### **ROBBINSVILLE PUBLIC SCHOOLS**

### **OFFICE OF CURRICULUM AND INSTRUCTION**

## Mathematics Department AP CALCULUS BC

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### BOARD OF EDUCATION INITIAL ADOPTION DATE:

### Course Philosophy

Every individual develops an understanding of the concepts of calculus and gains experience with its methods and applications. This understanding and experience is achieved through a multi-representational approach, which emphasizes the relationships between graphical, numerical, analytical, and verbal representations of concepts, problems, and solutions. The connections among the multiple representations of concepts and functions are strengthened by the regular use of technology by students and teacher. Technology also plays a key role in verifying written work and in aiding in the interpretation of experimentation results. Several unifying themes serve to bring cohesion to the course. These themes include derivatives, integrals, limits, approximation, and applications and modeling. They are developed using all of the major mathematical functions with which students are already familiar.

#### **Course Description**

The AP Calculus BC course is designed to prepare the students for the Advanced Placement BC test and to give them an opportunity to earn credit for a college level calculus course. The material covered is based on the Advanced Placement Calculus course description. The course will cover all calculus topics ranging from the concepts of limits, derivatives, integration, and all of the applications that apply to each. Students entering this course should have a strong background in geometric concepts and a mastery of the different families of functions. Students should also have an understanding of the language of higher mathematics as applied to functions. The use of a TI-84 is preferred since labs and other activities will be based on this calculator. Since the course is taught at a college level, students should understand that the workload is demanding and expectations are high.

### **Educational Technology**

### Standards: 8.1.12.A.2, 8.1.12.A.5

- <u>Technology Operations and Concepts:</u> Produce and edit a multi-page digital document for a commercial or professional audience and present it to peers and/or professionals in that related area for review.
- <u>Technology Operations and Concepts:</u> Create a report from a relational database consisting of at least two tables and describe the process, and explain the report results.

**Example:** Students can electronically submit a slide presentation on a research topic of their choosing and present it to their peers and teachers. They must include at least 2 tables and 3 graphs in their presentation and use them to help explain their results.

### **Career Ready Practices**

### Standards: (CRP1, CRP2, CRP4, CRP8, CRP11)

**<u>CRP1.</u>** Act as a responsible and contributing citizen and employee. Career-ready individuals understand the obligations and responsibilities of being a member of a community, and they demonstrate this understanding every day through their interactions with others. They are conscientious of the impacts of their decisions on others and the environment around them. They think about the near-term and long-term consequences of their actions and seek to act in ways that contribute to the betterment of their teams, families, community and workplace. They are reliable and consistent in going beyond the minimum expectation and in participating in activities that serve the greater good.

**Example:** Students will demonstrate the responsibilities associated with being a member of a community when engaging collaboratively during sharing in pairs/trios, and participating in whole group discussions. Examples may include jigsaw and fishbowl activities, as well as group projects and assessments.

**<u>CRP2.</u>** Apply appropriate academic and technical skills.

**Example:** Students will demonstrate the skills learned in AP Calculus when engaging collaboratively during sharing in pairs/trios, and participating in whole group discussions. Examples may include jigsaw and fishbowl activities, as well as projects and formal assessments.

**<u>CRP4.</u>** Communicate clearly and effectively and with reason. Communication is a key factor in AP Calculus. Students are conscious that their words and techniques they use to convey their thoughts are crucial to audience understanding.

Example: Students will demonstrate clear and effective communication through written and oral assignments and assessments.

**<u>CRP8.</u>**Utilize critical thinking to make sense of problems and persevere in solving them.

**Example:** Students will demonstrate critical thinking as they determine the best methods to solve free response and multiple choice problems.

**<u>CRP11.</u>** Use technology to enhance productivity.

**Example:** Students will use technology to enhance productivity on a regular basis as they use graphing calculators and other graphing software to complete calculus problems.

### Robbinsville Ready 21st Century Skill Integration

The following skills will be embedded throughout the curriculum and instruction of this course.

**Collaborative Team Member:** Robbinsville students will learn more by working together than in isolation. As educational theorist Lev Vygotsky advocated, learning is a social process. Many workplaces today encourage employees to work in teams to solicit diverse perspectives, brainstorm new ideas and/or products, and solve problems. Further, collaboration fosters interpersonal relationships, self-management skills, cooperation, and a sense of collective responsibility. Collaborative team members are able to work with diverse groups of people who hold a variety of perspectives.

**Effective Communicator:** Robbinsville students must be able to clearly articulate their ideas orally, in writing, and across various media in order to successfully connect to the world around them. As the world becomes increasingly globalized, communication is more than just sharing one's ideas. Effective communicators are able to communicate their convictions, actively listen and analyze others' work to identify perspective and/or potential bias.

**Emotionally Intelligent Learner:** Robbinsville students who are emotionally intelligent learn to be empathetic, demonstrate integrity and ethical behavior, are kind, are self-aware, willing to change, and practice self-care. They are better able to cope with the demands of the 21st century digital society and workplace because they are reliable, responsible, form stable and healthy relationships, and seek to grow personally and professionally. Emotionally intelligent people are able to manage their emotions, work effectively on teams and are leaders who can grow and help to develop others.

Informed and Involved Citizen: Robbinsville students need to be digital citizens who are civically and globally aware. The concept of what it means to be "literate" has evolved along with 21st century technological and cultural shifts. Our progressive vision of literacy entails having our students explore real world problems in the classroom. Informed and involved citizens are able to safely and accurately communicate with people all around the world and are financially, environmentally and informationally literate.

Innovative Thinker: Robbinsville students must encompass innovative thinking skills in order be successful lifelong learners in the 21st century world. As stated by Karl Fisch and Scott McLeod in the short film Shift Happens, "We are currently preparing students for jobs that don't yet exist . . . using technologies that haven't been invented . . . in order to solve problems we don't even know are problems yet." Innovative thinkers are able to think analytically, solve problems critically, creatively engage in curiosity and tinkering, and demonstrate originality.

**Resilient and Self-Directed Learner:** Robbinsville students need to take risks and ultimately make independent and informed decisions in an ever-changing world. Author of Life, the Truth, and Being Free, Steve Maraboli stated, "Life doesn't get easier or more forgiving, we get stronger and more resilient." Self-directed scholars of the 21st century are able to set goals, initiate resolutions by seeking creative approaches, and adjust their thinking in light of difficult situations. Resilient students are able to take risks without fear of failure and overcome setbacks by utilizing experiences to confront new challenges. Resilient and self directed scholars will consistently embrace opportunities to initiate solutions and overcome obstacles.

### Interdisciplinary Connections

AP Calculus connects widely to the following Career & Technical Education (CTE) Content Area: 21st Century Life and Careers Standards:

### 9.3.12.BM-MGT.2 Access, evaluate and disseminate information for business decision making.

Students will access, evaluate, and disseminate information for decision making on a regular basis as they apply the skills learned in AP Calculus.

### 9.3.12.ED.2 Demonstrate effective oral, written and multimedia communication in multiple formats and contexts.

Students will present oral and multimedia projects to the class in a wide variety of contexts througout the course. They regularly communicate in writing as they practice AP Free Response quesitons each unit.

# 9.3.12.AG-ENV.4 Develop proposed solutions to environmental issues, problems and applications using scientific principles of meteorology, soil science, hydrology, microbiology, chemistry and ecology.

Many mathematical applications of AP Calculus are geared towards environmental phenomena, such as population growth and rates of change of sand, water, and other natural resources.

General Differentiated Instruction Strategies				
• Leveled texts	Repeat, reword directions			
Chunking texts	Brain breaks and movement breaks			
Choice board	Brief and concrete directions			
Socratic Seminar	• Checklists for tasks			
Tiered Instruction	Graphic organizers			
• Small group instruction	• Assistive technology (spell check, voice to type)			
Guided Reading	• Study guides			
• Sentence starters/frames	Tiered learning stations			
• Writing scaffolds	Tiered questioning			
• Tangible items/pictures	Data-driven student partnerships			
Adjust length of assignment	• Extra time			

Possible Additional Strategies for Special Education Students, At-Risk Students, and English Language Learners (ELLs)								
Time/GeneralProcessingComprehensionRecall								
<ul> <li>Extra time for assigned tasks</li> <li>Adjust length of assignment</li> <li>Timeline with due dates for</li> </ul>	<ul> <li>Extra Response time</li> <li>Have students verbalize steps</li> </ul>	<ul><li>Precise step-by-step directions</li><li>Short manageable tasks</li></ul>	<ul> <li>Teacher-made checklist</li> <li>Use visual graphic organizers</li> </ul>					

reports and projects <ul> <li>Communication system</li> <li>between home and school</li> </ul> <li>Provide lecture <ul> <li>notes/outline</li> </ul></li>	<ul> <li>Repeat, clarify or reword directions</li> <li>Mini-breaks between tasks</li> <li>Provide a warning for transitions</li> <li>Reading partners</li> </ul>	<ul> <li>Brief and concrete directions</li> <li>Provide immediate feedback</li> <li>Small group instruction</li> <li>Emphasize multi-sensory learning</li> </ul>	<ul> <li>Reference resources to promote independence</li> <li>Visual and verbal reminders</li> <li>Graphic organizers</li> </ul>	
Assistive Technology	Assessments and Grading	Behavior/Attention	Organization	
<ul> <li>Computer/whiteboard</li> <li>Tape recorder</li> <li>Spell-checker</li> <li>Audio-taped books</li> </ul>	<ul> <li>Extended time</li> <li>Study guides</li> <li>Shortened tests</li> <li>Read directions aloud</li> </ul>	<ul> <li>Consistent daily structured routine</li> <li>Simple and clear classroom rules</li> </ul>	<ul> <li>Individual daily planner</li> <li>Display a written agenda</li> <li>Note-taking assistance</li> <li>Color code materials</li> </ul>	

### Enrichment

The goal of Enrichment is to provide learners with the opportunity to participate in extension activities that are differentiated and enhance the curriculum. All enrichment decisions will be based upon individual student needs.

- Show a high degree of intellectual, creative and/or artistic ability and demonstrate this ability in multiple ways.
- Pose questions and exhibit sincere curiosity about principles and how things work.
- The ability to grasp concepts and make real world and cross-curricular connections.
- Generate theories and hypotheses and pursue methods of inquiry.
- Produce products that express insight, creativity, and excellence.
- Possess exceptional leadership skills.
- Evaluate vocabulary
- Elevate Text Complexity
- Inquiry based assignments and projects
- Independent student options
- Tiered/Multi-level activities

- Purposeful Learning Center
- Open-ended activities and projects
- Form and build on learning communities
- Providing pupils with experiences outside the 'regular' curriculum
- Altering the pace the student uses to cover regular curriculum in order to explore topics of interest in greater depth/breadth within their own grade level
- A higher quality of work than the norm for the given age group.
- The promotion of a higher level of thinking and making connections.
- The inclusion of additional subject areas and/or activities (cross-curricular).
- Using supplementary materials in addition to the normal range of resources.

### Curriculum Map

### **AP Calculus BC**

Relevant Standards	Standards Unpacked Skill / Concept / Process?	Enduring Understandings / Unit Goals	Essential Questions	Unit Title / Suggested Timeline
Unit #1: MP.2 S-ID.7 A-APR.3	Students will study limits of functions and will develop an intuitive understanding of the limiting process, the ability to calculate limits using algebra, and estimate limits from graphs or tables of data	<ul> <li>The concept of limit is one of the ideas that distinguish calculus from algebra and trigonometry.</li> <li>One of the uses of limits is to test functions for continuity.</li> <li>1. The student will define and calculate limits of function values.</li> <li>2. The student will determine the continuity of functions.</li> </ul>	<ul><li>What does it mean for a function to have a limit at a point?</li><li>Why are limits important in the study of calculus?</li><li>What does it mean for a function to be continuous?</li><li>Why is it important to determine if a function is continuous?</li></ul>	Limits and Continuity (Summer work plus 2 blocks)

Unit #2: MP.5 A-APR.3 F-TF.6-9 S-ID.7	Students will evaluate derivatives of functions, including trigonometric functions and their inverses. Students will use graphs to evaluate derivatives Interpret the slope (rate of change) and the intercept (constant term) of a model in the context of the data.	<ul> <li>The derivative gives the instantaneous rate of change of a function.</li> <li>There are simple formulas for calculating derivatives of all types of functions.</li> <li>1. The student will apply differentiation rules to calculate derivatives of various types of functions.</li> </ul>	<ul><li>What does the derivative of a function represent?</li><li>How are derivatives calculated using limits?</li><li>Is there a way to calculate derivatives that does not involve limits?</li></ul>	Derivatives (12 blocks)
Unit #3: MP.3 MP.4 MP.5 A-APR.3 A-REI.1	-Students will study the analysis of graphs of functions and their derivatives. With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function -Students will sove related rate and optimatization problems including those involving unit pricing and constant speed.	<ul> <li>Derivatives provide information about the general shape of a function's graph.</li> <li>Derivatives are applied to model and solve a variety of real-life problems.</li> <li>1. The student will find a function given only its first derivative and its value at a single point.</li> <li>2. The student will use derivatives to model and solve real-life application problems.</li> </ul>	<ul> <li>What information does a derivative reveal about the graph of the original function?</li> <li>How are the graphs of the first and second derivative related to the graph of the original function?</li> <li>What types of real-life problems can derivatives be used to solve? How are derivatives applied in these problems?</li> </ul>	Applications of Derivatives (9 blocks)
Unit #4: MP.2 MP.5 A-APR.3 A-APR.6 A-APR.7	-Use algebraic techniques to integrate functions, including long division and partial fractions -Use graphs to evaluate a definite integral	<ul> <li>The integral describes how instantaneous changes accumulate over an interval to produce a function.</li> <li>Derivatives and integrals are closely related through the Fundamental Theorem of Calculus.</li> <li>1. The student will calculate integrals using the rules of integration and a graphing calculator.</li> </ul>	What does a definite integral represent? What are several methods of calculating a definite integral? What is the connection between derivatives and integrals?	The Definite Integral (7 blocks)

Unit #5: MP.4 MP.5 A-APR.3 A-APR.6 A-APR.7 F-IF.5	-Students will study the analysis of graphs of slope fields and logistic growth. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.	<ul> <li>There are several methods for integrating functions for which there is no rule of integration.</li> <li>Differential equations are used to model various real-life situations and make predictions about future behavior.</li> <li>1. The student will use various integration methods to solve problems involving differential equations.</li> </ul>	How can a function be integrated if there are no rules of integration that apply to the function? What is a differential equation? How are differential equations related to problems involving growth and decay?	Differential Equations and Mathematical Modeling (8 blocks)
Unit #6: MP.3 MP.4 MP.5 A-APR.3 S-ID.7 G-GMD.1-4 A-REI.1	-Use graphs and functions to determine rate and amount of quantities of real-world problems -Interpret the slope (rate of change) and the intercept (constant term) of a model in the context of the data given a real-world problem -Explain volume formulas and use them to solve problems	<ul> <li>Integrals can be applied to model and solve a wide variety of mathematical and real-life problems.</li> <li>1. The student will solve problems by setting up an appropriate function and integrating it.</li> </ul>	For what types of problems can integrals be used to model and solve the problems?	Applications of Definite Integrals (7 blocks)
Unit #7: A-APR.3 A-APR.6 A-APR.7 F-BF.1 MP.8	Students will build functions that represent infinite seugences, and use limits, derivatives, and other algebraic techniques to evaluate, such as: -Perform arithmetic operations on polynomials -Understand the relationship between zeros and factors of polynomials -Use polynomial identities to solve problems	Derivatives are used to calculate limits involving indeterminate forms. There are several techniques to evaluate integrals whose limits of integration are infinite. The student will use l'Hopital's Rule to calculate limits involving indeterminate forms. The student will use various methods to evaluate improper integrals.	What is a limit of indeterminate form? How can such limits be calculated? What is an improper integral? How can such integrals be evaluated?	Sequences, L'Hopital's Rule, Improper Integrals (5 blocks)

Unit #8 A-APR.1 A-APR.3 A-APR.6 A-APR.7 A-SSE.4	<ul> <li>-Students will treat inifinite series as polynomials, and use techniques to analyze and determine convegence or divergence</li> <li>-Students will use tests for convergence to rewrite series as rational or radical functions.</li> <li>-Students will analyze geometric series</li> </ul>	<ul> <li>A convergent series is a number and can be treated as such.</li> <li>There are several methods for approximating a series by a polynomial.</li> <li>1. The student will apply a variety of techniques to test for the convergence of a series.</li> </ul>	<ul><li>What is an infinite series? How is an infinite series related to a function?</li><li>How can the convergence or divergence of a series be determined?</li><li>Why is it important to determine if a series converges?</li></ul>	Infinite Series (9 blocks)
Unit #9: N-VM.3 A-APR.3 F-LE.5	Students will analyze planar curves given in parametric form, polar form and vector form, including velocity and acceleration. Students will analyze parameters of functions in terms of a context	Single-variable calculus can be used to analyze some curves defined by two variables. 1. Students will apply previously learned calculus concepts to differentiate and integrate parametric, vector, and polar fuctions.	How can the calculus concepts learned throughout this course be applied to functions defined by two variables?	Vectors, and Parametric & Polar Functions (5 blocks)
Unit #10: MP.1 MP.2 MP.3 MP.5 MP.6		<ul> <li>Mastery of all topics from AP Calc</li> <li>BC is necessary for success on the</li> <li>AP Exam <ol> <li>Students will complete</li> <li>practice tests in groups and</li> <li>individually</li> </ol> </li> <li>Students will review topics <ol> <li>by unit</li> <li>Students will do practice</li> <li>problems in which they</li> <li>must incorporate several</li> <li>topics at once and</li> <li>determine the best method</li> <li>for solving</li> </ol> </li> </ul>	What can I do to prepare for the AP Exam? How can I determine the best methods for solving a problem?	Review for AP Exam (10 blocks)
Unit #11: MP.4 MP.5		Topics learned throughout the course can be used in fun and creative ways in real-world situtions. 1. Students will complete two projects and/or	How can I apply what I learned in Calculus to the real world?	Post-AP Exam Projects (6 blocks)

presentations on various	
topics in AP Calculus.	

### Robbinsville Public Schools Scope, Sequence, Pacing and Assessment

### AP Calculus BC

	Unit Understandings and Goals	Recommended	Benchmark Assessments		
Unit Title		Duration/ Pacing	Diagnostic (before)	Formative (during)	Summative (after)
Unit #1: Limits and Continuity	<ul> <li>The concept of limit is one of the ideas that distinguish calculus from algebra and trigonometry. One of the uses of limits is to test functions for continuity.</li> <li>1. The student will define and calculate limits of function values.</li> <li>2. The student will determine the continuity of functions.</li> </ul>	Summer work plus 2 blocks	Summer Assignment Oral Questions/ Discussion Entrance Slips	Chapter Test Exit Slips Observations	AP Test
Unit # 2: Derivatives	<ul><li>The derivative gives the instantaneous rate of change of a function.</li><li>There are simple formulas for calculating derivatives of all types of functions.</li><li>1. The student will apply differentiation rules to calculate derivatives of various types of functions.</li></ul>	12 blocks	Oral Questions/ Discussion Entrance Slips Student Survey	Written Assignments Oral Presentations Observations Exit Slips Quizzes Chapter Test	AP Test

Unit #3: Applications of Derivatives	<ul> <li>Derivatives provide information about the general shape of a function's graph.</li> <li>Derivatives are applied to model and solve a variety of real-life problems.</li> <li>1. The student will find a function given only its first derivative and its value at a single point.</li> <li>2. The student will use derivatives to model and solve real-life application problems.</li> </ul>	9 blocks	Oral Questions/ Discussion Entrance Slips Student Survey	Written Assignments Oral Presentations Observations Exit Slips Quizzes Chapter Test	AP Test
Unit #4: The Definite Integral	<ul> <li>The integral describes how instantaneous changes accumulate over an interval to produce a function.</li> <li>Derivatives and integrals are closely related through the Fundamental Theorem of Calculus.</li> <li>1. The student will calculate integrals using the rules of integration and a graphing calculator.</li> </ul>	7 blocks	Oral Questions/ Discussion Entrance Slips Student Survey	Written Assignments Oral Presentations Observations Exit Slips Quizzes Chapter Test	AP Test
Unit #5: Differentia l Equations and Mathemati cal Modeling	<ul> <li>There are several methods for integrating functions for which there is no rule of integration.</li> <li>Differential equations are used to model various real-life situations and make predictions about future behavior.</li> <li>1. The student will use various integration methods to solve problems involving differential equations.</li> </ul>	8 blocks	Oral Questions/ Discussion Entrance Slips Student Survey	Written Assignments Oral Presentations Observations Exit Slips Quizzes Chapter Test	AP Test

Unit #6: Applications of the Definite Integral	<ul><li>Integrals can be applied to model and solve a wide variety of mathematical and real-life problems.</li><li>1. The student will solve problems by setting up an appropriate function and integrating it.</li></ul>	7 blocks	Oral Questions/ Discussion Entrance Slips Student Survey	Written Assignments Oral Presentations Observations Exit Slips Quizzes Chapter Test	AP Test
Unit #7: Sequences, L'Hopital's Rule, Improper Integrals	<ul> <li>Derivatives are used to calculate limits involving indeterminate forms.</li> <li>There are several techniques to evaluate integrals whose limits of integration are infinite.</li> <li>1. The student will use l'Hopital's Rule to calculate limits involving indeterminate forms.</li> <li>2. The student will use various methods to evaluate improper integrals.</li> </ul>	5 blocks	Oral Questions/ Discussion Entrance Slips Student Survey	<ul> <li>Written Assignments</li> <li>Oral Presentations</li> <li>Observations</li> <li>Projects</li> <li>Participatory Rubrics</li> <li>Exit Slips</li> <li>Quizzes</li> <li>Chapter Test</li> </ul>	AP Test
Unit #8: Infinite Series	<ul> <li>A convergent series is a number and can be treated as such.</li> <li>There are several methods for approximating a series by a polynomial.</li> <li>1. The student will apply a variety of techniques to test for the convergence of a series.</li> </ul>	9 blocks	Oral Questions/ Discussion Entrance Slips Student Survey	Written Assignments Oral Presentations Observations Exit Slips Quizzes Chapter Test	AP Test

Unit #9: Vectors, and Parametric & Polar Functions	<ul> <li>Single-variable calculus can be used to analyze some curves defined by two variables.</li> <li>1. Students will apply previously learned calculus concepts to differentiate and integrate parametric, vector, and polar fuctions.</li> </ul>	5 blocks	Oral Questions/ Discussion Entrance Slips Student Survey	Written Assignments Oral Presentations Observations Quizzes Chapter Test Exit Slips	AP Test
Unit #10: Review for AP Exam	<ul> <li>Mastery of all topics from AP Calc BC is necessary for success on the AP Exam <ol> <li>Students will complete practice tests in groups and individually</li> <li>Students will review topics by unit</li> <li>Students will do practice problems in which they must incorporate several topics at once and determine the best method for solving</li> </ol></li></ul>	10 blocks	Oral Questions/ Discussion Entrance Slips Student Survey	Written Assignments Oral Presentations Observations Participatory Rubrics Quizzes Exit Slips	AP Test
Unit #11: Post-AP Exam Work	<ul> <li>Topics learned throughout the course can be used in fun and creative ways in real-world situtions.</li> <li>1. Students will complete two projects and/or presentations on various topics in AP Calculus</li> </ul>	6 blocks	Oral Questions/ Discussion Student Survey	Written Assignments Oral Presentations Observations Participatory Rubrics	Projects Final Exam End-of-Year course evaluation

### Unit #1: Limits and Continuity

Enduring Understandings:	Essential Questions:	
• The concept of limit is one of the ideas that distinguish calculus from	• What does it mean for a function to have a limit at a point?	
algebra and trigonometry.	• Why are limits important in the study of calculus?	
• One of the uses of limits is to test functions for continuity.	• What does it mean for a function to be continuous?	
	• Why is it important to determine if a function is continuous?	

	ng / Topical Questions th Specific Standards	Content, Themes, Concepts, and Skills	Teaching Strategies	Instructional Resources and Materials	Assessment Strategies
MP.2	What does the limit of a function represent? What are the various methods used for calculating limits of functions?	Calculate limits and apply the properties of limits.	Complete the summer assignment Class discussion in review of summer assignment	Textbooks Summer assignment packet Summer assignment	Class participation Homework / written assignments Student solutions to
A-AP R.3	What is a continuous function? How can a function be modified to remove discontinuities? What is the connection between limits and continuity?	Identify continuous functions and intervals of continuity.	of a function.	answer packet Graphing calculators	summer assignment problems Written test
S-ID.7	How is the average rate of change of a function related to the slope of the curve?	Find the average rate of change of a function.			
S-ID.7	How can the slope of the curve be used to find the equation of the tangent line to the curve? How are the slope of the curve and the equation of the tangent line related to limits?	Find the slope of a curve and the equation of the tangent line to a curve.			

### Duration of Unit: Summer work plus 2 blocks

### Unit #2: Derivatives

Enduring Understandings:	Essential Questions		
• The derivative gives the instantaneous rate of change of a function.	• What does the derivative of a function represent?		
• There are simple formulas for calculating derivatives of all types of	• How are derivatives calculated using limits?		
functions.	• Is there a way to calculate derivatives that does not involve limits?		
	• How can derivatives be applied to solve real-life problems?		

#### Duration of Unit: 12 blocks

Guiding / Topical Questions with Specific Standards		Content, Themes, Concepts, and Skills	Teaching Strategies	Instructional Resources and Materials	Assessment Strategies
F-TF.6 -9	How does the derivative relate to limits and slope?	Find where a function is not differentiable.	Direct instruction	Textbooks	Written test / quiz
S-ID.7 A-AP	What causes a function to not be differentiable?		Class discussion (question and answer)	Graphing calculators	Cooperative activities (rubrics)
R.3			Discovery learning	www. illuminations.nctm.org	Notebooks
A-AP	What are the rules of	Use the rules of differentiation to calculate	Cooperative learning		
R.3	differentiation? When is it necessary to use the Chain	derivatives of various types of functions.	Investigations	www.apcentral.collegeb oard.com	Class participation
	Rule? When is it necessary to use Implicit Differentiation?		Use of instructional media	www. desmos.com	Homework / written assignments
S-ID.7	How is the derivative related to rates of change? How can the derivative be used to solve problems involving rates of change?	Use derivatives to solve problems involving rates of change.	Computer tutorials		

### Unit #3: Applications of Derivatives

Enduring Understandings:.	Essential Questions		
• Derivatives provide information about the general shape of a	• What information does a derivative reveal about the graph of the		
function's graph.	original function?		
• Derivatives are applied to model and solve a variety of real-life	• How are the graphs of the first and second derivative related to the		
problems.	graph of the original function?		
	• What types of real-life problems can derivatives be used to solve?		
	• How are derivatives applied in these problems?		

#### Duration of Unit: 9 blocks

	ing / Topical Questions th Specific Standards	Content, Themes, Concepts, and Skills	Teaching Strategies	Instructional Resources and Materials	Assessment Strategies
A-AP	What is the First Derivative Test? What is	Determine extreme values of a function.	Direct instruction	Textbooks	Written test / quiz
R.3	the Second Derivative Test? What information	Find the intervals on which a function increases or decreases.	Class discussion (question and answer)	Graphing calculators	Cooperative activities (rubrics)
	do the first and second		Discovery learning	www.	
	derivatives give about the original function?	Find the concavity of a function and the points of inflection.	Cooperative learning	illuminations.nctm.org	Notebooks
MP.4	How are derivatives used to solve real-life problems involving maximum and	Solve application problems involving minimum and maximum values.	Investigations	www.apcentral.collegeb oard.com	Class participation Homework / written
	minimum values? What strategies can be used to		Use of instructional media	www. desmos.com	assignments
	solve such problems?		Computer tutorials		Oral presentations
MP.5 A-REI .1	How are derivatives used to solve real-life problems involving related rates? What strategies can be used to solve such problems?	Solve related rates problems.			Special projects

### Unit #4: The Definite Integral

Enduring Understandings:	Essential Questions:		
• The integral describes how instantaneous changes accumulate over an	• What does a definite integral represent?		
interval to produce a function.	• What are several methods of calculating a definite integral?		
• Derivatives and integrals are closely related through the Fundamental	• What is the connection between derivatives and integrals?		
Theorem of Calculus.			

Guiding / Topical Questions with Specific Standards		Content, Themes, Concepts, and Skills	Teaching Strategies	Instructional Resources and Materials	Assessment Strategies
MP.4 A-AP	What are the several methods for	Approximate the area under the graph of a function.	Direct instruction	Textbooks	Written test / quiz
R.3	approximating the area under the graph of a		Class discussion (question and answer)	Graphing calculators	Cooperative activities (rubrics)
	function? How are they		Discovery learning	www.	
	similar? How are they			illuminations.nctm.org	Notebooks
	different?		Cooperative learning		
				www.apcentral.collegeb	Class participation
A-AP	What is a definite integral?	Express the area under the curve as a definite	Investigations	oard.com	Homework / written
R.6	How is it related to limits? How can it be used to	integral.	Use of instructional media	www. desmos.com	assignments
	calculate the area under a				0
	curve?		Computer tutorials		Oral presentations
A-AP	How can definite integrals be	Calculate definite integrals numerically and by			Special projects
<b>R.</b> 7	calculated numerically? How	applying rules for integrals.			
	can they be calculated				
	analytically? Which method				
	is more precise?				
MP.2	How is the definite integral	Apply the Fundamental Theorem of Calculus.	1		
	related to the derivative?				
	How is this connection				
	applied to calculate the value				
	of a definite integral				
	analytically?				1

#### Duration of Unit: 7 blocks

### Unit #5: Differential Equations and Mathematical Modeling

EnduringUnderstandings:	Essential Questions: :		
• There are several methods for integrating functions for which there is no rule	• How can a function be integrated if there are no rules of integration that		
of integration.	apply to the function?		
• Differential equations are used to model various real-life situations and make	• What is a differential equation?		
predictions about future behavior.	• How are differential equations related to problems involving growth and		
	decay?		

#### Duration of Unit: 8 blocks

Guiding / Topical Questions with Specific Standards		Content, Themes, Concepts, and Skills	Teaching Strategies	Instructional Resources and Materials	Assessment Strategies
A-AP R.3 MP.4	How do you solve an initial value problem? How does Euler's Method help you to	Solve initial value problems.	Direct instruction Class discussion (question and answer)	Textbooks Graphing calculators	Written test / quiz Cooperative activities
F-IF.5	graph the solution to an initial value problem?		Discovery learning	www.	(rubrics)
A-AP	What is the substitution method	Compute definite and indefinite integrals	Cooperative learning	illuminations.nctm.org	Notebooks
R.7	for evaluating integrals? When is it necessary to use the substitution method?	by substitution.	Investigations	www.apcentral.collegeb oard.com	Class participation Homework / written
A-AP R.7	What is integration by parts? When is it necessary to use	Use integration by parts to evaluate definite and indefinite integrals.	Use of instructional media	www. desmos.com	assignments
<b>R</b> . 7	integration by parts to evaluate an integral?		Computer tutorials		Oral presentations
A-AP R.3 MP.5	What is a separable differential equation? How is a separable differential equation solved? How does this apply to problems involving exponential growth and decay?	Solve problems involving exponential growth and decay.			Special projects
A-AP R.6	What is partial fraction decomposition? How does it apply to problems involving logistic growth?	Solve problems involving logistic population growth.			

### Unit #6: Applications of Definite Integrals

Enduring Understandings:		Essential Questions: :		
• Integrals can be applied to model and solve a wide variety of mathematical		•	• For what types of problems can integrals be used to model and solve the	
	and real-life problems.		problems?	

Guidin	g / Topical Questions with Specific Standards	Content, Themes, Concepts, and Skills	Teaching Strategies	Instructional Resources and Materials	Assessment Strategies
A-REI .1	How can integration be applied to a rate to determine	Use integration to find net change over time.	Direct instruction	Textbooks	Written test / quiz
A-AP R.3	net change over time?		Class discussion (question and answer)	Graphing calculators	Cooperative activities (rubrics)
S-ID.7			Discovery learning	www. illuminations.nctm.org	Notebooks
MP.4	How can integration be applied to calculate the area of a region	Use integration to calculate areas of regions in a plane.	Cooperative learning	www.apcentral.collegeb	Class participation
	in a plane?		Investigations	<u>oard.com</u>	Homework / written
MP.4	How can integration be applied	Use integration to calculate volumes and	Use of instructional media	www. desmos.com	assignments
G-GM D.1-4	to calculate the volume and surface area of a solid?	surface areas of solids.	Computer tutorials		Oral presentations
MP.5	How can integration be applied to calculate the length of a curve in a plane?	Use integration to calculate lengths of curves in a plane.			Special projects

#### Duration of Unit: 7 blocks

### Unit #7: Sequences, L'Hopital's Rule, and Improper Integrals

Enduring Understandings:	Essential Questions: :		
• Derivatives are used to calculate limits involving indeterminate forms.	• What is a limit of indeterminate form? How can such limits be		
• There are several techniques to evaluate integrals whose limits of integration	calculated?		
are infinite.	• What is an improper integral? How can such integrals be evaluated?		

Guidin	g / Topical Questions with Specific Standards	Content, Themes, Concepts, and Skills	Teaching Strategies	Instructional Resources and Materials	Assessment Strategies
F-BF.1 MP.8	What is a sequence? What are the two ways sequences are defined?	Define explicit and recursive rules for sequences.	Direct instruction Class discussion (question and answer) Discovery learning	Textbooks Graphing calculators www.	Written test / quiz Cooperative activities (rubrics)
A-AP R.3	What does it mean for a sequence to converge or diverge? How can we determine convergence or divergence?	Find the limit of a sequence and determine whether a sequence converges or diverges.	Cooperative learning Investigations	illuminations.nctm.org www.apcentral.collegeb oard.com	Notebooks Class participation Homework / written
A-AP R.6,7	What is l'Hopital's Rule? How can it be used to find the limits of indeterminate forms? How can it be used to compare the rates of growth of functions?	Find limits of indeterminate forms using l'Hopital's Rule. Use l'Hopital's Rule to compare the rates of growth of functions.	Use of instructional media Computer tutorials	www. desmos.com	assignments Oral presentations Special projects
F-BF.1	What is an improper integral? How can limits be used to evaluate an improper integral?	Use limits to evaluate improper integrals			

### Duration of Unit: 6 blocks

### Unit #8: Infinite Series

Enduring Understandings:	Essential Questions: :		
• A convergent series is a number and can be treated as such.	• What is an infinite series?		
• There are several methods for approximating a series by a polynomial.	• How is an infinite series related to a function?		
	• How can the convergence or divergence of a series be determined?		
	• Why is it important to determine if a series converges?		

Duration of Unit: 9 blocks

Guidin	g / Topical Questions with Specific Standards	Content, Themes, Concepts, and Skills	Teaching Strategies	Instructional Resources and Materials	Assessment Strategies
A-AP R.1	How do you represent a function by a series? How do you find a series by differentiating or integrating?	Differentiate or integrate a known power series to find additional power series representations.	Direct instruction Class discussion (question and answer) Discovery learning	Textbooks Graphing calculators www. illuminations.nctm.org	Written test / quiz Cooperative activities (rubrics) Notebooks
A-APR. 3	What is a Maclaurin series? What is a Taylor series? How are these series generated?	Find the Maclaurin series or Taylor series generated by a differentiable function.	Cooperative learning Investigations	www.apcentral.collegeb oard.com	Class participation Homework / written
A-APR. 6,7	What is a Taylor polynomial? What is the truncation error of a Taylor polynomial? How is the error found?	Approximate a function with a Taylor polynomial.	Use of instructional media Computer tutorials	www. desmos.com	assignments
A-SSE .4	What are some tests that can be used to determine the convergence or divergence of a series? How are these tests applied?	Determine the convergence or divergence of a series of numbers.			

### Unit #9: Parametric, Vector, and Polar Functions

Enduring Understandings:	Essential Questions: :		
• Single-variable calculus can be used to analyze some curves defined by two	• How can the calculus concepts learned throughout this course be applied		
variables.	to functions defined by two variables?		

#### Duration of Unit: 5 blocks

Guidin	g / Topical Questions with Specific Standards	Content, Themes, Concepts, and Skills	Teaching Strategies	Instructional Resources and Materials	Assessment Strategies
F-LE.5	How do you differentiate a parametric function? How	Find derivatives and integrals of parametrically defined functions.	Direct instruction	Textbooks	Written test / quiz
	do you integrate a parametric function? How can this be		Class discussion (question and answer)	Graphing calculators	Cooperative activities (rubrics)
	applied to find arc length?		Discovery learning	www. illuminations.nctm.org	Notebooks
N-VM.	What is a vector? What does a	Use vectors to solve application problems.	Cooperative learning	0	
3	vector represent? How can			www.apcentral.collegeb	Class participation
	vectors be used to model		Investigations	<u>oard.com</u>	
	motion? How do you				Homework / written
	differentiate and integrate vector		Use of instructional media	www. desmos.com	assignments
	functions?				
A-APR.	What are polar coordinates?	Calculate slopes and areas of regions	Computer tutorials		
3	How do you convert between	determined by polar curves.			
	rectangular and polar				
	coordinates? How do you				
	differentiate and integrate polar				
	functions?				

### Unit #10: Review for AP Exam

Enduring Understandings:	Essential Questions: :	
• A deep understanding of differentiable & integral calculus and their applications	• What can I do to prepare for the AP Exam?	
is cruicial to a successful AP Exam.		

#### **Duration of Unit:** 10 blocks

Guidir	ng / Topical Questions with Specific Standards	Content, Themes, Concepts, and Skills	Teaching Strategies	Instructional Resources and Materials	Assessment Strategies
MP.1 MP.5	How can I determine the best methods for solving a problem?	Review all topics from the curriculum	Direct instruction (review) Class discussion (question and answer)	Textbooks Graphing calculators	Partner/group quizzes Practice Tests
			Discovery learning	www. illuminations.nctm.org	Cooperative activities (rubrics)
MP.6	What is the best strategy for multiple choice problems?	Take practice tests	Cooperative learning Investigations	www.apcentral.collegeb oard.com	Notebooks Class participation
			Use of instructional media	www. desmos.com	Homework / written
MP.2 MP.3	How can I make sure I explain my work correctly and thoroughly on the free response section?	Complete previous years' free response problems	Computer tutorials Afterschool and Evening Review Sessions	Previous AP Exams AP Review Books	assignments Oral presentations Special projects

### Unit #11: Post-AP Exam Work

Enduring Understandings:	Essential Questions: :	
• Calculus is all around us	• Where is calculus found in the real world?	

Duration of Unit: 6 blocks

Guidir	ng / Topical Questions with Specific Standards	Content, Themes, Concepts, and Skills	Teaching Strategies	Instructional Resources and Materials	Assessment Strategies
MP.4 MP.5	How can I apply what I learned in Calculus to the real world?	<ul> <li>Choose two projects to complete that use single-variable calculus in real-world situations. Examples include: <ol> <li>"Cooking with Calculus":Students rewrite numbers in a recipe using expressions and equations from the course</li> <li>Volume Model Project: Students use integrals to express the volume of a solid, then construct the actual solid out of any material they like</li> <li>Research Project: Students choose a topic in math (like Math in Football or Counting Cards) and write a research paper, then give a presentation to the class</li> <li>Flashcards: Students make flashcards or a matching game about a topic from calculus for</li> </ol> </li> </ul>	Student-directed research Independent study/work time Group work	Materials         Desmos.com         Online resources         All worksheets/tests         from the year         Textbooks         Graphing Calculators	Project Final Exam
		future AP Calc students to use to study			

### English Language Learner (ELL) Resources

- Learning style quiz for students- http://www.educationplanner.org/students/self-assessments/learning-styles-quiz.shtml
- "Word clouds" from text that you provide-http://www.wordle.net/
- Bilingual website for students, parents and educators: http://www.colorincolorado.org/
- Learn a language for FREE-www.Duolingo.com
- Time on task for students-http://www.online-stopwatch.com/
- Differentiation activities for students based on their Lexile-www.Mobymax.com
- WIDA-http://www.wida.us/
- Everything ESL http://www.everythingESL.net
- ELL Tool Box Suggestion Sitehttp://www.wallwisher.com/wall/elltoolbox
- Hope4Education http://www.hope4education.com
- Learning the Language http://blogs.edweek.org/edweek/learning-the-language/
- FLENJ (Foreign Language Educators of NJ) 'E-Verse' wiki: http://www.flenj.org/Publications/?page=135
- OELA http://www.ed.gov/offices/OBEMLA
- New Jersey Department of Education-Bilingual Education information http://www.state.nj.us/education/bilingual/

### **Special Education Resources**

- Animoto -Animoto provides tools for making videos by using animation to pull together a series of images and combining with audio. Animoto videos or presentations are easy to publish and share. https://animoto.com
- Bookbuilder -Use this site to create, share, publish, and read digital books that engage and support diverse learners according to their individual needs, interests, and skills. http://bookbuilder.cast.org/
- CAST -CAST is a non-profit research and development organization dedicated to Universal Design for Learning (UDL). UDL research demonstrates that the challenge of diversity can and must be met by making curriculum flexible and responsive to learner differences. http://www.cast.org

- CoSketch -CoSketch is a multi-user online whiteboard designed to give you the ability to quickly visualize and share your ideas as images. http://www.cosketch.com/
- Crayon -The Crayon.net site offers an electronic template for students to create their own newspapers. The site allows you to bring multiple sources together, thus creating an individualized and customized newspaper. http://crayon.net/ Education Oasis -Education Oasis offers a collection of graphic organizers to help students organize and retain knowledge cause and effect, character and story, compare and contrast, and more! http://www.educationoasis.com/printables/graphic-organizers/
- Edutopia -A comprehensive website and online community that increases knowledge, sharing, and adoption of what works in K-12 education. We emphasize core strategies: project-based learning, comprehensive assessment, integrated studies, social and emotional learning, educational leadership and teacher development, and technology integration. <u>http://www.edutopia.org/</u>
- Glogster -Glogster allows you to create "interactive posters" to communicate ideas. Students can embedded media links, sound, and video, and then share their posters with friends. http://edu.glogster.com/?ref=personal
- Interactives Elements of a Story -This interactive breaks down the important elements of a story. Students go through the series of steps for constructing a story including: Setting, Characters, Sequence, Exposition, Conflict, Climax, and Resolution. http://www.learner.org/interactives/story/index.html
- National Writing Project (NWP) -Unique in breadth and scale, the NWP is a network of sites anchored at colleges and universities and serving teachers across disciplines and at all levels, early childhood through university. We provide professional development, develop resources, generate research, and act on knowledge to improve the teaching of writing and learning in schools and communities. http://www.nwp.org
- Pacecar -Vocab Ahead offers videos that give an active demonstration of vocabulary with audio repeating the pronunciation, definition, various uses, and synonyms. Students can also go through flash cards which give a written definition and visual representation of the word. http://pacecar.missingmethod.com/