Name Key Period___Date________

**AP Calculus: Chapter 6 Test Outline – Applications of Integration**

I. For parts A – E: **Sketch** the region. You must use the method suggested in each problem to set up the integral that would be used to find the answer. **Simplify, but do not evaluate the integrals.** Choose 9 total problems under the given restrictions. Circle the problem numbers you want graded.

*A. Area between Curves (Choose at least 2)*

1. Find the area bounded by $y = 2x^3 + 3$, $y = -x - 2$, $x = 0$, and $x = 1$.
2. Find the area bounded by $y = (x + 1)^3$ and $y = x + 1$.
3. Find the area bounded by $x = 3 - y^2$ and $x = y + 1$.

*B. Disk (Choose at least 1)*

1. Use the disk method to find the volume of the solid bounded by $y = 5 - x^2$ and y =4 rotating about the line y=4.
2. Use the disk method to find the volume of the solid bounded by $x = 2 - y^2$ and $x = 0$ revolving about the $y$-axis.

*C. Washer (Choose at least 1)*

1. Use the washer method to find the volume of the solid bounded by $y = x^{3/2}$, $x = 0$, and y=3 revolving about the line x=5
2. Use the washer method to find the volume of the solid bounded by $y = \sqrt{x} + 2$ and $y = x^2 + 2$ revolving about the $x$-axis.

*D. Shell (Choose at least 1)*

1. Use the shell method to find the volume of the solid bounded by $y=-3x^2+2x$ ~~$y = -3x^2 + 2x$~~ and $y = 0$ revolving about the $y$-axis.
2. Use the shell method to find the volume of the solid bounded by, $y = 2x^3$, $x = 1$ revolving about the line $x = 2$ .

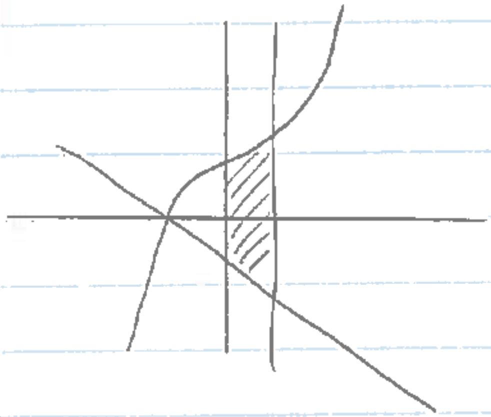
*E. Arc Length/Surface Area (Choose at least 2)*

1. Find the arc length of the curve $y = 4x^{3/2} + 2$ over the interval [0, 2].
2. Find the surface area formed by revolving $y = x^5$ on the interval [1, 3] about the $y$-axis.
3. Find the surface area formed by revolving $y = \sqrt[3]{x}$ on the interval [0, 5] about the $x$-axis.

II. Short Answer

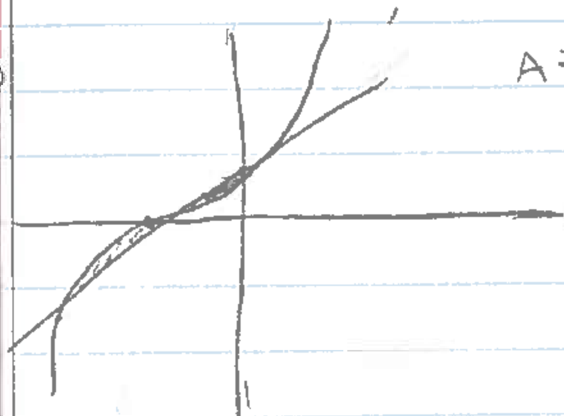
Chapter 6 Practice

(A)

1) 

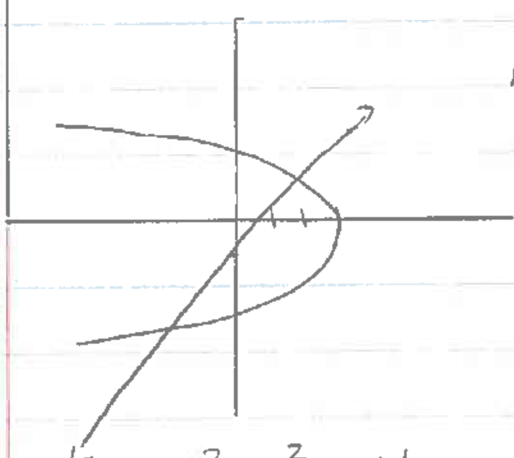
$$A = \int_0^1 (2x^3+3)-(-x-2)\,dx$$

$$= \boxed{\int_0^1 2x^2 + x + 5 \ dx}$$

2) 

$$A = 2\int_{-1}^{0} (x+1)^3 - (x+1)\,dx$$

$$= \boxed{2\int_{-1}^{0} (x^3+3x^2+2x)\,dx}$$

3) 

$$A = \int_{-2}^{1} (3-y^2)-(y+1)\,dy$$

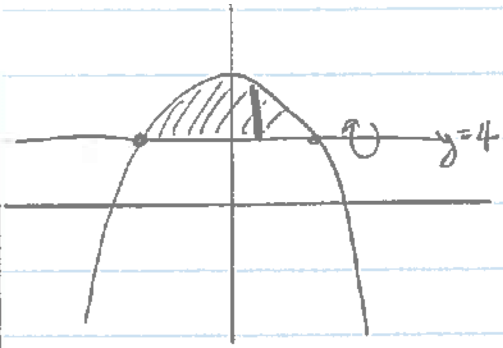
$$= \boxed{\int_{-2}^{1} -y^2 - y + 2 \ dy}$$

$3-y^2 = y+1$

$0 = y^2+y-2$

$0 = (y+2)(y-1)$

(B) (1)

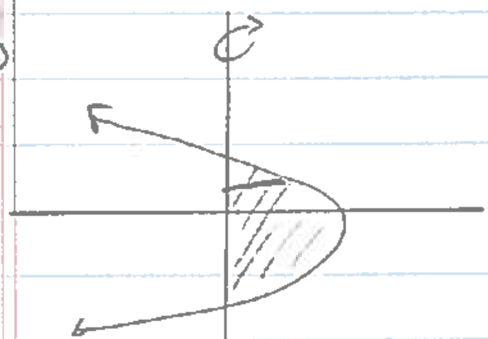



$R = 5 - x^2 - 4$
$\quad = 1 - x^2$

$$V = \pi \int_{-1}^{1} (1-x^2)^2 \, dx$$

$$= \boxed{\pi \int_{-1}^{1} (1 - 2x^2 + x^4) \, dx}$$

(2)

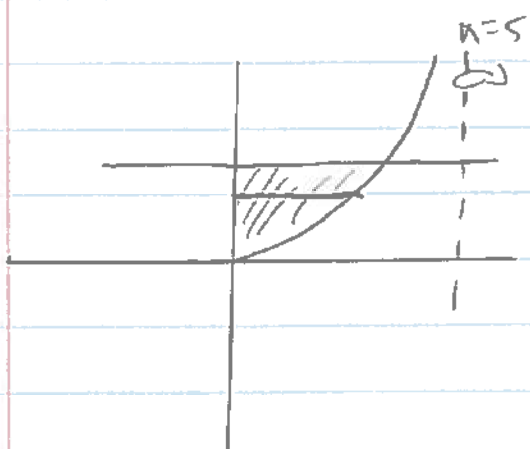


$R = 2 - y^2$

$$V = \pi \int_{-\sqrt{2}}^{\sqrt{2}} (2-y^2)^2 \, dy$$

$$= \boxed{\pi \int_{-\sqrt{2}}^{\sqrt{2}} (4 - 4y^2 + y^4) \, dy}$$

(C) (1)

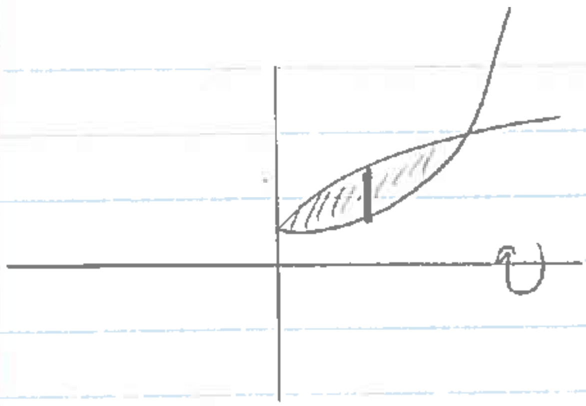



$R = 5$
$r = 5 - y^{2/3}$

$$V = \pi \int_{0}^{3} (5)^2 - \underbrace{(5 - y^{2/3})^2}_{25 - 10y^{2/3} + y^{4/3}} \, dy$$

$$= \boxed{\pi \int_{0}^{3} 10y^{2/3} - y^{4/3} \, dy}$$

C 2



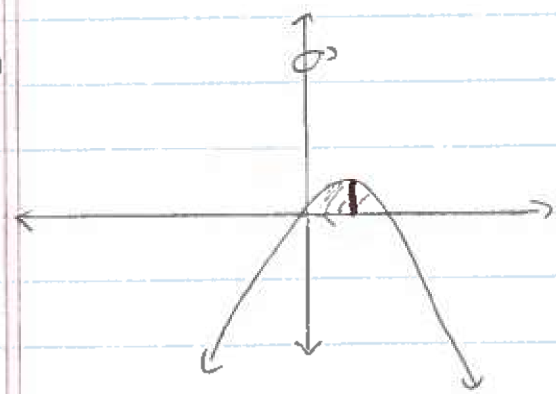
$R = \sqrt{x} + 2$

$r = x^2 + 2$

$$V = \pi \int_0^1 (\sqrt{x}+2)^2 - (x^2+2)^2 \, dx$$

$$= \boxed{\pi \int_0^1 x^{\frac{1}{4}} + 2x^{\frac{1}{2}} - x^4 - 4x^2 \, dx}$$

D 1



$r = x$

$h = -3x^2 + 2x$

$$V = 2\pi \int_0^{2/3} x(-3x^2+2x) \, dx$$

$-3x^2+2x=0$

$-x(3x-2)=0$

$$= \boxed{2\pi \int_0^{2/3} -3x^3 + 2x^2 \, dx}$$

2


$r = 2 - x$

$h = 2x^3$

$$V = 2\pi \int_0^1 (2-x)(2x^3) \, dx$$

$$= \boxed{2\pi \int_0^1 4x^3 - 2x^4 \, dx}$$

(E) (1) $L = \int_0^2 \sqrt{1+(6x^{\frac{1}{2}})^2}\,dx$

$= \boxed{\int_0^2 \sqrt{1+36x}\,dx}$

(2) $S = 2\pi \int_1^3 x\sqrt{1+(5x^4)^2}\,dx$

$= \boxed{2\pi \int_1^3 x\sqrt{1+25x^8}\,dx}$

(3) $S = 2\pi \int_0^5 \sqrt[3]{x}\sqrt{1+\left(\frac{1}{3}x^{-2/3}\right)^2}\,dx$

$= \boxed{2\pi \int_0^5 \sqrt[3]{x}\sqrt{1+\frac{1}{9}x^{-4/3}}\,dx}$