

- * 31. M&M's. The Masterfoods company says that before the introduction of purple, yellow candies made up 20% of their plain M&M's, red another 20%, and orange, blue, and green each made up 10%. The rest were brown.

b) If you pick three M&M's in a row, what is the probability that

1) they are all brown?

$$P(\text{BROWN}) = (.3)(.3)(.3) = .027 = 2.7\%$$

2) the third one is the first one that's red? = $P(3^{\text{rd}} \text{ RED}) = (.8)(.8)(.2) = .128 = 12.8\%$

3) none are yellow? $P(\text{NONE YELLOW}) = (.8)^3 = .512 = 51.2\%$

4) at least one is green?

$$\rightarrow P(\text{at least 1 Green}) = 1 - (.9)^3 = .271 = 27.1\%$$

20% YELLOW

20% RED

10% Orange,

10% blue,

10% green

30% OTHER BROWN

SAMPLING WITHOUT REPLACEMENT

- * 16. Another hand. You pick three cards at random from a deck. Find the probability of each event described below.

a) You get no aces.

b) You get all hearts.

c) The third card is your first red card.

d) You have at least one diamond.

$$\textcircled{C} P(3^{\text{rd}} \text{ CARD RED}) = \left(\frac{26}{52}\right)\left(\frac{25}{51}\right)\left(\frac{24}{50}\right) = .127$$

$$\textcircled{A} P(\text{NO ACE}) = \frac{52-4}{52} = \frac{48}{52} \left(\frac{47}{51}\right) \left(\frac{46}{50}\right) = .783$$

$$\textcircled{D} P(\text{AT LEAST 1 DIAMOND}) = 1 - \left(\frac{39}{52}\right)\left(\frac{38}{51}\right)\left(\frac{37}{50}\right) = .318$$

$$\textcircled{B} P(\text{ALL HEARTS}) = \frac{13}{52} \left(\frac{12}{51}\right) \left(\frac{11}{50}\right) = .013$$

OPTIONAL - MORE PRACTICE

- ✓ 18. Shirts. The soccer team's shirts have arrived in a big box, and people just start grabbing them, looking for the right size. The box contains 4 medium, 10 large, and 6 extra-large shirts. You want a medium for you and one for your sister. Find the probability of each event described.

a) The first two you grab are the wrong sizes.

b) The first medium shirt you find is the third one you check.

c) The first four shirts you pick are all extra-large.

d) At least one of the first four shirts you check is a medium.

$$M=4 \quad L=10 \quad XL=6 \quad \text{TOTAL}=20 \quad \text{NEED 2 MEDIUM}$$

$$\textcircled{A} P(\text{NOT M}) P(\text{NOT M}) = \left(\frac{16}{20}\right) \left(\frac{15}{19}\right) = .632$$

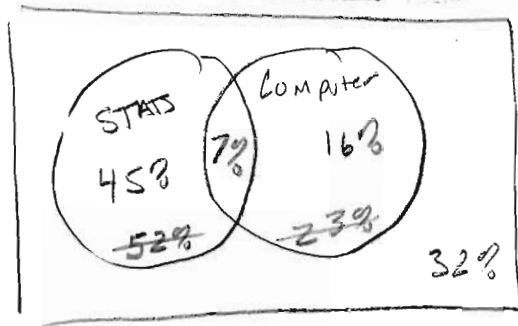
$$\textcircled{B} P(\text{NOT M}) (P(\text{NOT M}) P(M)) = \left(\frac{16}{20}\right) \left(\frac{15}{19}\right) \left(\frac{4}{18}\right) = .140$$

$$\textcircled{C} P(L) P(XL) P(XL) P(XL) = \left(\frac{10}{20}\right) \left(\frac{5}{19}\right) \left(\frac{4}{18}\right) \left(\frac{3}{17}\right) = .003$$

$$\textcircled{D} 1 - \left(\frac{16}{20}\right) \left(\frac{15}{19}\right) \left(\frac{14}{18}\right) \left(\frac{13}{17}\right) = .624$$

DISJOINT AND INDEPENDENCE

- * 19. Eligibility. A university requires its biology majors to take a course called BioResearch. The prerequisite for this course is that students must have taken either a Statistics course or a computer course. By the time they are juniors, 52% of the Biology majors have taken Statistics, 23% have had a computer course, and 7% have done both.
- What percent of the junior Biology majors are ineligible for BioResearch?
 - What's the probability that a junior Biology major who has taken Statistics has also taken a computer course?
 - Are taking these two courses disjoint events? Explain.
 - Are taking these two courses independent events? Explain.



(a) $P(\text{INELIGIBLE}) = P(\text{NEITHER}) = 32\%$

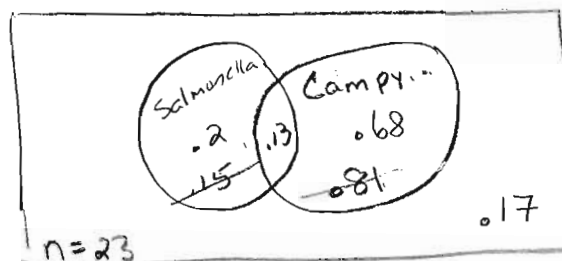
(b) $P(\text{COMPUTER} | \text{STATS}) = \frac{.07}{.52} = .135 = 13.5\%$

(c) TAKING THE 2 COURSES IS NOT DISJOINT, SINCE THEY HAVE OUTCOMES IN COMMON (7% TOOK BOTH)

(d) TAKING THE 2 COURSES ARE NOT INDEPENDENT. CHECKED $P(A) = P(A|B)$
 $P(\text{COMPUTER}) = P(\text{COMPUTER} | \text{STAT})$ OR $P(\text{STAT}) = P(\text{STAT} | \text{COMPUTER})$
 $.23 \neq .135$ OR $.52 \neq .07/.23 = .30$

OPTIONAL: MORE PRACTICE:

25. Unsafe food. Early in 2007 Consumer Reports published the results of an extensive investigation of broiler chickens purchased from food stores in 23 states. Tests for bacteria in the meat showed that 81% of the chickens were contaminated with campylobacter, 15% with salmonella, and 13% with both.
- What's the probability that a tested chicken was not contaminated with either kind of bacteria?
 - Are contamination with the two kinds of bacteria disjoint? Explain.
 - Are contamination with the two kinds of bacteria independent? Explain.



(a) $P(\text{NEITHER}) = 17\%$ (see Venn diagram)

(b) NOT DISJOINT. 13% OVERLAP. AND HAVE BOTH

(c) NOT INDEPENDENT.

$$P(\text{Salm...}) = P(\text{SALM} | \text{campy})$$

$$.15 = .13 / .81$$

$$.15 \neq .16$$

$$P(\text{Camp...}) = P(\text{Camp...} | \text{SALM})$$

$$.68 = .13 / .15$$

$$.68 \neq .87$$

- * 24. **Pets again.** The local animal shelter reported that it currently has 24 dogs and 18 cats available for adoption; 8 of the dogs and 6 of the cats are male. Are the species and sex of the animals independent? Explain.
(TIP MAKE A TABLE)

	CATS	DOGS	TOTAL
M	6 ^{.33}	8 ^{.33}	14
F	12 ^{.67}	16 ^{.67}	28
TOTAL	18 ^{1.00}	24 ^{1.00}	42

INDEPENDENT

① LOOK AT TABLE %'S

1/3 OF DOGS MALE

1/3 OF FEMALES MALE

$$\begin{aligned} \textcircled{2} P(\text{CATS}) &= P(\text{CATS} | \text{MALE}) \\ 18/42 &= 6/14 \\ .43 &= .43 \end{aligned}$$

- * 28. **Politics.** Given the table of probabilities are party affiliation and position on the death penalty independent? Explain.

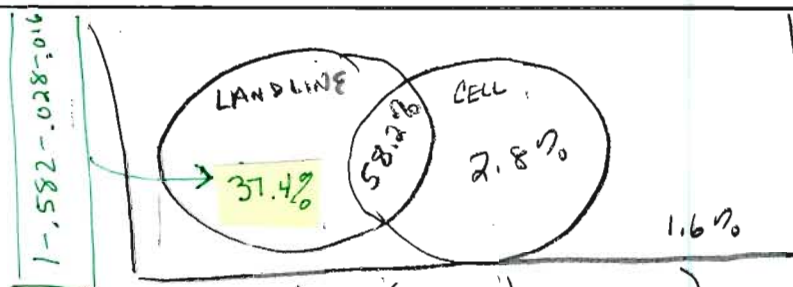
	Death Penalty	
	Favor	Oppose
Republican	0.26	0.04
Democrat	0.12	0.24
Other	0.24	0.10

NOT INDEPENDENT

* IF THE EVENTS WERE INDEPENDENT THEN THESE PERCENTAGES WOULD BE THE SAME.

- * 29. **Phone service.** According to estimates from the federal government's 2003 National Health Interview Survey, based on face-to-face interviews in 16,677 households, approximately 58.2% of U.S. adults have both a landline in their residence and a cell phone, 2.8% have only cell phone service but no landline, and 1.6% have no telephone service at all.

• ARE HAVING A CELL PHONE + LANDLINE INDEPENDENT?



$$\begin{aligned} P(\text{CELL}) &= P(\text{CELL} | \text{LANDLINE}) \\ .582 + .028 &= .582 / (1.582 + .374) \\ .610 &\approx .609 \end{aligned}$$

* THE PROBABILITIES ARE ROUGHLY THE SAME, SO IT APPEARS CELL + LANDLINE ARE INDEPENDENT

- * 31. **Montana.** A 1992 poll conducted by the University of Montana classified respondents by sex and political party, as shown in the table. Is party affiliation independent of the respondents' sex? Explain.

	Democrat	Republican	Independent
Male	36	45	24
Female	48	33	16

84 78 40 202
41.6% 38.6% 19.8% 100%

NOT INDEPENDENT

41.6% - Democratic Overall
36/105 = P(DEM | MALE)
34.2% = OF MALES, ONLY
34.2% DEM AND
SHOULD BE 41.6% TO BE INDEPENDENT.

TREE DIAGRAMS - REVERSE CONDITION

- * 34. **Graduation.** A private college report contains these statistics:

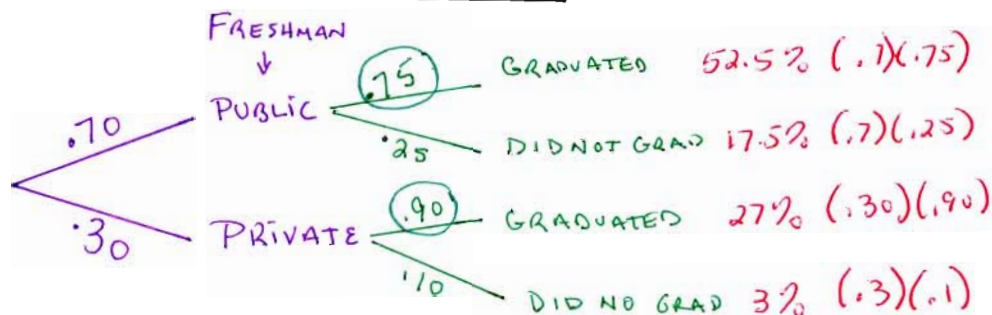
70% of incoming freshmen attended public schools.
75% of public school students who enroll as freshmen eventually graduate.
90% of other freshmen eventually graduate.

- a) Is there any evidence that a freshman's chances to graduate may depend upon what kind of high school the student attended? Explain.
b) What percent of freshmen eventually graduate?

$$\rightarrow P(\text{GRADUATE}) = 52.5\% + 27\% = 79.5\%$$

YES, THERE IS EVIDENCE THAT TYPE OF SCHOOL AND GRADUATION ARE NOT INDEPENDENT. IF THEY WERE INDEPENDENT THEN GRADUATION RATES WOULD BE THE SAME. BUT 75% GRADUATION RATE AT PUBLIC COMPARED TO GRADUATION RATE OF 90% AT PRIVATE.

34. CREATE A TREE:



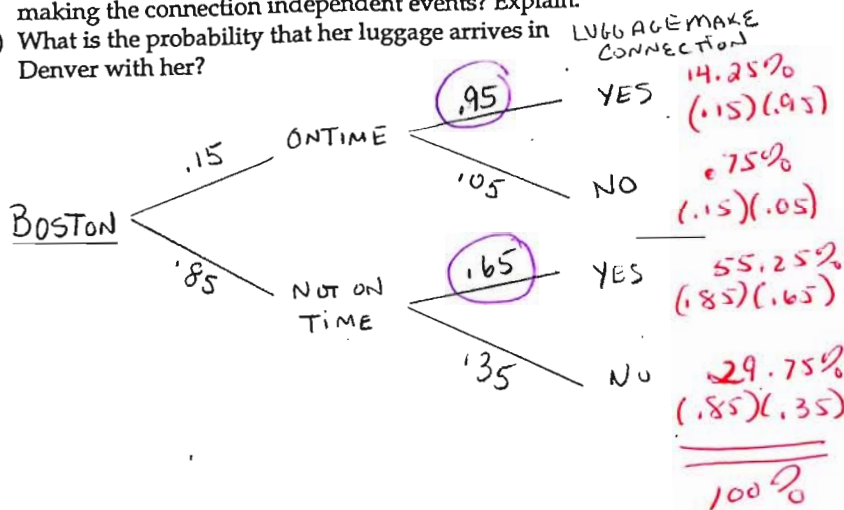
- * 36. **Graduation, part II.** What percent of students who graduate from the college in Exercise 34 attended a public high school?

$$P(\text{PUBLIC} | \text{GRADUATED}) = \frac{P(\text{PUBLIC} \cap \text{GRADUATED})}{P(\text{GRADUATED})} = \frac{.525}{.525 + .27} = .660$$

ABOUT 66% who graduated from college attended a public high school

33. **Luggage.** Leah is flying from Boston to Denver with a connection in Chicago. The probability her first flight leaves on time is 0.15. If the flight is on time, the probability that her luggage will make the connecting flight in Chicago is 0.95, but if the first flight is delayed, the probability that the luggage will make it is only 0.65.

- a) Are the first flight leaving on time and the luggage making the connection independent events? Explain.
b) What is the probability that her luggage arrives in Denver with her?



(a) NOT INDEPENDENT.

The probability is .95 if the flight is ON TIME, AND ONLY .65 if it is NOT ON TIME.

(b) $P(\text{LUGGAGE ARRIVES IN DENVER}) = 14.25\% + 55.25\% = 69.5\%$

35. **Late luggage.** Remember Leah (Exercise 33)? Suppose you pick her up at the Denver airport, and her luggage is

not there. What is the probability that Leah's first flight was delayed?

$$\begin{aligned}
 P(\text{LATE} \mid \text{MISSING LUGGAGE}) &= \frac{P(\text{LATE} \cap \text{MISSING LUGGAGE})}{P(\text{MISSING LUGGAGE})} \\
 &= \frac{29.75}{.75 + 29.75} \\
 &= .975
 \end{aligned}$$

The probability Leah first flight was delayed given her luggage is missing is 97.5%