1. If $y = \left(x^3 + 1\right)^2$, then $\frac{dy}{dx} =$

(A) $\left(3x^2\right)^2$ (B) $2\left(x^3 + 1\right)$ (C) $2\left(3x^2 + 1\right)$ (D) $3x^2\left(x^3 + 1\right)$ $\boxed{\text{(E) } 6x^2\left(x^3 + 1\right)}$

$2(x^3+1)\cdot 3x^2$

$6x^2(x^3+1)$

---

2. $\int_0^1 e^{-4x}\,dx =$

(A) $\frac{-e^{-4}}{4}$ (B) $-4e^{-4}$ (C) $e^{-4} - 1$ $\boxed{\text{(D) } \frac{1}{4} - \frac{e^{-4}}{4}}$ (E) $4 - 4e^{-4}$

$$-\frac{1}{4}\cdot e^{-4x}\Big|_0^1 = -\frac{1}{4}e^{-4} - -\frac{1}{4}e^{0}$$
$$= -\frac{1}{4}e^{-4} + \frac{1}{4}$$

3. For $x \geq 0$, the horizontal line $y = 2$ is an asymptote for the graph of the function f. Which of the following statements must be true?

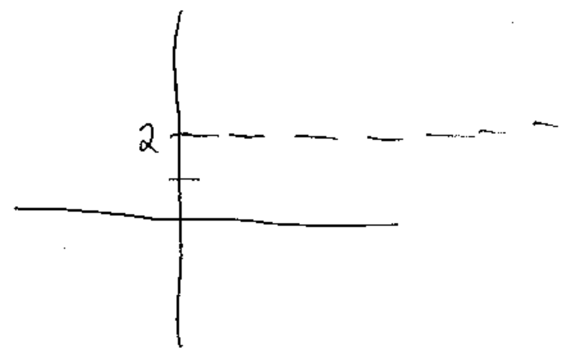
(A) $f(0) = 2$

(B) $f(x) \neq 2$ for all $x \geq 0$

(C) $f(2)$ is undefined.

(D) $\lim_{x \to 2} f(x) = \infty$

(E) $\lim_{x \to \infty} f(x) = 2$



4. If $y = \dfrac{2x+3}{3x+2}$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{12x+13}{(3x+2)^2}$ (B) $\dfrac{12x-13}{(3x+2)^2}$ (C) $\dfrac{5}{(3x+2)^2}$ (D) $\dfrac{-5}{(3x+2)^2}$ (E) $\dfrac{2}{3}$

$$y' = \frac{2(3x+2) - 3(2x+3)}{(3x+2)^2} = \frac{\cancel{6x} + 4 - \cancel{6x} - 9}{(3x+2)^2}$$

5. $\int_0^{\frac{\pi}{4}} \sin x\, dx =$

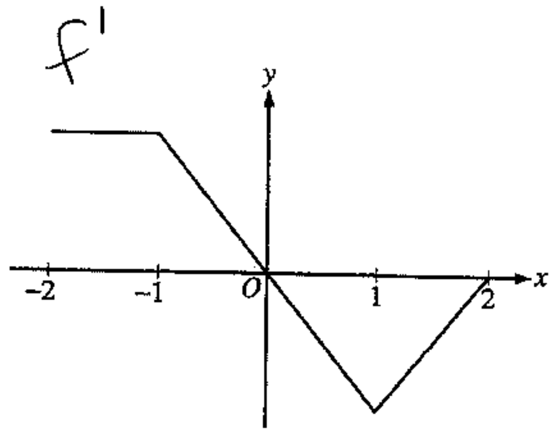
(A) $-\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{2}}{2}$ (C) $-\frac{\sqrt{2}}{2}-1$ $\boxed{\text{(D) } -\frac{\sqrt{2}}{2}+1}$ (E) $\frac{\sqrt{2}}{2}-1$

$$-\cos x \Big|_0^{\frac{\pi}{4}} = -\cos\left(\frac{\pi}{4}\right) - -\cos(0)$$

$$-\frac{\sqrt{2}}{2}+1$$

6. $\lim_{x\to\infty} \frac{x^3 - 2x^2 + 3x - 4}{4x^3 - 3x^2 + 2x - 1} =$ $\frac{1}{4}$

(A) 4 (B) 1 $\boxed{\text{(C) } \frac{1}{4}}$ (D) 0 (E) -1



Graph of f'

7. The graph of f', the derivative of the function f, is shown above. Which of the following statements is true about f ?

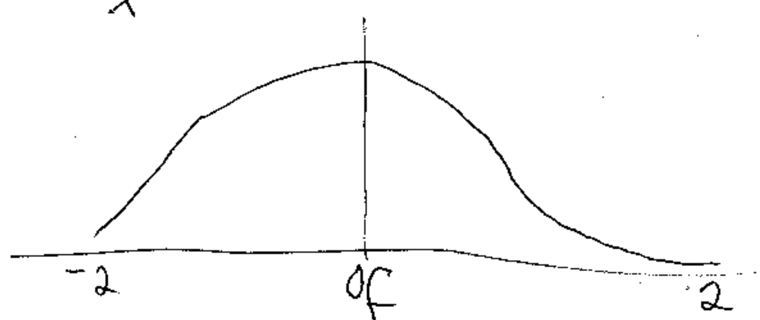
(A) f is decreasing for $-1 \leq x \leq 1$.

(B) f is increasing for $-2 \leq x \leq 0$.

(C) f is increasing for $1 \leq x \leq 2$.

(D) f has a local minimum at $x = 0$.

(E) f is not differentiable at $x = -1$ and $x = 1$.



8. $\int x^2 \cos\left(x^3\right) dx =$

(A) $-\frac{1}{3}\sin\left(x^3\right) + C$

(B) $\frac{1}{3}\sin\left(x^3\right) + C$

(C) $-\frac{x^3}{3}\sin\left(x^3\right) + C$

(D) $\frac{x^3}{3}\sin\left(x^3\right) + C$

(E) $\frac{x^3}{3}\sin\left(\frac{x^4}{4}\right) + C$

$\frac{1}{3}\sin\left(x^3\right) + C$

9. If $f(x) = \ln\left(x + 4 + e^{-3x}\right)$, then $f'(0)$ is

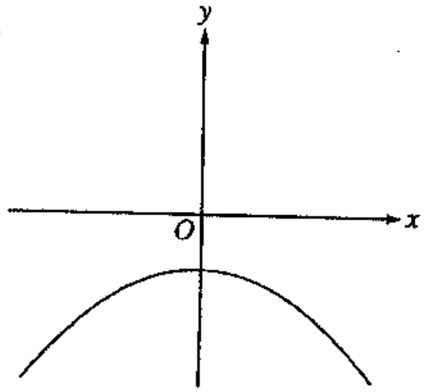
(A) $-\frac{2}{5}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$ (E) nonexistent

$$f'(x) = \frac{1}{x+4+e^{-3x}} \cdot \left(1 + -3e^{-3x}\right)$$

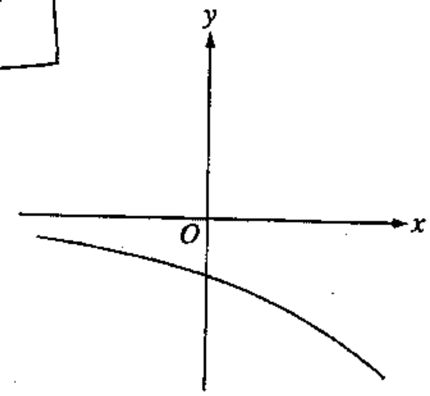
$$f'(0) = \frac{1}{5} \cdot (1 + -3) = -\frac{2}{5}$$

10. The function f has the property that $f(x)$, $f'(x)$, and $f''(x)$ are negative for all real values x. Which of the following could be the graph of f ?

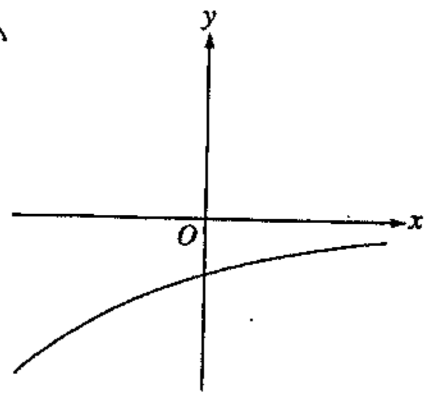
(A)



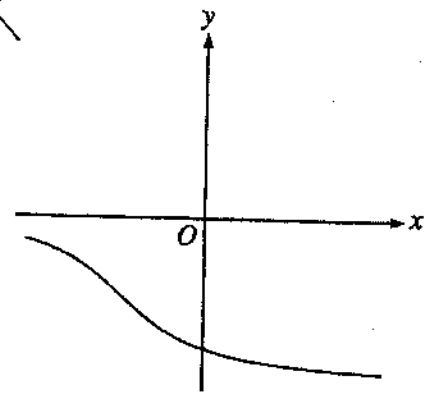
(B)



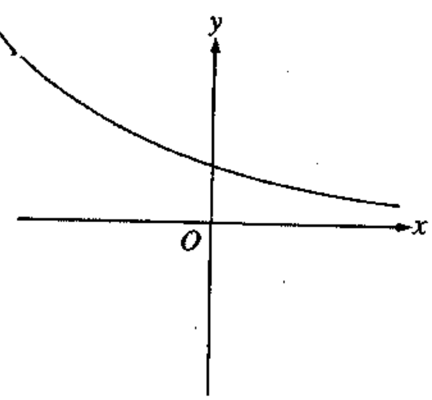
(C)



(D)



(E)



11. Using the substitution $u = 2x + 1$, $\int_0^2 \sqrt{2x+1}\,dx$ is equivalent to

(A) $\frac{1}{2}\int_{-1/2}^{1/2} \sqrt{u}\,du$ (B) $\frac{1}{2}\int_0^2 \sqrt{u}\,du$ (C) $\boxed{\frac{1}{2}\int_1^5 \sqrt{u}\,du}$ (D) $\int_0^2 \sqrt{u}\,du$ (E) $\int_1^5 \sqrt{u}\,du$

limits of integ : $2(0)+1 = 1$ $2(2)+1 = 5$

$\int_1^5 u^{\frac{1}{2}} \cdot \frac{1}{2}\,du$

12. The rate of change of the volume, V, of water in a tank with respect to time, t, is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship?

(A) $V(t) = k\sqrt{t}$

(B) $V(t) = k\sqrt{V}$

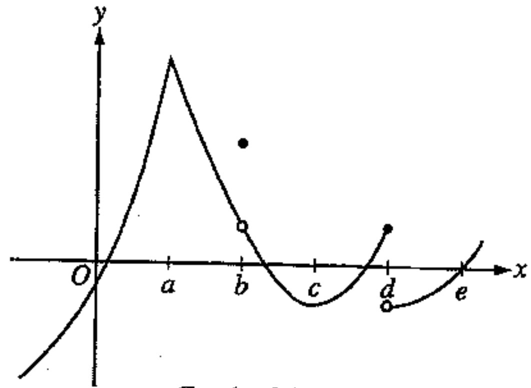
(C) $\frac{dV}{dt} = k\sqrt{t}$

(D) $\frac{dV}{dt} = \frac{k}{\sqrt{V}}$

(E) $\boxed{\frac{dV}{dt} = k\sqrt{V}}$

$\frac{dV}{dt} = k\sqrt{V}$

2003 Released Exam Excerpt



Graph of f

13. The graph of a function f is shown above. At which value of x is f continuous, but not differentiable?

(A) a (B) b (C) c (D) d (E) e

14. If $y = x^2 \sin 2x$, then $\frac{dy}{dx} =$

(A) $2x \cos 2x$

(B) $4x \cos 2x$

(C) $2x(\sin 2x + \cos 2x)$

(D) $2x(\sin 2x - x \cos 2x)$

(E) $2x(\sin 2x + x \cos 2x)$

$2x \cdot \sin(2x) + x^2 \cdot \cos(2x) \cdot 2$

$2x(\sin 2x + x \cos 2x)$

15. Let f be the function with derivative given by $f'(x) = x^2 - \frac{2}{x}$. On which of the following intervals is f decreasing?

(A) $(-\infty, -1]$ only

(B) $(-\infty, 0)$

(C) $[-1, 0)$ only

(D) $(0, \sqrt[3]{2}]$

(E) $[\sqrt[3]{2}, \infty)$

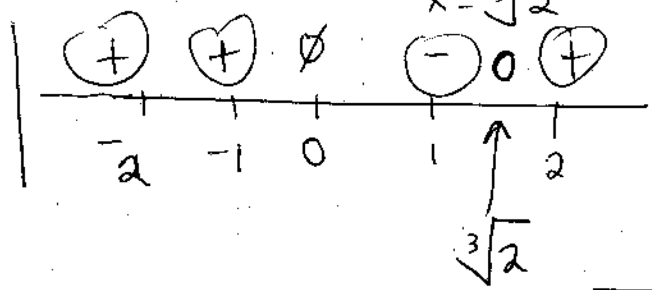
$x^2 - \frac{2}{x} = 0$

$f'(x)$ is neg.

$\frac{2}{x} = x^2$

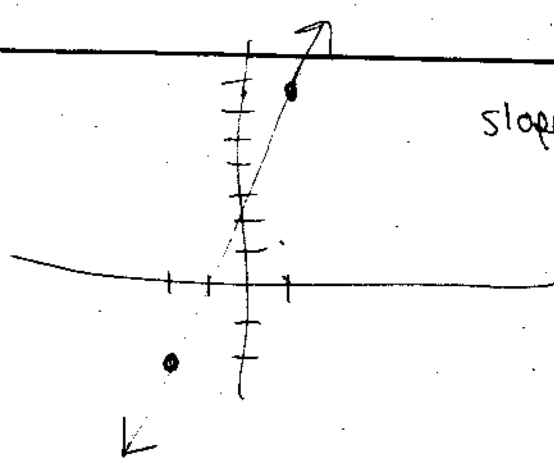
$2 = x^3$

$x = \sqrt[3]{2}$



16. If the line tangent to the graph of the function f at the point $(1, 7)$ passes through the point $(-2, -2)$, then $f'(1)$ is

(A) -5 (B) 1 (C) 3 (D) 7 (E) undefined



$\text{slope} = \frac{9}{3} = 3$

17. Let f be the function given by $f(x) = 2xe^x$. The graph of f is concave down when

(A) $x < -2$ (B) $x > -2$ (C) $x < -1$ (D) $x > -1$ (E) $x < 0$

$f''(x)$ $-$ $+$ -3 -2 0

$f'(x) = 2e^x + 2xe^x$

$f''(x) = 2e^x + 2e^x + 2xe^x = 4e^x + 2xe^x$

$0 = e^x(4+2x) \rightarrow x = -2$

---

x	-4	-3	-2	-1	0	1	2	3	4
$g'(x)$	2	3	0	-3	-2	-1	0	3	2

18. The derivative g' of a function g is continuous and has exactly two zeros. Selected values of g' are given in the table above. If the domain of g is the set of all real numbers, then g is decreasing on which of the following intervals?

(A) $-2 \le x \le 2$ only

(B) $-1 \le x \le 1$ only

(C) $x \ge -2$

(D) $x \ge 2$ only

(E) $x \le -2$ or $x \ge 2$

19. A curve has slope $2x + 3$ at each point (x, y) on the curve. Which of the following is an equation for this curve if it passes through the point $(1, 2)$?

(A) $y = 5x - 3$

(B) $y = x^2 + 1$

(C) $y = x^2 + 3x$

(D) $\boxed{y = x^2 + 3x - 2}$

(E) $y = 2x^2 + 3x - 3$

$f(x) = x^2 + 3x + C$

$f'(x) = 2x + 3$

$2 = (1)^2 + 3(1) + C$

$-2 = C$

$$f(x) = \begin{cases} x + 2 & \text{if } x \le 3 \\ 4x - 7 & \text{if } x > 3 \end{cases}$$

(3, 5) ✓ continuous
(3, 5)

20. Let f be the function given above. Which of the following statements are true about f ?

I. $\lim_{x \to 3} f(x)$ exists. ✓

II. f is continuous at $x = 3$. ✓

III. f is differentiable at $x = 3$. X

$f'(x) = \begin{cases} 1 \\ 4 \end{cases}$ not differentiable

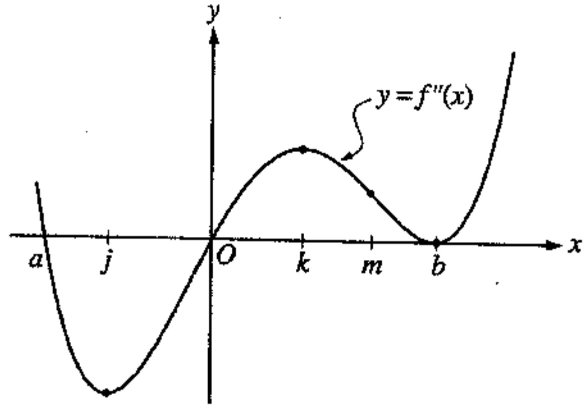
(A) None

(B) I only

(C) II only

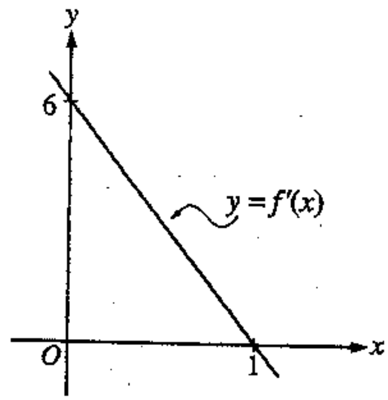
(D) $\boxed{\text{I and II only}}$

(E) I, II, and III



21. The second derivative of the function f is given by $f''(x) = x(x-a)(x-b)^2$. The graph of f'' is shown above. For what values of x does the graph of f have a point of inflection?

(A) 0 and a only (B) 0 and m only (C) b and j only (D) 0, a, and b (E) b, j, and k



$f'(x) = -6x + 6$
$f(x) = -3x^2 + 6x + C$
$5 = C$

22. The graph of f', the derivative of f, is the line shown in the figure above. If $f(0) = 5$, then $f(1) =$

(A) 0 (B) 3 (C) 6 $\boxed{\text{(D) } 8}$ (E) 11

$f(x) = -3x^2 + 6x + 5$
$f(1) = -3 + 6 + 5$

23. $\frac{d}{dx}\left(\int_0^{x^2} \sin(t^3)\, dt\right) =$

(A) $-\cos(x^6)$ (B) $\sin(x^3)$ (C) $\sin(x^6)$ (D) $2x\sin(x^3)$ $\boxed{\text{(E) } 2x\sin(x^6)}$

$\sin((x^2)^3) \cdot 2x = 2x\sin(x^6)$

24. Let f be the function defined by $f(x) = 4x^3 - 5x + 3$. Which of the following is an equation of the line tangent to the graph of f at the point where $x = -1$?

(A) $y = 7x - 3$

(B) $y = 7x + 7$

(C) $y = 7x + 11$

(D) $y = -5x - 1$

(E) $y = -5x - 5$

$f(-1) = -4 + 5 + 3 = 4$ $(-1, 4)$

$f'(x) = 12x^2 - 5$

$f'(-1) = 12 - 5 = 7$

$y - 4 = 7(x+1)$

$y = 7x + 11$

25. A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time t is the particle at rest?

(A) $t = 1$ only

(B) $t = 3$ only

(C) $t = \frac{7}{2}$ only

(D) $t = 3$ and $t = \frac{7}{2}$

(E) $t = 3$ and $t = 4$

$v(t) = x'(t) = 6t^2 - 42t + 72$

$0 = 6(t^2 - 7t + 12)$

$0 = 6(t-4)(t-3)$

$t = 4,\ t = 3$

26. What is the slope of the line tangent to the curve $3y^2 - 2x^2 = 6 - 2xy$ at the point $(3, 2)$?

(A) 0 $\boxed{\text{(B) } \frac{4}{9}}$ (C) $\frac{7}{9}$ (D) $\frac{6}{7}$ (E) $\frac{5}{3}$

$$6y\frac{dy}{dx} - 4x = -2y - 2x\frac{dy}{dx}$$

$$12\frac{dy}{dx} - 12 = -4 - 6\frac{dy}{dx}$$

$$18\frac{dy}{dx} = 8$$

$$\frac{dy}{dx} = \frac{8}{18} \quad \frac{4}{9}$$

27. Let f be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of $g'(2)$?

(A) $\frac{1}{13}$ $\boxed{\text{(B) } \frac{1}{4}}$ (C) $\frac{7}{4}$ (D) 4 (E) 13

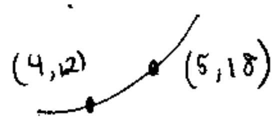
$g(x)$

$$x = y^3 + y$$

$$1 = 3y^2\frac{dy}{dx} + \frac{dy}{dx}$$

$$\frac{1}{3y^2+1} = \frac{dy}{dx}$$

$$\frac{1}{4} = \frac{dy}{dx}$$



28. Let g be a twice-differentiable function with $g'(x) > 0$ and $g''(x) > 0$ for all real numbers x, such that $g(4) = 12$ and $g(5) = 18$. Of the following, which is a possible value for $g(6)$?

(A) 15 (B) 18 (C) 21 (D) 24 (E) 27

END OF PART A OF SECTION I