br><br>**Build new functions from existing functions**<br>3. Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x), f(kx),* and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.<br><br>**Construct and compare linear, quadratic, and exponential models and solve problems**<br>1. Distinguish between situations that can be modeled with linear functions and with exponential functions.<br><br>**Interpret expressions for functions in terms of the situation they model**<br>5. Interpret the parameters in a linear or exponential function in terms of a context.<br><br>**Model periodic phenomena with trigonometric functions**<br>5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.<br><br>**Interpret linear models**<br>7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. | • Slope of a curve<br>• Tangent to a curve<br>• Normal to a curve | |

## Unit 1- Limits and Rates of Change
## Stage 2 – Evidence

| Evaluative Criteria | Assessment Evidence |
|---|---|
| The following rubric will be applied to each problem on any summative assessment.<br><br>6 Complete and correct solution including documentation of all concepts. Appropriate units will be included as needed.<br>5 Solution contains one minor arithmetic or algebraic error but demonstrates complete understanding of necessary concepts. OR Complete and correct solution without units.<br>4 Solution contains more than one minor arithmetic or algebraic error but demonstrates partial understanding of necessary concepts.<br>3 Solution contains one conceptual error or one or more major arithmetic or algebraic errors while demonstrating partial understanding of necessary concepts.<br>2 Limited understanding of necessary concepts while demonstrating proper mathematical skills.<br>1 Incorrect answer with no understanding of necessary concepts.<br>0 No answer is given. | PERFORMANCE TASK(S):<br>Summative assessment questions should be open-ended and model the format established by the College Board.<br><br>• Students will demonstrate ability to evaluate limits numerically, analytically and graphically; use limits to determine continuity of a function at a given point, find vertical and horizontal asymptotes, and identify end behavior models. Students will solve real life problems using limits.<br><br>• Students will demonstrate mastery of concepts on previous assessments in addition to ability to write equations of tangent and normal lines to a curve at a given point, sketching the graph of a function given conditions involving limits, and using limits to solve application problems. |
| The following is a student rubric to assess individual understanding during class activities.<br><br>4 - I understand completely and I can teach it to a classmate.<br>3 - I understand the concept but I do not think I can explain it to a classmate.<br>2 - I can complete the task with assistance.<br>1 - I need help! | OTHER EVIDENCE:<br>• **Pre-assessment-**Using limits to analyze a function<br>• Throughout each lesson student understanding will be assessed through informal questioning, formative assessments, and student's self-assessment.<br>• Given a function find the slopes of the secant line through different points. Examine the change in slopes as the interval decreases and the secant approaches a tangent at a given point. Make a conclusion about the slope of the tangent to the curve at the given point<br>• **Post-assessment-** Using limits to analyze a function |

# Unit 1- Limits and Rates of Change
# Stage 3 – Learning Plan

*Summary of Key Learning Events and Instruction*

- **Exploration:** Explore properties of limits of two functions by evaluating each one separately as $x \to c$ and comparing it to the limit of their product as $x \to c$
- Analyze a piecewise function by drawing its graph and using one-sided limits to determine whether a limit exists at a given point.
- **Activity**: Using geometric applications and one-sided limits to reinforce the Sandwich Theorem, students will evaluate the $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$ .
- **Technology- Computer and/or iPad:** Use Calculus In Motion with Geometer's Sketchpad to explore limits at a constant and limits at infinity.
- The teacher and students will evaluate limits using various methods including graphical, analytical and numerical.
- Sketch the graph of a function that satisfies given conditions involving limits, including asymptotes.
- Evaluate limits using end-behavior models and evaluating continuity at $x = c$ .
- **Exploration:** Explore the use of factoring to remove a discontinuity
- **Technology- Computer and/or iPad:** Use Calculus In Motion with Geometer's Sketchpad to explore The Intermediate Value Theorem.
- Given a salary guide, transportation or parking fees defined using a greatest integer or piecewise function, graph the function and determine the intervals of continuity. Discuss the financial consequences of different time intervals.
- **Activity:** Given a function find the slopes of the secant line through different points. Examine the change in slopes as the interval decreases and the secant approaches a tangent at a given point. Make a conclusion about the slope of the tangent to the curve at the given point.
- Given the equation for the free fall of an object, find the speed of the object at a given time.
- Students will be encouraged to use alternative resources to prepare for the Advanced Placement Exam.

**Resources:**

**Activities have been created by Susan Gasko and Susan Okulewicz and can be found on the Mathematics shared teachers folder.**

**Technology:**
*Calculus In Motion*™ software, Animations of calculus for the Geometer's Sketchpad v4, Audrey Weeks
*Wolfram APP*
Khan Academy www.**khan**academy.org/
The College Board www.**collegeboard**.org/

**Explorations can be found in the following text:**
Calculus: Graphical, Numerical, Algebraic
Finney, Ross L.; Demana. Franklin D.; Waits, Bert K.; Kennedy, Daniel;
Pearson Education, Inc., 2007.
Teachers Resources;

- Teacher's Guide with Answers.

- Teacher's AP Correlations and Preparation Guide
- Student Practice Workbook

Multiple-Choice & Free-Response Questions in Preparation for the AP Calculus (AB) Examination, 9th Edition, Lederman, David, D & S Marketing Systems, Inc, 2012.
Barron's AP Calculus, 11th Edition, Barron's Educational Series, Inc, 2012.
Barron's AP Calculus Flash Cards, Barron's Educational Series, Inc, 2008.
5 Steps to a 5 AP Calculus AB & BC, 2012-2013 edition, William Ma, McGraw Hill Companies, 2011.
Teaching AP Calculus, Lin McMulllin, D & S Marketing Systems, Inc, 2005.

# Unit 2-Derivatives
# Stage 1 Desired Results

ESTABLISHED GOALS
Students will be able to find and graph the derivative of a function.

**<u>Common Core State Standards for Mathematics</u>**

**Interpret functions that arise in applications in terms of the context**

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums symmetries; end behavior; and periodicity*
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

**Analyze functions using different representations**
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
8. Write a function defined by an expression in different but equivalent forms to reveal and

***Transfer***

*Students will be able to independently use their learning to…*
Solve real world problems involving instantaneous rates of change, simple harmonic motion, motion along a line, sensitivity to change, growth and decay, and economics.

***Meaning***

UNDERSTANDINGS
*Students will understand that…*
- The connection between a function and its derivative can be explored using graphs, tables and analytical methods.

ESSENTIAL QUESTIONS
- What is a derivative?
- How will derivatives be used to model real life phenomena?

***Acquisition***

*Students will know…*
- The formal definition of a derivative
- The alternate definition of a derivative
- Derivative notation
- Relationships between the graph of a function and its derivative
- One-sided derivatives
- How a derivative might fail to exist at a point
- Differentiability implies local linearity
- The symmetric difference quotient
- Differentiability implies continuity
- The Intermediate Value Theorem for Derivatives
- Rules for differentiation
- Instantaneous rate of change
- Displacement
- Average velocity
- Instantaneous velocity
- Speed
- Acceleration
- Jerk
- Marginal cost

*Students will be skilled at…*
- Calculating derivatives using the definition of a derivative
- Calculating the slope of a curve at a point
- Graphing the derivative of a function given the graph of the original function
- Graphing the function given the graph of its derivative
- Graphing the derivative of a function given numerical data
- Identifying differentiability of a function at a point
- Identifying corners, cusps, discontinuities, vertical and horizontal tangents, and justifying their existence using limits
- Analyzing differentiable functions using a graphing calculator to explore local linearity
- Identifying various difference quotients that can be used to evaluate the derivative of a function
- Approximating derivatives numerically

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| explain different properties of the function.<br>9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).<br><br>**Build new functions from existing functions**<br>3. Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x), f(kx),* and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.<br><br>**Construct and compare linear, quadratic, and exponential models and solve problems**<br>1. Distinguish between situations that can be modeled with linear functions and with exponential functions.<br><br>**Interpret expressions for functions in terms of the situation they model**<br>5. Interpret the parameters in a linear or exponential function in terms of a context.<br><br>**Model periodic phenomena with trigonometric functions**<br>5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.<br><br>**Interpret linear models**<br>7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. | • Marginal revenue<br>• Derivatives of trigonometric functions and their inverses<br>• Chain rule<br>• Slope of a parametric function<br>• Implicit differentiation<br>• Derivatives of exponential and logarithmic functions | and graphically<br>• Applying The Intermediate Value Theorem<br>• Finding the derivative of a constant function<br>• Applying the power rule<br>• Applying the constant multiple rule<br>• Applying the sum and difference rule<br>• Applying the product and quotient rules<br>• Finding higher order derivatives<br>• Analyzing particle motion along a line<br>• Differentiating trigonometric functions and their inverses<br>• Differentiating composite functions using the chain rule<br>• Using implicit differentiation<br>• Differentiating exponential and logarithmic functions |

| Unit 2-Derivatives<br>Stage 2 - Evidence | |
|---|---|
| **Evaluative Criteria** | **Assessment Evidence** |
| The following rubric will be applied to each problem on any summative assessment.<br><br>6 Complete and correct solution including documentation of all concepts. Appropriate units will be included as needed.<br>5 Solution contains one minor arithmetic or algebraic error but demonstrates complete understanding of necessary concepts. OR Complete and correct solution without units.<br>4 Solution contains more than one minor arithmetic or algebraic error but demonstrates partial understanding of necessary concepts.<br>3 Solution contains one conceptual error or one or more major arithmetic or algebraic errors while demonstrating partial understanding of necessary concepts.<br>2 Limited understanding of necessary concepts while demonstrating proper mathematical skills.<br>1 Incorrect answer with no understanding of necessary concepts.<br>0 No answer is given. | PERFORMANCE TASK(S):<br>Summative assessment questions should be open-ended and model the format established by the College Board.<br><br>• Students will demonstrate ability to apply the definition of a derivative and to demonstrate understanding of differentiability.<br>• Students will demonstrate ability to apply the basic rules for differentiation, calculate derivatives of trigonometric functions, and apply derivatives to applications involving particle motion, sensitivity to change and economics.<br>• Students will demonstrate mastery of concepts on previous assessments in addition to ability to apply the chain rule to these concepts.<br>• Students will demonstrate mastery of all rules of differentiation including the use of chain rule and implicit differentiation; apply rules for differentiating inverse trigonometric functions, exponential functions and logarithmic functions. |
| The following is a student rubric to assess individual understanding during class activities.<br><br>4 - I understand completely and I can teach it to a classmate.<br>3 - I understand the concept but I do not think I can explain it to a classmate.<br>2 - I can complete the task with assistance.<br>1 - I need help! | OTHER EVIDENCE:<br>Throughout each lesson student understanding will be assessed through informal questioning, formative assessments, and student's self-assessment.<br><br>**Pre-assessment-** Evaluating and graphing rates of change of a function<br><br>Formative assessments should include but not be limited to the following concepts:<br><br>• Given the position function for a particle moving along a line the student will<br><br>a) Find the displacement during a given time interval |

b) Find the average velocity on the same interval
c) Find the instantaneous velocity at a given time
d) Find the acceleration at the same time
e) Determine the direction of the particle at any time on the interval
f) Determine the location of the particle when the position function is at a minimum or maximum.

- Given a position function of a body moving in simple harmonic motion:
  a) Find the body's velocity, speed, acceleration and jerk at time t
  b) Describe the motion of the body

- Suppose the functions f and g and their first derivatives have the following values

| $x$ | $f(x)$ | $g(x)$ | $f'(x)$ | $g'(x)$ |
|---|---|---|---|---|
| $-1$ | 0 | $-1$ | 2 | 1 |
| 0 | $-1$ | $-3$ | $-2$ | 4 |

Find the first derivative of the following combinations at the given value of x.

$a)\ \ 3f(x)-g(x), \qquad x=-1$

$b)\ \ f^2(x)g^3(x), \qquad x=0$

$c)\ \ g(f(x)), \qquad x=-1$

$d)\ \ f(g(x)), \qquad x=-1$

$e)\ \ \dfrac{f(x)}{g(x)+2}, \qquad x=0$

$f)\ \ g(x+f(x)), \qquad x=0$

**Post-assessment-** Evaluating and graphing rates of change of a function

## Unit 2-Derivatives
## Stage 3 – Learning Plan

*Summary of Key Learning Events and Instruction*

- **Activity:** Students will establish the relationship between the graph of the function and the graph of its derivative by calculating the derivative of the function at given points using the definition of a derivative.
- **Technology- Computer and/or iPad:** Use Calculus In Motion with Geometer's Sketchpad to explore the definition of derivative.
- Use one-sided derivatives to determine if a piecewise function has a derivative at a given point.

- **Activity:** FLASH CARDS Given the graph of a function, students will find the graph of its derivative.
- **Exploration:** Using the graphing calculator to explore differentiability at a given point.
- Sketch the graph of a continuous function whose derivative is a piecewise function stated as the combination of two constant functions.
- **Exploration:** Comparing the definition of a derivative and the symmetric difference quotient.
- Compare left-hand and right-hand derivatives with limits to show that a function is not differentiable at a given point.
- Using the rules for differentiation, determine whether or not a function has a horizontal tangent.
- Apply the rules for differentiation given values of two functions and their derivatives where no function is given.
- Apply the rules of differentiation to write the equations of lines that are perpendicular, parallel, or tangent to a curve.
- Apply the rules of differentiation to find the point on a curve where the tangent is parallel or perpendicular to a given function.
- Given a formula for a real world situation, find the indicated rate of change.
- **Exploration:** Explore Rates of Change using the rate of change of the area of a cross section of a tree with respect to its radius.
- **Exploration / Technology:** Using the graphing calculator, students will model horizontal motion using parametric equations.
- **Exploration / Technology:** Use the graphing calculator to compare the graph of a function and the graph of its derivative.
- **Activity:** Students will explore the relationship between velocity, speed and acceleration through graphs.
- Describe the motion of a particle given the graph of its velocity or position.
- Calculate marginal cost or marginal revenue and discuss the meaning of the results in terms of production.
- Describe simple harmonic motion using velocity, speed, acceleration and jerk.
- Find the equation of the line tangent to a given parametric function at a point defined by a given value of t.
- Apply implicit differentiation to find the tangent line and normal line to an ellipse at a given point.
- Given a function in exponential or logarithmic form representing a real world example of growth or decay, apply rules of differentiation to explain growth rates and when maximum or minimum growth will occur.
- Students will be encouraged to use alternative resources to prepare for the Advanced Placement Exam.

**Resources:**

**Activities have been created by Susan Gasko and Susan Okulewicz and can be found on the Mathematics shared teachers folder.**

**Technology:**
*Calculus In Motion*™ software, Animations of calculus for the Geometer's Sketchpad v4, Audrey Weeks
*Wolfram APP*
Khan Academy www.**khan**academy.org/
The College Board www.**collegeboard**.org/

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Pearson Education, Inc., 2007.
Teachers Resources;
- Teacher's Guide with Answers.

- Teacher's AP Correlations and Preparation Guide
- Student Practice Workbook

Multiple-Choice & Free-Response Questions in Preparation for the AP Calculus (AB) Examination, 9th Edition, Lederman, David, D & S Marketing Systems, Inc, 2012.
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Barron's AP Calculus Flash Cards, Barron's Educational Series, Inc, 2008.
5 Steps to a 5 AP Calculus AB & BC, 2012-2013 edition, William Ma, McGraw Hill Companies, 2011.
Teaching AP Calculus, Lin McMulllin, D & S Marketing Systems, Inc, 2005.

# Unit 3-Applications of Derivatives
# Stage 1 Desired Results

ESTABLISHED GOALS

Students will be able to apply derivatives in modeling real-life situations.

**<u>Common Core State Standards for Mathematics</u>**

**Interpret functions that arise in applications in terms of the context**

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums symmetries; end behavior; and periodicity*
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

**Analyze functions using different representations**

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
8. Write a function defined by an expression in different but equivalent forms to reveal and

***Transfer***

*Students will be able to independently use their learning to…*

Model discrete phenomena to solve real world problems involving optimization, approximation and related rates.

***Meaning***

UNDERSTANDINGS

*Students will understand that…*

- Rates of change and extreme values play a critical role in solving real world applications.
- Approximation plays an important role in the engineering and science fields.

ESSENTIAL QUESTIONS

- How are derivatives used to solve real world applications?

***Acquisition***

*Students will know…*

- Absolute and local extreme values
- The Extreme Value Theorem
- The Local Extreme Value Theorem
- Definition of critical point
- The Mean Value Theorem for Derivatives
- Increasing and decreasing functions
- Definition of antiderivative
- First Derivative Test for Local Extrema
- Definition of concavity
- Concavity test
- Definition of point of inflection
- Second Derivative Test for Local Extrema
- Strategy for solving maximum and minimum problems
- Definition of linearization
- Newton's Method

*Students will be skilled at…*

- Finding extreme values
- Applying the Extreme Value Theorem
- Identifying critical points and endpoints of a function within a given interval
- Applying the Mean Value Theorem for Derivatives
- Finding the intervals on which a function is increasing or decreasing
- Applying The First Derivative Test For Local Extrema to find local maximum and minimum vales of a function
- Determining the concavity of a function
- Identifying points of inflection
- Applying the Second Derivative Test for Local Extrema
- Graphing a function given information about its first and second derivatives
- Modeling geometric and economic applications of optimization

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| explain different properties of the function.<br>9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).<br><br>**Build new functions from existing functions**<br>3. Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x), f(kx),* and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.<br><br>**Construct and compare linear, quadratic, and exponential models and solve problems**<br>1. Distinguish between situations that can be modeled with linear functions and with exponential functions.<br><br>**Interpret expressions for functions in terms of the situation they model**<br>5. Interpret the parameters in a linear or exponential function in terms of a context.<br><br>**Model periodic phenomena with trigonometric functions**<br>5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.<br><br>**Interpret linear models**<br>7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. | • Definition of differentials<br>• Differential estimate of change<br>• Absolute, relative and percentage change<br>• Solution strategy for solving related rate problems | • Applying linearization to approximate the values of a function near a given point<br>• Applying Newton's Method to approximate real solutions of an equation<br>• Applying differentials to estimate change<br>• Applying differentials to estimate maximum error<br>• Analyzing the accuracy of estimates<br>• Solving related rate application problems |

## Unit 3-Applications of Derivatives
## Stage 2 – Evidence

| Evaluative Criteria | Assessment Evidence |
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| The following rubric will be applied to each problem on any summative assessment.<br><br>6 Complete and correct solution including documentation of all concepts. Appropriate units will be included as needed.<br>5 Solution contains one minor arithmetic or algebraic error but demonstrates complete understanding of necessary concepts. OR Complete and correct solution without units.<br>4 Solution contains more than one minor arithmetic or algebraic error but demonstrates partial understanding of necessary concepts.<br>3 Solution contains one conceptual error or one or more major arithmetic or algebraic errors while demonstrating partial understanding of necessary concepts.<br>2 Limited understanding of necessary concepts while demonstrating proper mathematical skills.<br>1 Incorrect answer with no understanding of necessary concepts.<br>0 No answer given. | PERFORMANCE TASK(S):<br>Summative assessment questions should be open-ended and model the format established by the College Board.<br>• Students will demonstrate ability to use analytical methods to find local extreme values, intervals on which the function is increasing or decreasing, inflection points, intervals on which the function is concave up or concave down**;** make connections between a function and its first and second derivatives both graphically and analytically, and to apply The Mean Value Theorem for Derivatives.<br>• Students will demonstrate ability to solve application problems involving optimization, approximation, and related rates.<br>• Students will demonstrate mastery of all concepts on both previous assessments using numerical, graphical and analytical approaches.<br>• **Optimization Project:** Given specific dimensions for poster board, students will calculate the dimensions of a box with a lid having the maximum volume possible. Students will then design and construct the box with the calculated dimensions. |
| The following is a student rubric to assess individual understanding during class activities.<br><br>4 - I understand completely and I can teach it to a classmate.<br>3 - I understand the concept but I do not think I can explain it to a classmate.<br>2 - I can complete the task with assistance.<br>1 - I need help! | OTHER EVIDENCE:<br>Throughout each lesson student understanding will be assessed through informal questioning, formative assessments, and student's self-assessment.<br><br>**Pre-assessment-** Applying rates of change to model real-life situations<br><br>Formative assessments should include but not be limited to the following concepts:<br>• Given a function find each of the following:<br>a) local extrema<br>b) intervals on which the function is increasing or decreasing<br>c) inflection points |

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| | d) intervals on which the function is concave up or concave down<br>• Example: Sketch a possible smooth curve $y = f(x)$ through the origin with the following properties:<br><br>$f'(x) < 0 \text{ for } x \le 2$<br>$f'(x) > 0 \text{ for } x \ge 2$<br>$f''(x) < 0 \text{ for } x < -1$<br>$f''(x) > 0 \text{ for } x > -1$<br><br>**Post-assessment-** Applying rates of change to model real-life situations |

## Unit 3-Applications of Derivatives
## Stage 3 – Learning Plan

*Summary of Key Learning Events and Instruction*

- **Technology- Computer and/or iPad:** Use Calculus In Motion with Geometer's Sketchpad to explore The Mean Value Theorem.
- **Exploration:** Sketching a possible graph of a function given information about its first and second derivative.
- Apply the First and Second Derivative Tests to find local extrema of a given function. Discuss the appropriate use of each method to obtain the same results.
- Apply the strategy for solving optimization problems to solve problems involving area, volume, and motion.
- Apply the strategy for solving optimization problems that address maximum profit and minimum cost. Students will discuss the economic implications of their results.
- **Exploration / Technology:** Using a graphing calculator to explore the local linearity of a function.
- Apply differentials to approximate the change in surface area or volume of a solid given a small change to one of its dimensions.
- Discuss the various aspects of error analysis in the context of approximation.
- **Exploration / Technology:** Using a graphing calculator to apply Newton's Method to estimate real solutions of a given equation.
- **Activity:** Students will complete a tiered activity involving related rates for a sliding ladder. Students will find the rate of change in position of the ladder, the rate of change of the angle formed by the ladder with the ground, or the rate of change of the area enclosed by the ladder the ground and the building.
- **Technology- Computer and/or iPad:** Use Calculus In Motion with Geometer's Sketchpad to explore a variety of related rate problems.
- Apply the strategy for solving related rate applications to solve problems involving area of geometric figures and solids.
- Apply the strategy for solving related rate applications to solve problems involving volumes of spheres, cones, cylindrical drums, and prisms.
- Apply the strategy for solving related rate applications to solve problems involving moving objects.
- Students will be encouraged to use alternative resources to prepare for the Advanced Placement Exam.

**Resources:**

**Activities have been created by Susan Gasko and Susan Okulewicz and can be found on the Mathematics shared teachers folder.**

**Technology:**
*Calculus In Motion*™ software, Animations of calculus for the Geometer's Sketchpad v4, Audrey Weeks
*Wolfram APP*
Khan Academy www.**khan**academy.org/
The College Board www.**collegeboard**.org/

**Explorations can be found in the following text:**
Calculus: Graphical, Numerical, Algebraic
Finney, Ross L.; Demana. Franklin D.; Waits, Bert K.; Kennedy, Daniel;
Pearson Education, Inc., 2007.
Teachers Resources;

- Teacher's Guide with Answers.
- Teacher's AP Correlations and Preparation Guide
- Student Practice Workbook

Multiple-Choice & Free-Response Questions in Preparation for the AP Calculus (AB) Examination, 9th Edition, Lederman, David, D & S Marketing Systems, Inc, 2012.
Barron's AP Calculus, 11th Edition, Barron's Educational Series, Inc, 2012.
Barron's AP Calculus Flash Cards, Barron's Educational Series, Inc, 2008.
5 Steps to a 5 AP Calculus AB & BC, 2012-2013 edition, William Ma, McGraw Hill Companies, 2011.
Teaching AP Calculus, Lin McMulllin, D & S Marketing Systems, Inc, 2005.

# Unit 4- Integrals
# Stage 1 Desired Results

ESTABLISHED GOALS

Students will be able to develop, understand and apply indefinite and definite integrals.

**<u>Common Core State Standards for Mathematics</u>**

**Interpret functions that arise in applications in terms of the context**

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums symmetries; end behavior; and periodicity*
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

**Analyze functions using different representations**

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
8. Write a function defined by an expression in different but equivalent forms to reveal and

***Transfer***

*Students will be able to independently use their learning to…*

- Apply approximation methods to solve real world problems where an analytical function is not available.
- Apply the Fundamental Theorem of Calculus to calculate quantities given rates of change in the context of Geometry, Physics and Economics.

***Meaning***

UNDERSTANDINGS

*Students will understand that…*

- There is a relationship between the derivative and definite integral as expressed in both parts of The Fundamental Theorem of Calculus.

ESSENTIAL QUESTIONS

- What is a definite integral and how is it different from the area under a curve?
- When is an approximation method appropriate to solve real world application problems?

***Acquisition***

*Students will know…*

- Area under a curve
- Distance traveled
- Rectangular Approximation Method (RAM) (LRAM)(RRAM)(MRAM)
- Volume of a sphere
- Riemann Sum
- Definite integral as a limit of Riemann Sums
- Integration notation
- Area under a curve as a definite integral
- Integrals on a calculator (numerical integral)
- Discontinuous integrable functions
- Properties of definite integrals
- Average value of a function
- Mean Value Theorem for Definite Integrals
- The Fundamental Theorem of Calculus

*Students will be skilled at…*

- Approximating the area under a curve of a nonnegative continuous function using RAM
- Calculating a particle's position given a function for its velocity or a table of values
- Applying RAM to approximate volumes of solids
- Expressing a limit of Riemann Sums as a definite integral
- Identifying intervals on which a function is above or below the *x*-axis
- Evaluating definite integrals using area formulas
- Expressing a quantity as a definite integral of a constant rate function
- Calculating the numerical integral using a graphing calculator
- Identifying points of discontinuity when

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| explain different properties of the function.<br>9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).<br><br>**Build new functions from existing functions**<br>3. Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x), f(kx),* and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.<br><br>**Construct and compare linear, quadratic, and exponential models and solve problems**<br>1. Distinguish between situations that can be modeled with linear functions and with exponential functions.<br><br>**Interpret expressions for functions in terms of the situation they model**<br>5. Interpret the parameters in a linear or exponential function in terms of a context.<br><br>**Model periodic phenomena with trigonometric functions**<br>5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.<br><br>**Interpret linear models**<br>7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. | (Part 1)<br>• Definition of antiderivative<br>• The Fundamental Theorem of Calculus (Part 2)<br>• Trapezoidal Rule<br>• Simpson's Rule | calculating definite integrals<br>• Applying the properties of definite integrals<br>• Calculating the average value of a function over a given interval<br>• Applying the Mean Value Theorem for Definite Integrals<br>• Applying The Fundamental Theorem of Calculus Part 1<br>• Identifying all possible functions with a given derivative (+constant)<br>• Applying The Fundamental Theorem of Calculus Part 2<br>• Calculating total area of a function over a closed interval analytically<br>• Calculating total area of a function over a closed interval numerically, using a graphing calculator<br>• Analyzing antiderivatives graphically<br>• Approximating the value of a definite integral using the Trapezoidal Rule<br>• Approximating the value of a definite integral using Simpson's Rule<br>• Determining if the approximation of a definite integral is an overestimate or underestimate |

# Unit 4-Integrals
## Stage 2 - Evidence

| Evaluative Criteria | Assessment Evidence |
|---|---|
| The following rubric will be applied to each problem on any summative assessment.<br><br>6 Complete and correct solution including documentation of all concepts. Appropriate units will be included as needed.<br>5 Solution contains one minor arithmetic or algebraic error but demonstrates complete understanding of necessary concepts. OR Complete and correct solution without units.<br>4 Solution contains more than one minor arithmetic or algebraic error but demonstrates partial understanding of necessary concepts.<br>3 Solution contains one conceptual error or one or more major arithmetic or algebraic errors while demonstrating partial understanding of necessary concepts.<br>2 Limited understanding of necessary concepts while demonstrating proper mathematical skills.<br>1 Incorrect answer with no understanding of necessary concepts.<br>0 No answer given. | PERFORMANCE TASK(S):<br>Summative assessment questions should be open-ended and model the format established by the College Board.<br><br>• Students will demonstrate the ability to apply RAM to approximate areas and volumes, to express a Riemann Sum as a definite integral, to evaluate definite integrals using area, to apply the properties of definite integrals, to calculate the average value of a function, to evaluate definite integrals using antiderivatives.<br>• Students will demonstrate mastery of concepts on previous assessments in addition to applying The Fundamental Theorem of Calculus and the Trapezoidal Rule. |
| The following is a student rubric to assess individual understanding during class activities.<br><br>4 - I understand completely and I can teach it to a classmate.<br>3 - I understand the concept but I do not think I can explain it to a classmate.<br>2 - I can complete the task with assistance.<br>1 - I need help! | OTHER EVIDENCE:<br>Throughout each lesson student understanding will be assessed through informal questioning, formative assessments, and student's self-assessment.<br><br>**Pre-assessment-** Applying definite and indefinite integrals<br><br>Formative assessments should include but not be limited to the following concepts:<br><br>• Given the graph of $f(x)$ where position is $s = \int_0^t f(x)\,dx$ |

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| | Find the following:<br>a) The particle's velocity at a given time<br>b) Whether the particle's acceleration is positive or negative at a given time<br>c) The particle's position at a given time<br>d) When the position has the largest value<br>e) When the acceleration is zero<br>f) When the particle is moving toward or away from the origin<br><br>• Given a table of values for the rate of change of a function over a closed interval, approximate the value of a definite integral and the average value of the function using MRAM and The Trapezoidal Rule. Discuss accuracy of your results in terms of overestimates or underestimates.<br><br>**Post-assessment-** Applying definite and indefinite integrals |

## Unit 4-Integrals
## Stage 3 – Learning Plan

*Summary of Key Learning Events and Instruction*

- **Technology- Computer and/or iPad:** Use Calculus In Motion with Geometer's Sketchpad to explore Riemann Sums.
- **Activity:** Students will complete a tiered activity to explore the differences between finding area with RRAM, MMRAM, or using a shape other than a rectangle.
- **Exploration:** Examine the relationship between RRAM, MRAM, and LRAM, and whether a function is increasing or decreasing.
- **Technology- Computer and/or iPad:** Use Calculus In Motion with Geometer's Sketchpad to explore using definite integrals to calculate area under a curve.
- **Exploration:** Finding the value of definite integrals using transformations of a given integral.
- Apply the integral of a constant function to express a quantity, given a rate of change.
- **Technology- Computer and/or iPad:** Use Calculus In Motion with Geometer's Sketchpad to explore The Mean Value Theorem for Integrals.
- Apply the Properties of Definite Integrals to perform operations on given definite integrals by substituting numerical values for definite integrals.
- Calculate the average value of a function on a given integral using area formulas.
- Apply The Fundamental Theorem of Calculus to find the derivative of a definite integral of a function over the given interval from a to x.
- Apply The Fundamental Theorem of Calculus Part 2 to evaluate a definite integral analytically.
- Calculate the total area of a region between a function and the *x*-axis using The Fundamental Theorem of Calculus Part 2.
- Students will be encouraged to use alternative resources to prepare for the Advanced Placement Exam.

**Resources:**

**Activities have been created by Susan Gasko and Susan Okulewicz and can be found on the Mathematics shared teachers folder.**

**Technology:**
*Calculus In Motion*™ software, Animations of calculus for the Geometer's Sketchpad v4, Audrey Weeks
*Wolfram APP*
Khan Academy www.**khan**academy.org/
The College Board www.**collegeboard**.org/

**Explorations can be found in the following text:**
Calculus: Graphical, Numerical, Algebraic
Finney, Ross L.; Demana. Franklin D.; Waits, Bert K.; Kennedy, Daniel;
Pearson Education, Inc., 2007.
Teachers Resources;

- Teacher's Guide with Answers.
- Teacher's AP Correlations and Preparation Guide
- Student Practice Workbook

Multiple-Choice & Free-Response Questions in Preparation for the AP Calculus (AB) Examination, 9th Edition, Lederman, David, D & S Marketing Systems, Inc, 2012.
Barron's AP Calculus, 11th Edition, Barron's Educational Series, Inc, 2012.
Barron's AP Calculus Flash Cards , Barron's Educational Series, Inc, 2008.
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Teaching AP Calculus, Lin McMulllin, D & S Marketing Systems, Inc, 2005.

# Unit 5-Differential Equations
# Stage 1 Desired Results

ESTABLISHED GOALS

Students will be able to solve problems using differential equations and mathematical modeling.

**<u>Common Core State Standards for Mathematics</u>**

**Interpret functions that arise in applications in terms of the context**

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums symmetries; end behavior; and periodicity*

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

**Analyze functions using different representations**

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

8. Write a function defined by an expression in

***Transfer***

*Students will be able to independently use their learning to…*

Apply advanced methods of integration to solve real world applications in Economics and Science.

***Meaning***

UNDERSTANDINGS

*Students will understand that…*

- There are various methods of integration not all applicable to every problem.
- The solution to a differential equation, given an initial condition, is a function.
- Calculus plays a key role in real-world applications of exponential growth and decay.

ESSENTIAL QUESTIONS

- What are the various methods of integration available to us and how do we choose the correct method for a given problem?
- How are differential equations used to solve real-world applications?

***Acquisition***

*Students will know…*

- Definition of differential equations
- Initial value problem
- Slope fields
- Euler's Method
- Definition of indefinite integral
- Properties of indefinite integrals
- Integration by Substitution
- Integration by Parts Formula
- Tabular Integration
- Properties of Logarithmic and Exponential Functions
- Definition of separable differential equations
- The Law of Exponential Change
- Compound interest formulas
- Definition of half-life
- Newton's Law of Cooling

*Students will be skilled at…*

- Finding the general solution to a differential equation
- Finding the particular solution to differential equation given an initial value
- Constructing a slope field for a differential equation
- Sketching the graph of a particular solution to a differential equation on a slope field
- Applying Euler's Method to approximate the value of a function at a particular point
- Evaluating an indefinite integral
- Applying properties of indefinite integrals
- Applying Integration by Substitution to evaluate indefinite integrals

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| different but equivalent forms to reveal and explain different properties of the function.<br>9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).<br><br>**Build new functions from existing functions**<br>3. Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x), f(kx),* and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.<br><br>**Construct and compare linear, quadratic, and exponential models and solve problems**<br>1. Distinguish between situations that can be modeled with linear functions and with exponential functions.<br><br>**Interpret expressions for functions in terms of the situation they model**<br>5. Interpret the parameters in a linear or exponential function in terms of a context.<br><br>**Model periodic phenomena with trigonometric functions**<br>5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.<br><br>**Interpret linear models**<br>7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. | • Partial Fraction Decomposition | • Restating a definite integral using a u-substitution<br>• Evaluating a definite integral using integration by substitution<br>• Applying Integration by Parts to evaluate integrals<br>• Choosing the appropriate parts for use in the Integration by Parts Formula using LIPET. (natural **L**ogarithm, **I**nverse trigonometric, **P**olynomial, **E**xponential, and **T**rigonometric functions)<br>• Solving for the unknown integral<br>• Applying Tabular Integration to evaluate integrals<br>• Evaluating integrals using Properties of Logarithmic and Exponential functions<br>• Applying Separation of Variables to solve Differential Equations<br>• Applying The Law of Exponential Change to solve real-world applications involving exponential growth and decay in Economics and Science<br>• Evaluating integrals using Partial Fraction Decomposition |

| Unit 5-Differential Equations<br>Stage 2 - Evidence | |
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| **Evaluative Criteria** | **Assessment Evidence** |
| The following rubric will be applied to each problem on any summative assessment.<br><br>6 Complete and correct solution including documentation of all concepts. Appropriate units will be included as needed.<br>5 Solution contains one minor arithmetic or algebraic error but demonstrates complete understanding of necessary concepts. OR Complete and correct solution without units.<br>4 Solution contains more than one minor arithmetic or algebraic error but demonstrates partial understanding of necessary concepts.<br>3 Solution contains one conceptual error or one or more major arithmetic or algebraic errors while demonstrating partial understanding of necessary concepts.<br>2 Limited understanding of necessary concepts while demonstrating proper mathematical skills.<br>1 Incorrect answer with no understanding of necessary concepts.<br>0 No answer given. | PERFORMANCE TASK(S):<br>Summative assessment questions should be open-ended and model the format established by the College Board.<br><br>• Students will demonstrate the ability to evaluate definite and indefinite integrals analytically using properties of integrals and Integration by Substitution.<br>• Students will demonstrate the ability to evaluate definite and indefinite integrals analytically using properties of integration, Integration by Substitution, and Integration by Parts, solve initial value problems and construct slope fields for differential equations.<br>• Students will demonstrate ability to evaluate definite and indefinite integrals using partial fraction decomposition. |
| The following is a student rubric to assess individual understanding during class activities.<br><br>4 - I understand completely and I can teach it to a classmate.<br>3 - I understand the concept but I do not think I can explain it to a classmate.<br>2 - I can complete the task with assistance.<br>1 - I need help! | OTHER EVIDENCE:<br>Throughout each lesson student understanding will be assessed through informal questioning, formative assessments, and student's self-assessment.<br><br>**Pre-assessment-** Solving problems using differential equations and mathematical modeling<br><br>Formative assessments should include but not be limited to the following concepts:<br>• Choose the appropriate method of evaluating each integral. Explain your reasoning. |

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| | a) $\int_0^1 \frac{dx}{3(1+x)^2}$ b) $\int_0^1 \frac{dx}{3x^2}$ c) $\int e^x \cos x\, dx$ d) $\int x^2 \sin 4x\, dx$<br><br>• Students will verify solutions to complex integration problems using *Wolfram* APP on their iPad.<br>• Apply the Law of Exponential Change to solve differential equations resulting in specific formulas for compound interest, radioactive decay, half-life, Newton's Law of Cooling, and population growth.<br><br>**Post-assessment-** Solving problems using differential equations and mathematical modeling |

## Unit 5-Differential Equations
## Stage 3 – Learning Plan

*Summary of Key Learning Events and Instruction*

- **Technology- Computer and/or iPad:** Use Calculus In Motion with Geometer's Sketchpad to explore Slope Fields and Euler's Method.
- **Activity:** Given a differential equation, students will create a slope field, sketch a particular solution to the differential equation for a given value on their slope field, solve the differential equation using Separation of Variables, and compare solutions from the two methods.
- **Activity:** Students will complete a tiered activity to apply Separation of Variables to solve initial value problems
- Apply the method of u-substitution for various types of functions including polynomial, trigonometric, exponential and logarithmic
- **Exploration:** Given an integral, explore the choice of parts that leads to a successful solution using Integration by Parts.
- Use the Law of Exponential Change to solve differential equations resulting in specific formulas for compound interest, radioactive decay, half-life, Newton's Law of Cooling, and population growth.
- Students will be encouraged to use alternative resources to prepare for the Advanced Placement Exam.

**Resources:**

**Activities have been created by Susan Gasko and Susan Okulewicz and can be found on the Mathematics shared teachers folder.**

**Technology:**
*Calculus In Motion*™ software, Animations of calculus for the Geometer's Sketchpad v4, Audrey Weeks
*Wolfram APP*
Khan Academy www.**khan**academy.org/
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Finney, Ross L.; Demana. Franklin D.; Waits, Bert K.; Kennedy, Daniel;
Pearson Education, Inc., 2007.
Teachers Resources;

- Teacher's Guide with Answers.
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Barron's AP Calculus, 11th Edition, Barron's Educational Series, Inc, 2012.
Barron's AP Calculus Flash Cards , Barron's Educational Series, Inc, 2008.
5 Steps to a 5 AP Calculus AB & BC , 2012-2013 edition, William Ma, McGraw Hill Companies, 2011.
Teaching AP Calculus, Lin McMulllin, D & S Marketing Systems, Inc, 2005.

# Unit 6- Applications of Definite Integrals
## Stage 1 Desired Results

ESTABLISHED GOALS

Students will be able to apply definite integrals to model real-life situations mathematically.

**Common Core State Standards for Mathematics**

**Interpret functions that arise in applications in terms of the context**

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums symmetries; end behavior; and periodicity*
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

**Analyze functions using different representations**

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
8. Write a function defined by an expression in different but equivalent forms to reveal and

***Transfer***

*Students will be able to independently use their learning to…*

Apply their knowledge of integral calculus to model problems involving rates of change.

***Meaning***

UNDERSTANDINGS

*Students will understand that…*

- The integral of a rate gives the net change.
- Applications involving area and volume play an important role in the fields of Engineering and Business.

ESSENTIAL QUESTIONS

- How do we determine the appropriate method of integration for a given problem?
- What role does Integral Calculus play in Engineering, Manufacturing and Business?

***Acquisition***

*Students will know…*

- Position
- Displacement
- Total distance
- Relationship between position, velocity and acceleration
- Relationship between Net Change and Total Accumulation
- Definition of area between two curves
- Definition of boundaries with changing functions
- Definition of volume of a solid
- Strategy for calculating volume of a solid by slicing
- Strategies for calculating the volume of a solid of revolution

*Students will be skilled at…*

- Describing the motion of a particle given its velocity
- Calculating a particle's displacement and final position given its velocity on an interval
- Calculating the total distance traveled by a particle given its velocity on an interval
- Calculating total consumption given a rate of change analytically
- Approximating total consumption given data representing the rate of change
- Calculating the area between two curves on a given interval analytically
- Identifying boundaries when calculating area between two curves
- Calculating area using subregions
- Calculate the area of a region by integrating with respect to either axis
- Applying the slicing Method to calculate the volume of a solid with a given cross

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| explain different properties of the function.<br>9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).<br><br>**Build new functions from existing functions**<br>3. Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x), f(kx),* and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.<br><br>**Construct and compare linear, quadratic, and exponential models and solve problems**<br>1. Distinguish between situations that can be modeled with linear functions and with exponential functions.<br><br>**Interpret expressions for functions in terms of the situation they model**<br>5. Interpret the parameters in a linear or exponential function in terms of a context.<br><br>**Model periodic phenomena with trigonometric functions**<br>5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.<br><br>**Interpret linear models**<br>7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. | | section<br><br>• Calculating the volume of a solid by revolving a particular region about a given axis<br>• Recognizing when a cross section is a disk or a washer when finding volumes for solids of revolution<br>• Calculating the volume of a solid using The Cylindrical Shell Method<br>• Visualizing solids that are formed when a cross section of a solid is perpendicular to the axis over a given interval<br>• Visualizing solids formed by rotating a plane region about a given axis |

# Unit 6- Applications of Definite Integrals
## Stage 2 – Evidence

| Evaluative Criteria | Assessment Evidence |
|---|---|
| The following rubric will be applied to each problem on any summative assessment.<br><br>6 Complete and correct solution including documentation of all concepts. Appropriate units will be included as needed.<br>5 Solution contains one minor arithmetic or algebraic error but demonstrates complete understanding of necessary concepts. OR Complete and correct solution without units.<br>4 Solution contains more than one minor arithmetic or algebraic error but demonstrates partial understanding of necessary concepts.<br>3 Solution contains one conceptual error or one or more major arithmetic or algebraic errors while demonstrating partial understanding of necessary concepts.<br>2 Limited understanding of necessary concepts while demonstrating proper mathematical skills.<br>1 Incorrect answer with no understanding of necessary concepts.<br>0 No answer given. | PERFORMANCE TASK(S):<br>Summative assessment questions should be open-ended and model the format established by the College Board.<br><br>• Students will demonstrate the ability to describe particle motion in the context of position, displacement and total distance given its velocity; understand the connection between net change and total accumulation, and calculate the area of a region.<br>• Students will demonstrate mastery of all concepts on the previous assessment in addition to calculating volumes of solids using slicing and rotation methods.<br>• **Project:** Solids of Revolution in our Diet. Students will choose a food item or container with a circular cross section and calculate its volume by creating functions to model the boundary and rotating the functions about an axis. Students will present their findings to the class in a creative display. |
| The following is a student rubric to assess individual understanding during class activities.<br><br>4 - I understand completely and I can teach it to a classmate.<br>3 - I understand the concept but I do not think I can explain it to a classmate.<br>2 - I can complete the task with assistance.<br>1 - I need help! | OTHER EVIDENCE:<br>Throughout each lesson student understanding will be assessed through informal questioning, formative assessments, and student's self-assessment.<br><br>**Pre-assessment-** Applying definite integrals to solve applications involving area and volume<br><br>Formative assessments should include but not be limited to the following concepts:<br>• Given the function $v(t)$ in m/sec, of a particle moving along the *x*-axis on a specified interval at a specific starting position<br>a) Determine when the particle is moving to the right, left, and stopped.<br>b) Find the particle's displacement for the given time interval and its final position. |

| | |
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| | c) Find the total distance traveled by the particle.<br><br>• Given a continuous function on the closed interval $[0, 3]$ defined in the first quadrant, calculate the volume of the solid formed by rotating the function about the x-axis, the y-axis, the line x=4, and the line y=5.<br>• Students will work in small groups to calculate the volume of a ring pop. One half of the class will apply the slicing method while the other half of the class will apply the rotation method. Students will compare their results.<br><br>**Post-assessment-** Applying definite integrals to solve applications involving area and volume |

## Unit 6- Applications of Definite Integrals
## Stage 3 – Learning Plan

*Summary of Key Learning Events and Instruction*

- Students will calculate the volume of different shaped chocolate eggs by creating a function to model the boundary and rotating the function about an axis.
- **Technology- Computer and/or iPad:** Use Calculus In Motion with Geometer's Sketchpad to explore the area between two curves
- **Technology- Computer and/or iPad:** Use Calculus In Motion with Geometer's Sketchpad to explore the volume of a solid by slicing with a given cross section
- **Technology- Computer and/or iPad:** Use Calculus In Motion with Geometer's Sketchpad to explore to find the volume of a solid by revolution
- Students will be encouraged to use alternative resources to prepare for the Advanced Placement Exam.

**Resources:**

**Activities have been created by Susan Gasko and Susan Okulewicz and can be found on the Mathematics shared teachers folder.**

**Technology:**
*Calculus In Motion*™ software, Animations of calculus for the Geometer's Sketchpad v4, Audrey Weeks
*Wolfram APP*
Khan Academy www.**khan**academy.org/
The College Board www.**collegeboard**.org/

**Explorations can be found in the following text:**
Calculus: Graphical, Numerical, Algebraic
Finney, Ross L.; Demana. Franklin D.; Waits, Bert K.; Kennedy, Daniel;
Pearson Education, Inc., 2007.
Teachers Resources;
- Teacher's Guide with Answers.

- Teacher's AP Correlations and Preparation Guide
- Student Practice Workbook

Multiple-Choice & Free-Response Questions in Preparation for the AP Calculus (AB) Examination, 9th Edition, Lederman, David, D & S Marketing Systems, Inc, 2012.
Barron's AP Calculus, 11th Edition, Barron's Educational Series, Inc, 2012.
Barron's AP Calculus Flash Cards , Barron's Educational Series, Inc, 2008.
5 Steps to a 5 AP Calculus AB & BC , 2012-2013 edition, William Ma, McGraw Hill Companies, 2011.
Teaching AP Calculus, Lin McMulllin, D & S Marketing Systems, Inc, 2005.

# Unit 7- Advanced Applications of Derivatives and Integrals
# Stage 1 Desired Results

ESTABLISHED GOALS

Students will be able to apply concepts from differential and integral calculus to solve application problems numerically, analytically, and graphically.

**<u>Common Core State Standards for Mathematics</u>**

**Interpret functions that arise in applications in terms of the context**

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums symmetries; end behavior; and periodicity*
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

**Analyze functions using different representations**

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

## *Transfer*

*Students will be able to independently use their learning to…*

Apply problem-solving skills to unfamiliar situations involving derivatives and integrals.

## *Meaning*

UNDERSTANDINGS

*Students will understand that…*

- Calculus plays an integral role in our everyday lives.

ESSENTIAL QUESTIONS

- How do we recognize which mathematical concepts are necessary to solve elaborate application problems?

## *Acquisition*

*Students will know…*

- All definitions and properties associated with
  - Limits
  - Derivatives
  - Integrals
- All rules for differentiation
- All rules for integration

*Students will be skilled at…*

- Applying all definitions and properties to solve advanced calculus problems numerically, analytically, and graphically.
- Applying all definitions and properties to solve application problems in unfamiliar contexts, involving geometry, science, and economics.

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| 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.<br>9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).<br><br>**Build new functions from existing functions**<br>3. Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x), f(kx),* and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.<br><br>**Construct and compare linear, quadratic, and exponential models and solve problems**<br>1. Distinguish between situations that can be modeled with linear functions and with exponential functions.<br><br>**Interpret expressions for functions in terms of the situation they model**<br>5. Interpret the parameters in a linear or exponential function in terms of a context.<br><br>**Model periodic phenomena with trigonometric functions**<br>5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.<br><br>**Interpret linear models**<br>7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. | | |

## Unit 7- Advanced Applications of Derivatives and Integrals
## Stage 2 - Evidence

| Evaluative Criteria | Assessment Evidence |
|---|---|
| The following rubric will be applied to each problem on any summative assessment.<br><br>6 Complete and correct solution including documentation of all concepts. Appropriate units will be included as needed.<br>5 Solution contains one minor arithmetic or algebraic error but demonstrates complete understanding of necessary concepts. OR Complete and correct solution without units.<br>4 Solution contains more than one minor arithmetic or algebraic error but demonstrates partial understanding of necessary concepts.<br>3 Solution contains one conceptual error or one or more major arithmetic or algebraic errors while demonstrating partial understanding of necessary concepts.<br>2 Limited understanding of necessary concepts while demonstrating proper mathematical skills.<br>1 Incorrect answer with no understanding of necessary concepts.<br>0 No answer given. | PERFORMANCE TASK(S):<br>Summative assessment questions should be open-ended and model the format established by the College Board.<br><br>• Students will demonstrate their ability to apply their knowledge of calculus to successfully complete multiple choice and free response problems at the Advanced Placement level.<br>• ***Project:*** Given an economics situation, students will use differential and integral calculus to solve a task in a small business setting. Students will present their findings to the class using MathType, Keynote, and iMovie.<br>• Students will individually analyze the results of their project to make suggestions and improvements. |
| The following is a student rubric to assess individual understanding during class activities.<br><br>4 - I understand completely and I can teach it to a classmate.<br>3 - I understand the concept but I do not think I can explain it to a classmate.<br>2 - I can complete the task with assistance.<br>1 - I need help! | OTHER EVIDENCE:<br>Throughout each lesson student understanding will be assessed through informal questioning, formative assessments, and student's self-assessment.<br><br>Formative assessments should include but not be limited to the following:<br>• College Board released exams<br>• D&S Marketing Systems published practice exams |

# Unit 7- Advanced Applications of Derivatives and Integrals
# Stage 3 – Learning Plan

*Summary of Key Learning Events and Instruction*

- Students will complete the 2008 released College Board practice exam in a classroom setting within the designated timeframe.
- Students will analyze and make corrections in areas needing improvement.
- Blocks of time will be designated to implement strategies for successfully completing problems in the multiple-choice format.
- Blocks of time will be designated to implement strategies for successfully completing problems in the free response format.
- Free response questions will be discussed using the College Board released 9-point rubric.
- Students will organize their study materials to prepare for the Advanced Placement Exam.

**Resources:**

**Activities have been created by Susan Gasko and Susan Okulewicz and can be found on the Mathematics shared teachers folder.**

**Technology:**
*Calculus In Motion*™ software, Animations of calculus for the Geometer's Sketchpad v4, Audrey Weeks
*Wolfram APP*
Khan Academy www.**khan**academy.org/
The College Board www.**collegeboard**.org/

**Explorations can be found in the following text:**
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Pearson Education, Inc., 2007.
Teachers Resources;

- Teacher's Guide with Answers.
- Teacher's AP Correlations and Preparation Guide
- Student Practice Workbook

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Teaching AP Calculus, Lin McMulllin, D & S Marketing Systems, Inc, 2005.

## Advanced Placement Calculus AB
## Benchmark Assessment Quarter 1

1. Students will demonstrate ability to evaluate limits numerically, analytically and graphically.
2. Students will use limits to determine continuity of a function at a given point, to find vertical and horizontal asymptotes, and to identify end behavior models.
3. Students will be able to write equations of tangent and normal lines to a curve at a given point.
4. Students will be able to sketch the graph of a function given conditions involving limits.
5. Students will successfully use limits to solve application problems.
6. Students will demonstrate ability to apply the definition of a derivative and to demonstrate understanding of differentiability.
7. Students will demonstrate ability to apply the basic rules for differentiation.
8. Students will demonstrate ability to apply the chain rule and implicit differentiation to calculate derivatives.
9. Students will demonstrate ability to differentiate trigonometric functions, inverse trigonometric functions, exponential functions and logarithmic functions.
10. Students will successfully apply derivatives to applications involving particle motion, sensitivity to change and economics.

## Advanced Placement Calculus AB
## Benchmark Assessment Quarter 2

1. Students will demonstrate the ability to use analytical methods to find local extreme values, intervals on which a function is increasing or decreasing, points of inflection, and intervals on which a function is concave up or concave down.
2. Students will be able to make connections between a function and its first and second derivatives both graphically and analytically.
3. Students will successfully apply The Mean Value Theorem for Derivatives.
4. Students will successfully solve application problems involving optimization, approximation, and related rates.
5. Students will demonstrate the ability to approximate area and volume using RAM.
6. Students will be able to express a Riemann Sum as a definite integral.
7. Students will be able to evaluate definite integrals using area formulas and antiderivatives.
8. Students will demonstrate the ability to apply the properties of definite integrals to evaluate integrals and to calculate the average value of a function.
9. Students will successfully apply The Fundamental Theorem of Calculus.
10. Students will successfully apply The Trapezoidal Rule.

## Advanced Placement Calculus AB
## Benchmark Assessment Quarter 3

1. Students will be able to apply the properties of integrals to evaluate definite and indefinite integrals analytically.
2. Students will be able to evaluate definite and indefinite integrals analytically using Integration by Substitution.
3. Students will be able to evaluate definite and indefinite integrals analytically using Integration by Parts.
4. Students will successfully solve initial value problems and construct slope fields for differential equations.
5. Students will be able to evaluate definite and indefinite integrals analytically using Partial Fraction Decomposition.
6. Students will be able to describe particle motion in the context of position, displacement and total distance given its velocity.
7. Students will demonstrate an understanding of the connection between net change and total accumulation.
8. Students will be able to apply definite integrals to calculate the area of a region.
9. Students will be able to calculate volumes of solids using slicing and rotation methods.

## Advanced Placement Calculus AB
## Benchmark Assessment Quarter 4

1. Students will demonstrate the ability to apply their knowledge of differential and integral calculus to successfully complete multiple choice and free response problems at the Advanced Placement level.
2. Students will apply their knowledge of differential and integral calculus by completing and presenting an in-depth real-life application project.
3. Students will present their project to the class using MathType, Keynote, and iMovie.
4. Students will individually analyze the results of their project to make suggestions and improvements.