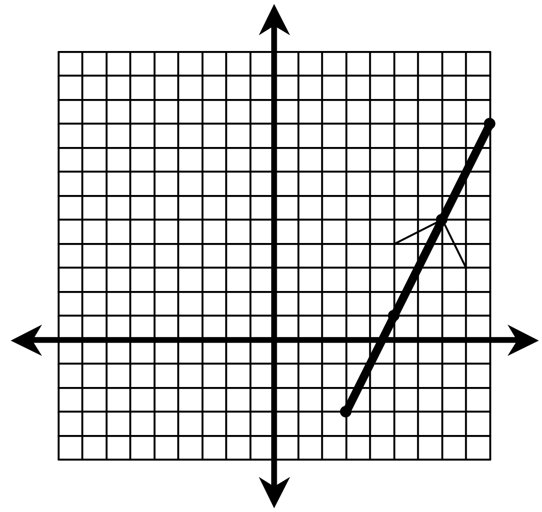
AP Calculus page 535ff Solutions

1)

t	0	1	2	3
x	3	5	7	9
y	-3	1	5	9

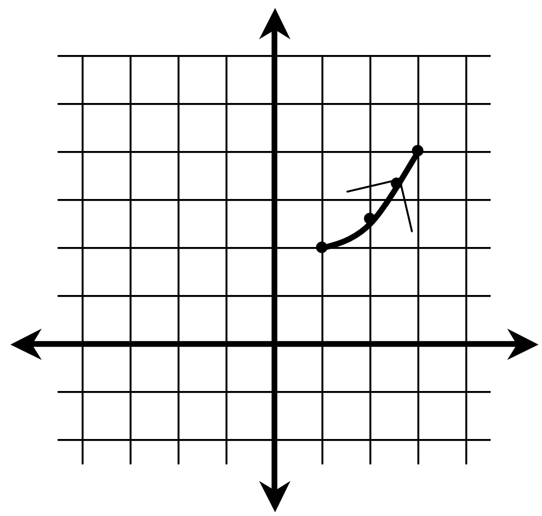


y is a function of x.

$t = \dfrac{x-3}{2}$ $\quad y = 4\left(\dfrac{x-3}{2}\right) - 3 = 2x - 9$, on $x \in [3,9]$

2)

t	3	6	8	11
x	1	2	√6	3
y	2	11/4	13/4	4

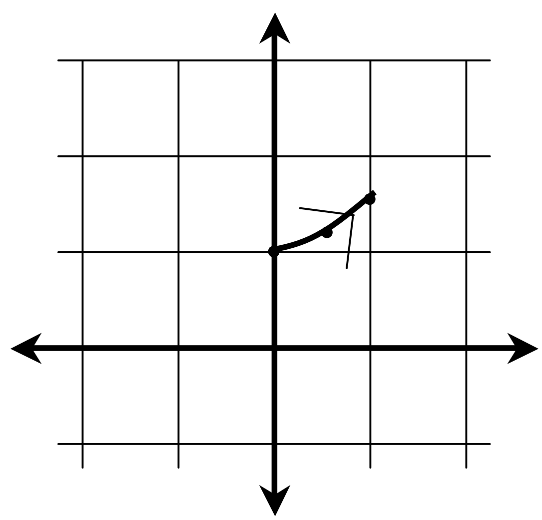


y is a function of x.

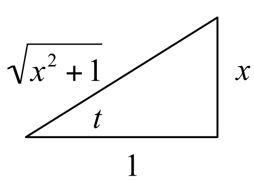
$t = x^2 + 2$ $\quad y = \dfrac{x^2+7}{4}$, on $x \in [1,3]$

3)

t	0	π/6	π/4
x	0	√3/3≈0.577	1
y	1	2√3/3≈1.155	√2≈1.414



y is a function of x.

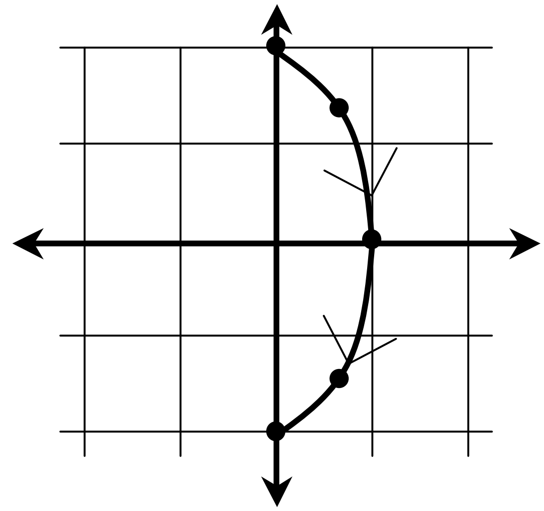


$x = \tan t$ $\quad y = \sec t = \sqrt{x^2+1}$, on $x \in [0,1]$

4)

t	0	π/4	π/2	3π/4	π
x	0	√2/2≈0.707	1	√2/2≈0.707	0
y	2	√2≈1.414	0	-√2≈ -1.414	-2

y is **not** a function of x.



$$\sin^2 t + \cos^2 t = 1$$

$$x^2 + \left(\frac{y}{2}\right)^2 = 1 \Rightarrow x^2 + \frac{y^2}{4} = 1 \text{ on } x \in [0,1]$$

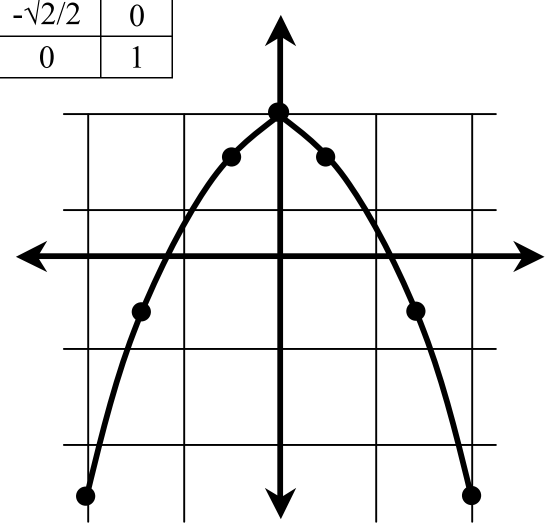
5)

t	0	π/6	π/4	π/2	3π/4	π	5π/4	3π/2	7π/4	2π
x	0	.5	√2/2	1	√2/2	0	-√2/2	-1	-√2/2	0
y	1	√3/2	0	-1	0	1	0	-1	0	1

y is a function of x.

$$\cos(2t) = 1 - 2\sin^2 t$$

$$y = 1 - 2x^2 \text{ on } x \in [-1,1]$$



[-1,1] by [-1,1]

7a) $\dfrac{dy}{dx}=\dfrac{-2\sin t}{4\cos t}=-\dfrac{1}{2}\tan t$

7b) $\dfrac{d^2y}{dx^2}=\dfrac{-\frac{1}{2}\sec^2 t}{4\cos t}=-\dfrac{1}{8}\sec^3 t$

8a) $\dfrac{dy}{dx}=\dfrac{-\sqrt{3}\sin t}{-\sin t}=\sqrt{3}$

8b) $\dfrac{d^2y}{dx^2}=\dfrac{0}{-\sin t}=0$

9a) $\dfrac{dy}{dx}=\dfrac{\frac{3}{2\sqrt{3t}}}{-\frac{1}{2\sqrt{t+1}}}=-3\sqrt{\dfrac{t+1}{3t}}$

9b) $\dfrac{d^2y}{dx^2}=\dfrac{\frac{1}{2t^2}\sqrt{\frac{3t}{t+1}}}{-\frac{1}{2\sqrt{t+1}}}=-\dfrac{\sqrt{3t}}{t^2}$

10a) $\dfrac{dy}{dx}=\dfrac{\frac{1}{t}}{-\frac{1}{t^2}}=-t$

10b) $\dfrac{d^2y}{dx^2}=\dfrac{-1}{-\frac{1}{t^2}}=t^2$

11a) $\dfrac{dy}{dx}=\dfrac{3t^2}{2t-3}$

11b) $\dfrac{d^2y}{dx^2}=\dfrac{\frac{(2t-3)6t-3t^2(2)}{(2t-3)^2}}{2t-3}=\dfrac{6t^2-18t}{(2t-3)^3}$

12a) $\dfrac{dy}{dx}=\dfrac{2t-1}{2t+1}$

12b) $\dfrac{d^2y}{dx^2}=\dfrac{\frac{(2t+1)2-(2t-1)(2)}{(2t+1)^2}}{2t+1}=\dfrac{4}{(2t+1)^3}$

13a) $\dfrac{dy}{dx}=\dfrac{\sec t\tan t}{\sec^2 t}=\dfrac{\tan t}{\sec t}=\sin t$

13b) $\dfrac{d^2y}{dx^2}=\dfrac{\cos t}{\sec^2 t}=\cos^3 t$

14a) $\dfrac{dy}{dx}=\dfrac{-2\sin(2t)}{-2\sin t}=\dfrac{\sin(2t)}{\sin t}=\dfrac{2\sin t\cos t}{\sin t}=2\cos t$

14b) $\dfrac{d^2y}{dx^2}=\dfrac{-2\sin t}{-2\sin t}=1$

15a) $\dfrac{dy}{dx}=\dfrac{\frac{1}{(3t)^4}\left(4(3t)^3\right)(3)}{\frac{1}{2t}(2)}=4$

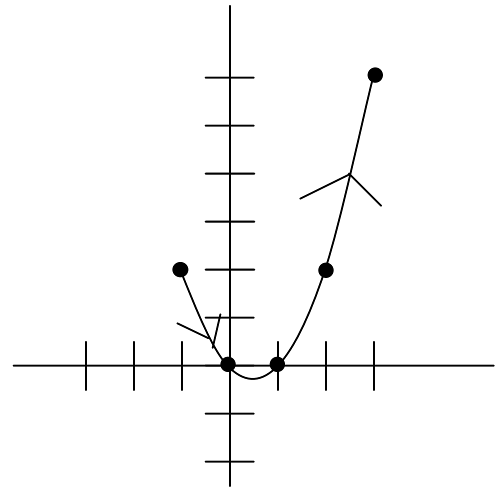
15b) $\dfrac{d^2y}{dx^2}=\dfrac{0}{\frac{1}{2t}(2)}=0$

16a) $\dfrac{dy}{dx}=\dfrac{5e^{5t}}{\frac{1}{5t}(5)}=5te^{5t}$

16b) $\dfrac{d^2y}{dx^2}=\dfrac{5t\left(5e^{5t}\right)+5e^{5t}}{\frac{1}{5t}(5)}=25t^2e^{5t}+5te^{5t}$

17a)

t	-2	-1	0	1	2
x	-1	0	1	2	3
y	2	0	0	2	6



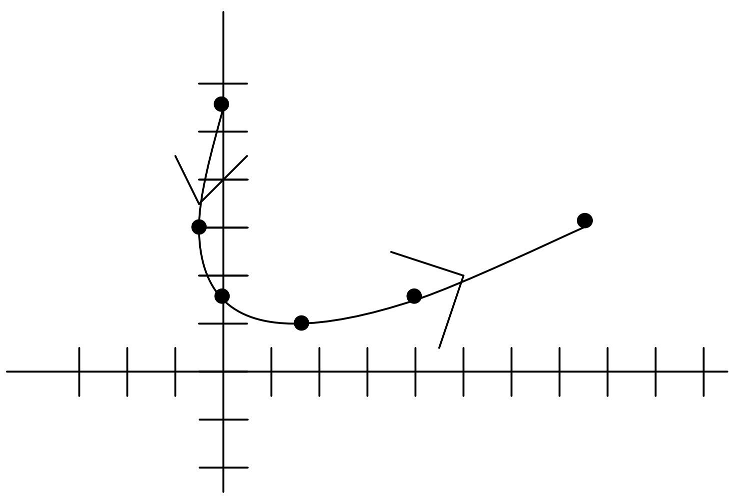
17b,c) $\frac{dy}{dt} = 2t + 1 \quad 2t + 1 = 0 \Rightarrow t = -\frac{1}{2}$

$\frac{d^2y}{dt^2} = 2$

$\left(\frac{1}{2}, -\frac{1}{4}\right)$ is a lowest point by the second deriv. test.

18a)

t	-2	-1	0	1	2	3
x	0	-1	0	3	8	15
y	11	6	3	2	3	6



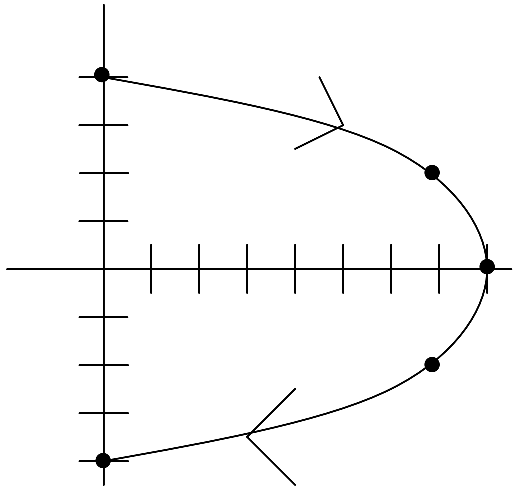
18b,c) $\frac{dx}{dt} = 2t + 2 \quad 2t + 2 = 0 \Rightarrow t = -1$

$\frac{d^2x}{dt^2} = 2$

$(-1, 6)$ is the leftmost value by the second deriv. test.

19a)

t	0	π/3	π/2	2π/3	π
x	0	√3	2	√3	0
y	1	0.5	0	-0.5	-1



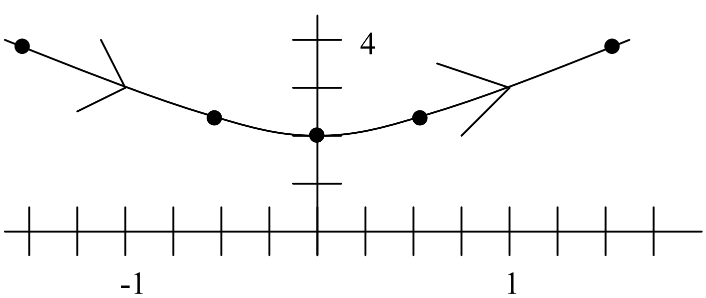
19 b,c) $\frac{dx}{dt} = 2\cos t \quad 2\cos t = 0 \Rightarrow \cos t = 0 \Rightarrow t = \frac{\pi}{2}$

$\frac{d^2x}{dt^2} = -2\sin t \quad \frac{d^2x}{dt^2}\left(\frac{\pi}{2}\right) = -2$

$(2,0)$ is the rightmost value by the second deriv. test.

20a)

t	-1	-1/2	0	1/2	1
x	-1.557	-0.546	0	0.546	1.557
y	3.702	2.279	2	2.279	3.702



20 b,c)

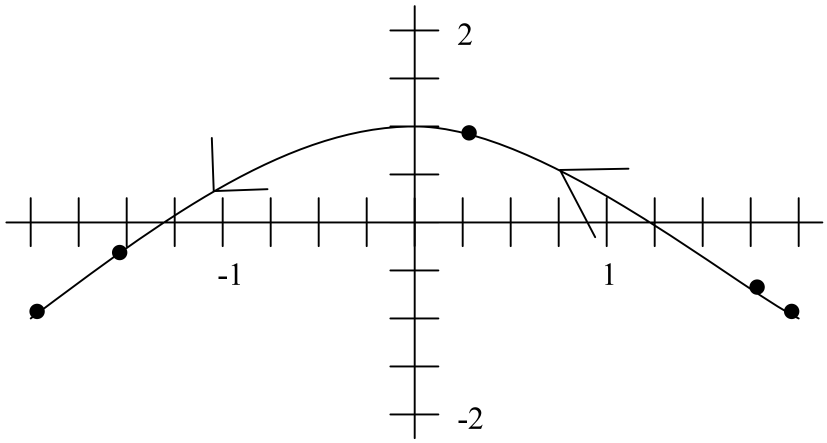
$\frac{dy}{dt} = 2\sec t \tan t \quad 2\sec t \tan t = 0 \Rightarrow \tan t = 0 \Rightarrow t = 0$

$\frac{d^2y}{dt^2} = 2\sec^3 t + 2\sec t \tan^2 t \quad \frac{d^2y}{dt^2}(0) = 2 + 0 = 2$

$(0,2)$ is the lowest value by the second deriv. test.

21a)

t	1.5	2	3	4	4.5
x	1.995	1.819	0.282	-1.514	-1.995
y	-0.990	-0.654	0.960	-0.146	-0.911



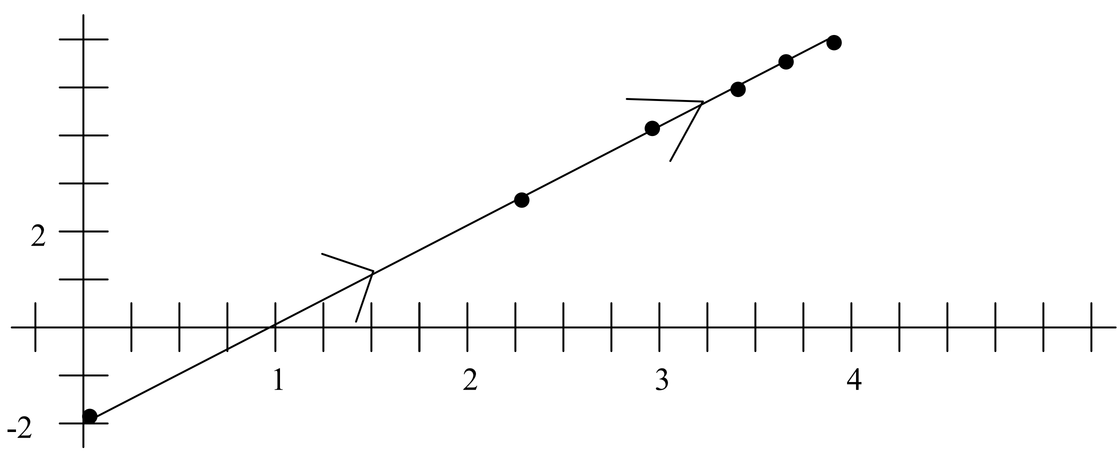
21 b, c) $\frac{dy}{dt} = -2\sin 2t \quad -2\sin 2t = 0 \Rightarrow t = \pi$

$\frac{d^2y}{dt^2} = -4\cos 2t \quad \frac{d^2y}{dt^2}(\pi) = -4$

$(0,1)$ is the highest point by the second deriv. test.

22a)

t	0.2	2	4	6	8	10
x	0	2.303	2.996	3.401	3.689	3.912
y	-1.833	2.773	4.159	4.970	5.545	5.991



22 b, c) $\frac{dx}{dt} = \frac{1}{5t}(5) = \frac{1}{t} \quad \frac{1}{t} \neq 0$

$(\ln(50), \ln(400))$ is the rightmost point by the second deriv. test.

23) $\frac{dy}{dx}=\frac{\cos t}{-\sin t}=-\cot t$

Horizontal: $-\cot t=0\Rightarrow t=\frac{\pi}{2}+n\pi \quad (2,0);(2,-2)$

Vertical: $-\cot t=\frac{1}{0}\Rightarrow -\tan t=0\Rightarrow t=n\pi \quad (3,-1);(1,-1)$

24) $\frac{dy}{dx}=\frac{\sec^2 t}{\sec t\tan t}=\frac{1}{\sin t}-\csc t$

Horizontal: $\frac{1}{\sin t}\neq 0,$ so there are no horizontal tangents

Vertical: $\frac{1}{\sin t}=\frac{1}{0}\Rightarrow \sin t=0\Rightarrow t=n\pi \quad (1,0);(-1,0)$

25) $\frac{dy}{dx}=\frac{3t^2-4}{-1}=4-3t^2$

Horizontal: $4-3t^2=0\Rightarrow t=\pm\frac{2}{\sqrt{3}} \quad \left(2+\frac{2}{\sqrt{3}},\frac{16}{3\sqrt{3}}\right);\left(2-\frac{2}{\sqrt{3}},-\frac{16}{3\sqrt{3}}\right)$

Vertical: $4-3t^2\neq\frac{1}{0},$ so there are no vertical tangents

26) $\frac{dy}{dx}=\frac{3\cos t}{-3\sin t}=-\cot t$

Horizontal: $-\cot t=0\Rightarrow t=\frac{\pi}{2}+n\pi \quad (-2,4);(-2,-2)$

Vertical: $-\cot t=\frac{1}{0}\Rightarrow t=n\pi \quad (1,1);(-5,1)$

27) $L=\int_0^{2\pi}\sqrt{(-\sin t)^2+(\cos t)^2}\,dt=\int_0^{2\pi}\sqrt{\sin^2 t+\cos^2 t}\,dt=\int_0^{2\pi}\sqrt{1}\,dt$

$=\left[t\right]_0^{2\pi}=2\pi-0=2\pi$

28) $L=\int_0^{\pi}\sqrt{(3\cos t)^2+(-3\sin t)^2}\,dt=\int_0^{\pi}\sqrt{9\cos^2 t+9\sin^2 t}\,dt=\int_0^{\pi}\sqrt{9}\,dt$

$=\left[3t\right]_0^{\pi}=3\pi-0=3\pi$

29) $L=\int_0^{\pi/2}\sqrt{(8t\cos t)^2+(8t\sin t)^2}\,dt=\int_0^{\pi/2}\sqrt{64t^2\cos^2 t+64t^2\sin^2 t}\,dt=\int_0^{\pi/2}\sqrt{64t^2}\,dt$

$=\int_0^{\pi/2}8t\,dt=\left[4t^2\right]_0^{\pi/2}=\frac{4\pi^2}{4}-0=\pi^2$

30) This problem is not correct, although it seems to be

$$L = \int_0^{2\pi} \sqrt{\left(-6\cos^2 t \sin t\right)^2 + \left(6\sin^2 t \cos t\right)^2}\, dt = \int_0^{2\pi} \sqrt{36\cos^4 t \sin^2 t + 36\sin^4 t \cos^2 t}\, dt$$

$$= \int_0^{2\pi} \sqrt{36\sin^2 t \cos^2 t\left(\cos^2 t + \sin^2 t\right)}\, dt = \int_0^{2\pi} 6\sin t \cos t\, dt \qquad \begin{matrix} u = \sin t \\ du = \cos t\, dt \end{matrix}$$

$$= \int_0^{0} 6u\,du = \left[4t^2\right]_0^{2\pi} = \frac{4\pi^2}{4} - 0 = \pi^2$$