Robbinsville High School
Mathematics Department
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Dear Students,

Welcome to AP Calculus BC! Attached you will find a summer packet for math reinforcement for the upcoming school year. This packet should be completed and returned to school on the ***first full day of school***. I will give you the answer key on the first day of school. The packet will be **collected** and **graded** as **a 10-point homework grade** based on **completion** and **effort**. Work is required for many of these problems, so unsupported answers will not receive credit.

The packet covers material from Honors PreCalc that is found in Chapters 1 and 2 and Section 3.1 in your textbook. You will probably need to use your textbook or other resources in order to complete this packet. In addition to completing the summer assignment, please review all material in Chapter 1, 2, 3.1, and 3.2.

The packet itself is only a sampling of concepts and questions that are prerequisite for entering AP Calc. In addition to the packet, Khan Academy is a great place to review limits and assess your understanding. The following lessons in the AP Calc AB course might be helpful to you:

1. **Limits and Continuity**- all lessons
2. **Differentiation: Definition and Basic Derivative Rules**
    - *Defining Average and Instantaneous Rates of Change at a Point*
    - *Defining the Derivative of a Function and Using Derivative Notation*
    - *Estimating Derivatives of a Function at a Point*
    - *Connecting Differentiability & Continuity*

Y**our first test will be sometime during the second week of school** and will cover material from this packet *and* Chapters 1,2 and Sections 3.1 and 3.2.

If you have any questions while completing the packet, please feel free to email me over the summer at ziomek.morgan@rvilleschools.org. I check my email every couple of weeks.

Have a great summer!

Mrs. Ziomek

PreCalc Review (Ch. 1)

1. Evaluate the following trig values without a calculator:

a) $\cos\frac{\pi}{2} =$ ________________

b) $\tan\frac{3\pi}{4} =$ ________________

c) $\sin\left(-\frac{\pi}{3}\right) =$ ________________

d) $\sin\frac{3\pi}{2} =$ ________________

e) $\cot\pi =$ ________________

f) $\csc\frac{11\pi}{6} =$ ________________

g) $\tan\frac{5\pi}{3} =$ ________________

h) $\sec\frac{\pi}{4} =$ ________________

i) $\tan\left(-\frac{\pi}{2}\right) =$ ________________

2. Evaluate the following without a calculator. Give all answers in radians.

a) $\arccos(1)$ __________

b) $\arcsin(-1)$ __________

c) $\arctan\left(\frac{\sqrt{3}}{3}\right)$ __________

d) $\arctan(-1)$ __________

e) $\arccos\left(-\frac{\sqrt{2}}{2}\right)$ __________

f) $\arcsin\left(-\frac{1}{2}\right)$ __________

g) $\arctan(0)$ __________

h) $\arccos\left(\frac{1}{2}\right)$ __________

i) $\arccos\left(-\frac{\sqrt{3}}{2}\right)$ __________

j) $\arcsin\left(\sin\frac{2\pi}{3}\right)$ _______

k) $\sin\left(\arcsin(-.25)\right)$ _______

l) $\arctan\left(\tan\left(-\frac{\pi}{6}\right)\right)$ _______

m) $\arccos\left(\cos\left(\frac{3\pi}{2}\right)\right)$ _______

n) $\cos\left(\arccos(-5)\right)$ _______

o) $\arctan\left(\tan\left(\frac{7\pi}{6}\right)\right)$ _______

Section 2.1: Rates of Change and Limits

Find the limits below. Be sure to show all work and give exact answers.

1) $\lim_{x\to 0}\frac{x^2}{x+5} =$

2) $\lim_{x\to 5} 6 =$

3) $\lim_{x\to 3}\frac{(x-4)^2}{x+3} =$

4) $\lim_{x\to 0} 3x\cos x =$

5) $\lim_{x\to 0}\frac{\sin 3x}{6x} =$

6) $\lim_{x\to -2}\frac{x^2+x-2}{x^2+5x+6} =$

7) $\lim_{x\to 7}\frac{\sqrt{2x-5}-3}{x-7} =$

8) $\lim_{x\to 0}\frac{\frac{1}{(x+5)}-\frac{1}{5}}{x} =$

9) $\lim_{x\to 0}\left(\ln(\cos(x))\right) =$

10) $\lim_{x\to \pi/2}\left(e^x\sin(x)\right) =$

11) $\lim_{x\to 1-}\operatorname{int}(x)$

12) $\lim_{x\to 5}\frac{x^2-7x+10}{x-5} =$

13) Use the limits $\lim_{x \to 3} f(x) = 5$ *and* $\lim_{x \to 3} g(x) = -2$ to answer the following:

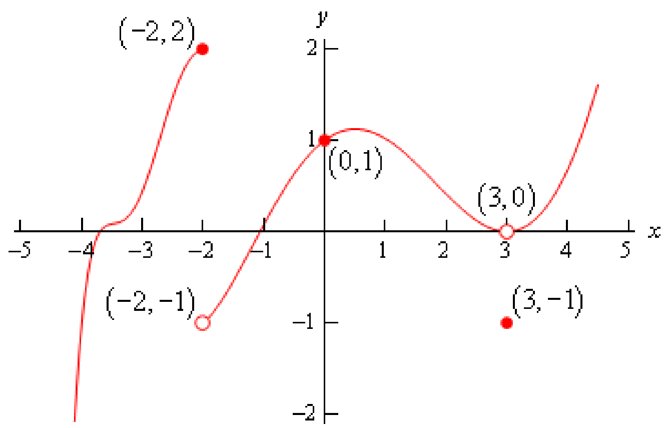
a) $\lim_{x \to 3} f(x) + g(x) =$

b) $\lim_{x \to 3} f(x) \bullet g(x) =$

c) $\lim_{x \to 3} 3f(x) - g(x) =$

d) $\lim_{x \to 3} \frac{f(x) - 5}{g(x)} =$

14) Use the following diagram to answer the questions:



a) f(3) =

b) $\lim_{x \to 3^-} f(x) =$

c) $\lim_{x \to 3^+} f(x) =$

d) $\lim_{x \to 3} f(x) =$

e) f(-2) =

f) $\lim_{x \to -2^-} f(x) =$

g) $\lim_{x \to -2^+} f(x) =$

h) $\lim_{x \to -2} f(x) =$

i) f(0) =

j) $\lim_{x \to 0^-} f(x) =$

k) $\lim_{x \to 0^+} f(x) =$

l) $\lim_{x \to 0} f(x) =$

m) $\lim_{x \to \infty} f(x) =$

n) $\lim_{x \to -\infty} f(x) =$

15) Given the information below, sketch a possible graph of *f(x)*.

a)
$f(x) = 0 \;\; at \;\; x = 2$
$\lim_{x \to 4} f(x) = DNE$
$crosses \; y - axis \; at \; y = -1$
$\lim_{x \to \infty} f(x) = \lim_{x \to -\infty} = -2$
$\lim_{x \to 4^+} = -\infty$
$f(x)$ DNE $at \; x = -1 and \; x = 4$

b)
$\lim_{x \to 2} f(x) = -1$
$\lim_{x \to 4^+} f(x) = -\infty$
$\lim_{x \to 4^-} f(x) = \infty$
$\lim_{x \to \infty} f(x) = \infty$
$\lim_{x \to -\infty} f(x) = 2$

Section 2.2: Limits Involving Infinity

Find the limits below.

1) $\lim_{x\to\infty} \ln x =$

2) $\lim_{x\to-\infty} e^{-x} =$

3) $\lim_{x\to\infty} \frac{4x^4-5x^3}{7x^4+9x^3} =$

4) $\lim_{x\to\infty} \frac{3x^3-x+1}{x+3} =$

5) $\lim_{x\to-\infty} \frac{\sqrt{x^2-2}}{x^2+6} =$

6) $\lim_{x\to-\infty} \frac{1-7x^2}{x+5} =$

7) $\lim_{x\to0} \frac{\sin x}{5x}$

8) $\lim_{x\to0} \frac{3(1-\cos x)}{x}$

9) $\lim_{x\to0} \frac{\cos x \tan x}{x}$

10) $\lim_{x\to2} \frac{3x^2-7x+2}{x^2+5x-14}$

11) $\lim_{x\to0} \frac{x^2-4}{x+2}$

12) $\lim_{x\to\infty} \frac{2x+\sin x}{x}$

13) $\lim_{x\to\infty} \frac{x-6}{x^2+2x-48}$

14) $\lim_{x\to-\infty} \frac{x^3+6x}{\sqrt{x^2+5}}$

15) $\lim_{x\to-4^+} \frac{1}{x+4}$

16) $\lim_{x\to-\infty} \sqrt[3]{\frac{8+x^2}{8x(x+1)}}$

17) $\lim_{x\to4} \frac{\sqrt{x^2+9}+-5}{x-4}$

18) $\lim_{x\to0} \frac{\tan x}{x}$

Section 2.3: Continuity

1) Find all points of discontinuity of the functions below and state the type of discontinuity. If the function has no points of discontinuity, then specify over what intervals it is continuous.

a) $f(x)=\dfrac{x+1}{x^2-4}$

b) $f(x)=\dfrac{x^2-8x+15}{x^2-25}$

c) $f(x)=3x+9$

d) $f(x)=\sqrt{2x-7}$

e) $f(x)=\dfrac{8-2x}{x^2-16}$

f) $f(x)=\dfrac{x}{|x|-3}$

2) At what x-coordinate on $f(x)=\dfrac{x^2-x-6}{x^2-9}$ is there a removable discontinuity?

3) Find a value for *a* so that function is continuous.

a) $f(\mathrm{x})=\begin{cases}4-x^2, & x<-1\\ ax^2-1, & x\geq -1\end{cases}$

b) $f(x)=\begin{cases}x^2+x+a, & x<1\\ x^3, & x\geq 1\end{cases}$

Section 2.4: Rates of Change and Tangent Lines

1) Find the average rate of change for the function $f(x) = 5x^2 - 3$ over the interval [0, 2].

2) Find the average rate of change for the function $f(x) = e^{2x-3} - x$ over the interval [1, 2]. Leave answer in terms of *e*.

3) Find the average rate of change for the function $f(x) = \cos x - 4$ over the interval $[0, \pi]$.

4) Consider the function $f(x) = x^2 - 4x$.

a) Find the slope of the line tangent to $f(x)$ at the point $(1, -3)$.

b) Write the equation for the line tangent to $f(x)$ at $(1, -3)$.

c) Write the equation for the line normal $f(x)$ at $(1, -3)$.

5) Consider the function $f(x) = \sqrt{x+4}$.

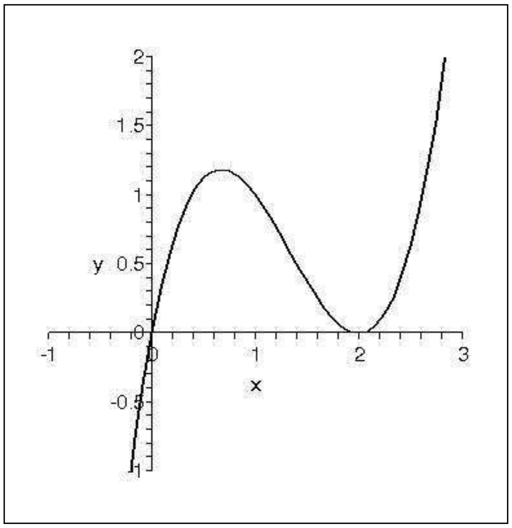
a) Write the equation for the line tangent to $f(x)$ at $x = 0$.

b) Write the equation for the line normal to $f(x)$ at $x = 0$.

6) A coffee shop opens at 5:00 am. The number of coffee cups, f(*x*), sold per hour can be modeled by the function $f(x) = 3x^3 - 2x^2 + 6$, where x is the number of hours the shop has been open. Find the average rate of change between the hours of 7:00am and 11:00am and **explain** what this rate represents.

7) Explain the difference between average rate of change and instantaneous rate of change.

8) Below is a graph of the function $f(x) = x(x-2)^2$.



a) Draw a line tangent to the $f(x)$ at $x = 1$. Is the slope of the tangent line positive or negative?

b) Draw a secant line through the points (1, 1) and (2.8, 1.792) on $f(x)$. Draw another secant line through the points (1, 1) and (0.6, 1.176). Which secant line is the best approximation of the line tangent to $f(x)$ at (1, 1)? Why?

9) Determine the slope of each curve at x = a.

a) $y = x^2 - x - 2$

b) $y = \dfrac{1}{x+2}$

10) At what points, if any, are the tangents to the graph $f(x) = x^2 - 3x$ horizontal?

11) Clara is hovering above the asteroid Elmy. She drops a rock, and the position (in feet) of the rock is modeled by the equation $f(x) = -3x^2 + 800$ (x measured in seconds)

a) What is the equation for the instantaneous velocity of the rock?

b) What is the instantaneous velocity of the rock at 8 seconds?

Section 3.1: Derivative of a Function

1) What are the two definitions you can use to find a derivative?

2) Use the definition of a derivative to find $f\,'(x)$ for each function (NO SHORTCUTS)

a) $f(x)=\dfrac{2}{x-3}$ at $x=2$.

b) $f(x)=4x^2+x-5$

c) $f\left(x\right)=\sqrt{x-3}$

3) Find the equations of the tangent and normal lines at x = 2, given the information below:

$$f\left(2\right)=6,\ f'\left(0\right)=4\ \textit{and}\ f'\left(2\right)=-3$$

4) Find the left-hand and right-hand derivatives in order to determine if the derivative exists when x = 1 for the function $f(x)=\begin{cases}2x^2+1, & x<1\\ 3x+6, & x\geq 1\end{cases}$

5) Find the derivative of $y=2x^2-13x+5$ and use it find the equation of the line tangent to the curve at x=3.

Section 3.2: Differentiability

This is NEW material! Refer to p.109-113 in your textbook to help fill in the blanks and complete the problems. Feel free to use other resource (like a friend or the internet) as well.

<u>How a derivative might fail to exist:</u>

- Corner occurs if the one-sided derivatives are ________________ numbers.
- Cusp occurs if one-side yields ______ and the other is ________
- Vertical tangents occur if BOTH one-sided limits are the same and both equal ± ______
- Discontinuity occurs at jumps or holes.

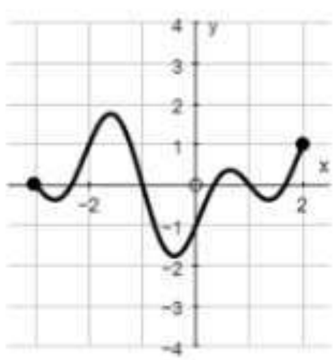
1) The functions below fail to be differentiable at x = 0. Tell whether the problem is a corner, a cusp, a vertical tangent, or a discontinuity. Graphs are helpful too.

a) $f(x) = x^{\frac{2}{5}}$ b) $f(x) = 6x - 2|x| + 3$ c) $f(x) = 2 - \sqrt{x}$

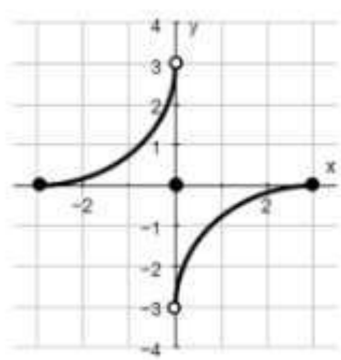
For #2-5, the graph of a function over a closed interval D is given. At what domain points does the function appear to be…

a) Differentiable?
b) Continuous but not differentiable?
c) Neither continuous nor differentiable?

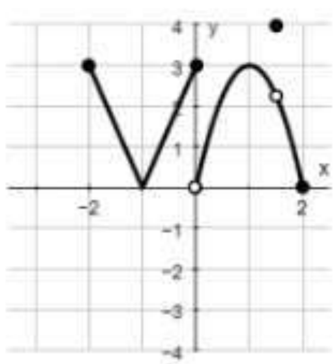
2. $D: \ -3 \leq x \leq 2$



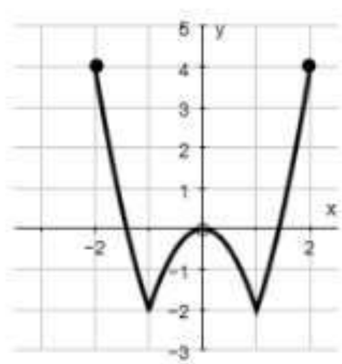
3. $D: \ -3 \leq x \leq 3$



4. $D: \ -2 \leq x \leq 2$

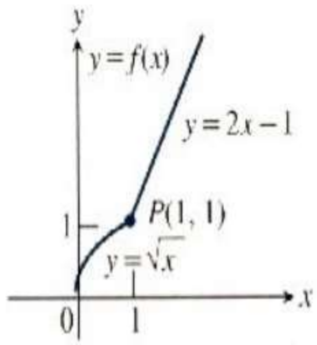


5. $D: \ -2 \leq x \leq 2$

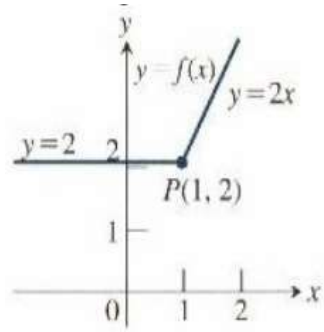


6) Compare the right-hand and left-hand derivatives (no shortcuts) to show that the function is not differentiable at point P.

a)



b)



7) Determine all values of x for which the function is differentiable. Graphs are helpful. Remember, if it has a derivative at $x = a$ then the graph MUST be continuous at $x = a$. For points at which the function is not differentiable, give a reason.

a) $f(x) = \lvert x-2 \rvert + 3$	b) $f(x) = \sqrt{x^2}$
c) $f(x) = \sqrt[3]{2x-4} + 10$	d) $f(x) = \dfrac{x+2}{x^2 - 8x + 15}$
e) $f(x) = \sqrt[5]{x^2} = \left(x^2\right)^{\frac{1}{5}} = x^{\frac{2}{5}}$	f) $f(x) = \begin{cases} 3x, & x \le 1 \\ x^3, & x > 1 \end{cases}$