Algebra I–Part 2
Unit 8: Quadratics

Time Frame: Approximately six weeks

Unit Description

This unit focuses on the understanding of how quadratic equations and graphs differ from linear equations and graphs, and how to determine the solutions to a quadratic equation by various methods.

Student Understandings

Students will understand how to determine the solutions to quadratic equations including how to solve quadratic equations by factoring, completing the square, and using the quadratic formula.

Guiding Questions

1. Can students solve quadratic equations by inspection (e.g., for $x^2 = 49$) taking square roots?
2. Can students use factoring in order to solve quadratic equations using the Zero-Product Property?
3. Can students relate factoring a polynomial to determining the zeros for the graph of a quadratic function?
4. Can students use completing the square to solve quadratic equations and develop the quadratic formula from completing the square of a general quadratic function?
5. Can students understand what it means for a solution to have imaginary solution and write these solution in $a \pm bi$ form with real numbers *a* and *b*?

Unit 8 Grade Level Expectations (GLEs) and Common Core State Standards (CCSS)

Grade Level Expectations	
GLE #	**GLE Text and Benchmarks**
Algebra	
15.	Translate among tabular, graphical, and algebraic representations of functions and real-life situations (A-3-H) (P-1-H) (P-2-H)
Data Analysis, Probability, and Discrete Math	
29.	Create a scatter plot from a set of data and determine if the relationship is linear or nonlinear (D-1-H) (D-6-H) (D-7-H)

Patterns, Relations, and Functions	
37.	Identify the domain and range of functions (P-1-H)

CCSS for Mathematical Content	
CCSS #	**CCSS Text**
Reasoning with Equations and Inequalities	
A-REI.4	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b*.

ELA CCSS	
CCSS #	**CCSS Text**
Reading Standards for Literacy in Science and Technical Subjects 6-12	
RST.9-10.3	Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.
Writing Standards for Literacy in History/Social Studies, Science and Technical Subjects 6-12	
WHST.9-10.10	Write routinely over extended time frames (time for reflection and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.

Sample Activities

Activity 1: Linear Graphs vs. Quadratic Graphs? (GLEs: 15, 29; CCSS: WHST.9-10.10)

Materials List: paper, pencil, graph paper, Linear Graphs vs. Quadratic Graphs BLM

In this activity, students will make a scatter plot for the two equations $y = x$ and $y = x^2$ and contrast/compare the two respective graphs. Make copies of the Linear Graphs vs. Quadratic Graphs BLM and have students work in groups of three to do the work. Students will also need graph paper and two different colored pens. Be sure to monitor student progress on the work and interject as needed if the class is in need of more direction. When students have completed the work, discuss the BLM completely. The goal for the activity is to ensure students understand how the graph of a quadratic relationship is nonlinear and forms a completely different shape (called a parabola), and how it is different than a linear relationship. Discuss some of the important points of the activity and talk about any new terminology that needs to be addressed (such as parabola and vertex).

At this point, utilize a modified form of *GISTing* (view literacy strategy descriptions) designed to help each student process content more effectively. *GISTing* is a way to focus student attention on key ideas, in this case having students to explain the difference between a linear graph and a quadratic graph. It requires the student to summarize (in an organized way) what has been learned in a few short, discrete sentences (or even a few words). In this case, have students write in their own words the difference between a linear graph and a quadratic graph using as many words as they like. For example, the student might write, *"Quadratic graphs are different than linear graphs in that they alternate from decreasing to increasing and have a highest or lowest point called a vertex."* Once students have written their statement, lead students to progressively refine their statement to use fewer and fewer words until the statement is summarized to its basic understanding. For example in this case, if the final "*GIST*" statement were limited to 10 words, the statement could be written as *"Quadratic is non-linear; isn't constantly increasing or decreasing, has vertex."* The goal is to have students understand in a very concise manner, how the two things (linear and quadratic graphs) are different and to fully develop their understanding through the use of this summarization strategy. After students have written their summary statements, pick up the students' *GISTs* and use them to see who fully has understood the lesson. Use this to guide your instruction on the rest of the activity.

Activity 2: Domain and Range for Quadratics (GLEs: 37)

Materials List: paper, pencil, graphing calculators, Domain and Range of Quadratics BLM

Students have learned previously how to identify domain and range for a function. In this activity, those concepts are re-visited. Do a quick review of what domain and range are and how to express the domain and range for a graph using interval notation. Make copies of the Domain and Range of Quadratics BLM and have students work in groups of three to do the worksheet. Students will also need graphing calculators. Monitor student work and provide assistance as needed as well as guide instruction. After students have worked on the problems in their groups, discuss the BLM as a class. Provide additional work on this skill as needed.

Note: Although the focus of the lesson is on determining the domain and range of quadratics, use this opportunity to discuss the vertex form for a quadratic equation (i.e., $y = a(x + h)^2 + k$) and the effects of a, h, and k on the fundamental graph of $y = x^2$.

2013-14
Activity 3: What number makes the equation true? (CCSS: A-REI.4)

Materials List: paper, pencil, a math textbook

Note: This activity addresses some new content based on CCSS and is to be taught in 2013-14. In this activity, students will be given simple quadratic equations to solve by "inspection" using mental math by looking at the equation using thinking (not manipulating the equation using algebraic methods). Write the equations provided below on the overhead/whiteboard and have students get into small groups to *brainstorm* (view literacy strategy descriptions) solutions to the equations provided. *Brainstorming* involves students working together to generate ideas. Students work in pairs or groups to freely exchange ideas in response to an open-ended question, statement, problem, or other prompt. Students try to generate as many ideas as possible, often building on a comment or idea from another participant. This supports creativity and leads to expanded possibilities. The process activates students' relevant prior knowledge, allows them to benefit from the knowledge and experience of others, and creates an anticipatory mental set for new learning. Once students have had the opportunity to *brainstorm* their ideas about the problems presented, have a class discussion on the ideas students came up with.

> Solve the following quadratic equations by inspection and be ready to discuss the answers you came up with as a class.
> Ex. 1: $x^2 = 49$
> Ex. 2: $x^2 = 100$
> Ex. 3: $x^2 = -25$
> Ex. 4: $2x^2 = 72$
> Ex. 5: $x^2 - 5 = 20$

Discuss the solutions to the problems. The teacher should act as a guide/facilitator to ultimately lead students to understand the main objective of this activity which is first to realize that in quadratic equations, typically there are two unique solutions (not just one as in a linear equation). It is also important that students realize that not all equations have a solution (as in Ex. 3 which has no real solution that works). Students should see that if they square any number, they will always get a positive answer, thus Ex. 3 does not have a real solution (imaginary solutions will be discussed later in another activity.) Provide additional examples of simple quadratic equations (equations which have perfect square numbers as solutions) and can be solved by "inspection" using a math textbook as a resource.

2013-14
Activity 4: Solving Simple Quadratic Equations by Square Roots (CCSS: A-REI.4)

Materials List: paper, pencil, a math textbook

Note: This activity addresses some new content based on CCSS and is to be taught in 2013-14. In the previous activity, students were given simple quadratic equations to solve using inspection. Answers to problems involved whole number answers and deal with perfect squares which made them easier to solve through mental math. In this activity, teach students how to solve simple quadratic equations by taking the square roots of both sides of the equation in cases when the quadratic equation is of the form $ax^2 + c = 0$ (when there is no middle "x" term). An example of the type of problems students need to be able to solve is shown at right. Do several examples of this type and teach students how to simplify radicals as the problems are done. Provide additional work on this skill using the math textbook as a resource.

> Solve: $3x^2 - 4 = 32$
>
> $$\begin{array}{rl} 3x^2 - 4 &= 32 \\ \underline{\quad + 4} & \underline{+4} \\ \dfrac{3x^2}{3} &= \dfrac{36}{3} \\ x^2 &= 12 \\ x &= \pm\sqrt{12} \\ x &= \pm 2\sqrt{3} \end{array}$$

2013-14
Activity 5: The Zero Product Property (CCSS: A-REI.4)

Materials List: paper, pencil, a math textbook

Note: This activity addresses some new content based on CCSS and is to be taught in 2013-14. In this activity, the goal is to introduce the zero product property which will be utilized when students are taught to use factoring to solve quadratic equations in the next activity. Start the activity by putting the following number sentence on the board and use it to talk about the solution to such a number sentence: $\triangle$ x 3 = 0. Students should easily recognize that the only number that could make the number sentence true if the $\triangle$ were equal to zero is "0."

Next, put this number sentence on the board: $\triangle$ x $\square$ = 0. Present the problem to students and have them attempt the problem individually first, then have students get in pairs to share their findings utilizing the *discussion* (view literacy strategy descriptions) strategy known as Think-Pair-Square-Share. Have each student pair up with another student to share their thoughts on the solution. Next, have pairs of students share with other pairs, forming small groups of four. Be sure to have students fully defend their answer and explain how they arrived at the answer they came up with. Once this process has taken place, gather oral responses to the solution for a full class *discussion* of the problem/solution. The goal of *discussion* is to provide a deeper processing of content and rehearsal of newly learned content. Students should realize that in this case, either the triangle or the rectangle must be zero in order for the product to equal zero. In fact, they could both be zero (unless there was a restriction on their being unique numbers). Explain that the number sentence shown displays what is referred to as the "zero product property" which simply states that if the product of two quantities is zero, then one or both of the quantities must be zero.

Finally, provide students with the following problem: $(x + 1)(x - 4) = 0$ and have students come up with the solution. Again, utilize Think-Pair-Square-Share in the *discussion* of the answer. Students should realize that this is essentially the same situation as the number sentence which had the triangle and rectangle. In this case, the two quantities $(x + 1)$ and $(x - 4)$ have a product equal to zero, thus one or both are equal to zero. Thus setting each quantity to zero, the solutions are $x = -1$ or $x = 4$. Make sure students understand how this method will be utilized to solve quadratic equations in future activities. Provide additional work as needed using a math textbook as a resource.

2013-14
Activity 6: Solving Quadratic Equations by Factoring I (CCSS: A-REI.4)

Materials List: paper, pencil, a math textbook

Note: This activity addresses some new content based on CCSS and is to be taught in 2013-14. In this activity, the goal is to utilize what was learned in the previous activity about the zero product property which will be utilized to solve quadratic equations using factoring. Thus far, only simple quadratics of the form $ax^2 + c = 0$ (when there is no middle "x" term) have been solved.

Now that the zero product property has been discussed, and since factoring was fully discussed in Unit 7, the two concepts converge into solving quadratics using factoring and the zero product property.

Begin the activity by explaining that in general, all quadratics are of the following form: $ax^2 + bx + c = 0$. Thus far, only those equations where b = 0 have been solved utilizing either inspection or by taking the square root to solve for *x*. Those methods fail when *a*, *b*, and *c* are real numbers not equal to 0. Explain that over the course of the rest of this unit, various methods will be taught and utilized to solve these types of quadratic equations.

Write on the board/overhead the following problem and have students think about a way they could solve it using any mathematics they may have learned about in the course thus far.

Ex 1: Solve $x^2 + 4x + 3 = 0$

Allow students the opportunity to solve/discuss the problem alone and in their groups and monitor progress to guide instruction. Hopefully, someone has come up with the solution using factoring and the zero product property. If not, then guide students to the solution through hints/guiding questions. Fully discuss how to solve problems of this type and provide additional examples. Limit the problems to those in which a = 1 (the coefficient of the x^2 term is 1). Once guided practice is complete, provide additional work on this concept using a math textbook as a resource.

2013-14
Activity 7: Solving Quadratic Equations by Factoring II (CCSS: A-REI.4)

Materials List: paper, pencil, a math textbook

Note: This activity addresses some new content based on CCSS and is to be taught in 2013-14. In this activity, the work with solving quadratic equations of the form $ax^2 + bx + c = 0$ is expanded to include equations where a > 1 and can be solved using factoring. Using a math textbook as a resource, have students solve quadratic equations of this type.

2013-14
Activity 8: Relating the Solutions to a Quadratic Equation Graphically (CCSS: A-REI.4)

Materials List: paper, pencil, graphing calculators

Note: This activity addresses some new content based on CCSS and is to be taught in 2013-14. In this activity, the goal is to help students connect finding the solutions to a quadratic equation to finding the zeros of a quadratic graph. Utilize graphing calculators to do the work in this activity. Lead students to understand that the solutions to a quadratic equation actually relate to the "*x*-intercepts" or the "zeros" of the graph of the parabola.

Begin by having students graph the function y = $x^2 + 4x + 3$ using graphing calculators. Lead students through the process of determining the x-intercepts for the graph and explain that these are sometimes called the "zeros" of the graph. Students should have found x-intercepts with linear graphs. Ask students if they remember how to find the x-intercepts of an equation, such as $3x + 4y = 12$. Hopefully, students should remember that to find that an x-intercept, they simply let $y = 0$ and solve for x. Explain that this process can be used to find any x-intercept. Therefore, relate finding the x-intercepts for the equation, $y = x^2 + 4x + 3$ by simply letting y = 0, resulting in: $0 = x^2 + 4x + 3$. Students have solved equations such as this by using factoring, so have students solve the equation. They should see that the solutions $x = -1$ and $x = -3$ which are the same as the zeros of the graph using the graphing calculator (i.e., (-1, 0) and (-3,0) as the coordinates for the zeros of the graph).

Provide additional examples of this type keeping in mind the goal is to connect what is happening graphically with the paper/pencil methods that students have used thus far in solving quadratic equations. The big idea that students need to get is that "zeros" of a function are the same as "x-intercepts" of that function which are the same as the "solutions" to the equation (when $y = 0$).

2013-14
Activity 9: Solving Quadratic Equations by Completing the Square (CCSS: A-REI.4; RST.9-10.3)

Materials List: paper, pencil, graphing calculators, a math textbook

Note: This activity addresses some new content based on CCSS and is to be taught in 2013-14. In this activity, students are presented with quadratic equations that cannot be solved by inspection or by simple factoring techniques. A new way to solve quadratic equations is introduced, the process of Completing the Square.

Begin first by asking students to try to solve the equation $x^2 + 6x + 2 = 0$. Give students the opportunity to work in small groups in trying to find the solution using methods that they have learned thus far (i.e., inspection and using factoring). After students have had the opportunity to work on the problem, talk about it as a class. Students should have realized that none of the methods learned thus far will produce a solution.

Next, relate graphically what is going on with this problem by having students use graphing calculators to find the zeros for the function $y = x^2 + 6x + 2$. Lead students to see that there are actually two zeros for this function and that those values are the solutions to the equation in question, the approximate values being x = -.35 and x = -5.64 if they are using the graphing calculator function. The question becomes, what are the exact solutions and how can this be done using paper/pencil methods?

Explain that equations of this type can be solved by a technique called "Completing the Square," and the goal is to take the equation and transform it into a quadratic that is a perfect square trinomial, i.e., the form $a^2 + 2ab + b^2 = (a + b)^2$. To do so, the coefficient of the x^2 term must be

1 (which it is in this case). In this particular case, if students look at $x^2 + 6x$ they can create a perfect square trinomial by taking ½ of the coefficient of the x term, and squaring it, thus getting $x^2 + 6x + 9$ which re-written becomes $(x + 3)^2$. This is at the heart of the "Completing the Square" process. Talk about the method of completing the square, following the steps below:

Completing the Square:

1. If the leading term is not 1, you should divide each side of the equation by this coefficient before completing the square.
   $x^2 + 6x + 2 = 0$
2. Transpose the constant term to one side of the equation.
   $x^2 + 6x \quad = -2$
3. Add a square number to both sides (the square of half the coefficient of the x term).
   $x^2 + 6x + 9 = -2 + 9$
4. The resulting polynomial side should now be a perfect square trinomial and should be rewritten in $(a + b)^2$ form.
   $(x + 3)^2 = 7$
5. To find final solution, solve for x by taking square root of both sides of resulting equation.
   $x + 3 = \pm\sqrt{7}$
   $x = -3 \pm\sqrt{7}$
6. The solutions that result should be simplified if possible (i.e. simplifying radicals).

Once students have done the process with guidance, talk about the resulting solution of the approximations found using the graphing calculator and the exact solutions of the pair of solutions $x = -3 \pm\sqrt{7}$ (which correspond to the approximations found using the calculator).

Provide additional examples for students to solve with guidance from the teacher (including equations that do not have a leading coefficient of 1), then use a math textbook as a resource for additional work of this type.

2013-14

Activity 10: The Quadratic Formula (CCSS: A-REI.4; RST.9-10.3)

Materials List: paper, pencil, a math textbook

Note: This activity addresses some new content based on CCSS and is to be taught in 2013-14. In this activity, the teacher will show students what the Quadratic Formula is and how it can be derived by using completing the square on the general form of a quadratic equation. Students will also understand how this formula can be used to solve quadratic equations.

Begin the activity by guiding students through the derivation of the quadratic formula through the completing the square process as shown in the box below.

Using Completing the Square to Derive the Quadratic Formula:

Consider a general quadratic equation: $ax^2 + bx + c = 0$ (where $a \neq 0$):

Following the completion of the square process students get the following:

1. Subtract c from both sides of the equation.
   $ax^2 + bx = -c$
2. Divide each side by a to get the leading coefficient to be 1.
   $$x^2 + \frac{b}{a}x = -\frac{c}{a}$$
3. Add the square of half the coefficient of the x term to each side.
   $$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$$
4. Write the left side as a perfect square.
   $$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$$
5. Use a common denominator to express the right side as a single fraction.
   $$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$$
6. Find the square root of each side. Include ± on the right side.
   $$\left(x + \frac{b}{2a}\right) = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$
7. Solve for x by subtracting the same term from each side.
   $$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$
8. Use a common denominator to express the right side as a single fraction.
   $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Once the quadratic formula is derived, explain the formula can be used on ANY quadratic equation to solve it. Verify this by using the example: $0 = x^2 + 4x + 3$. Have students solve the equation first by factoring and then by completing the square and finally by using the quadratic formula. Students should see that all three methods arrive at the same solution.

Provide additional examples for students to try in order to get used to using the formula. Explain that the formula is especially useful when trying to solve quadratics that cannot be factored. When students are ready, assign work on using the quadratic formula using a math textbook as a resource for problems. Limit the problems to only real solutions for now. Imaginary solutions will be dealt with in another activity.

2013-14
Activity 11: Solving Quadratics Using Various Methods (CCSS: A-REI.4)

Materials List: paper, pencil, a math textbook

Note: This activity addresses some new content based on CCSS and is to be taught in 2013-14. In this activity, have students get in groups of 3 and create quadratic equation problems for their fellow classmates, as well as the solutions, using any of the methods discussed (i.e., inspection, taking square roots, factoring, completing the square, or quadratic formula). Afterwards, have students participate in a version of *professor know-it-all* (view literacy strategy descriptions). In this particular use of the strategy, have each group of 3 students come up to the board and act as the professor. Call on groups randomly. Students should quiz the professors on how to determine the solutions to quadratic equations (based upon problems created by the questioners). Students ask the questions, the professors answer the questions and aren't allowed to sit until the class feels its question has been answered satisfactorily. Each "professor" in the group should be required to answer some part of the question. *Professor know-it-all* is a fun way to review a concept and also to see if the students really grasp the material being covered. The teacher should act as the facilitator during this time. One thing to keep in mind is to ask students why they chose a particular method and whether the method chosen is the most efficient for the particular problem being discussed.

2013-14
Activity 12: Imaginary Roots (CCSS: A-REI.4)

Materials List: paper, pencil, graphing calculators

Note: This activity addresses some new content based on CCSS and is to be taught in 2013-14. In this activity, talk about the fact that some quadratic equations have no real solutions and connect this graphically with a quadratic that would have no x-intercepts, then tie this in with the discriminant and what it tells us about the solutions for a quadratic equation.

Begin the discussion by having students graph the function, $y = x^2 + 6x + 12$, using graphing calculators. Ask students to find the x-intercepts or "roots" for the graph. Students should see that this graph has no x-intercepts. Next, have students try to use the quadratic formula to determine what would happen if they attempted to use paper/pencil methods to find the roots for the equation. Students should see that in this problem, the number which results in the formula, the part under the square root (the discriminant) actually results in trying to take the square root of a negative number ($\sqrt{-12}$). Talk about this problem with students and how it is impossible to have a number multiply by itself to get a negative answer (which essentially is what taking a square root is). Use this opportunity to discuss the difference between real number solutions and imaginary solutions. Explain that in math, the answer can be expressed by using the letter i which represents $\sqrt{-1}$ and how it can be written $\sqrt{-12}$ as $i\sqrt{12}$ which becomes $2i\sqrt{3}$ when simplified.

Ultimately, students need to be able to correctly write the solution in $a \pm bi$ form and connect this imaginary solution with its graphical counterpart, meaning no x-intercepts for the graph of the function. Essentially, students need to understand that anytime the discriminant is negative, imaginary solutions are a result.

Provide additional examples of this type as guided practice, then use a math textbook to provide additional practice in solving quadratic equations which have no real solution and writing the answer in $a \pm bi$ form.

Sample Assessments

General Guidelines

Performance assessments can be used to ascertain student achievement. Following are some examples:

General Assessments

- The student will explain what the quadratic formula is used for and where it comes from.
- The student will create portfolios containing samples of his/her activities.

Activity-Specific Assessments

- Activity 2: Have students identify the domain and range of the graph of a quadratic function.

- Activity4: Have students find the solution to the equation: $2x^2 - 4 = 32$. Students must show and explain each step in the process.

- Activity 9: Have students use completing the square to find the solution to the problem $x^2 + 4x + 2 = 0$.

- Activity 11: Have students use any method taught in this unit (i.e., inspection, taking square roots, factoring, completing the square, quadratic formula) to determine solutions to quadratic equations with real number solutions.