

Algebra I–Part 2
Unit 7: Polynomials and Factoring

Time Frame: Approximately four weeks

Unit Description

This unit focuses on the arithmetic operations on polynomial expressions as well as on basic factoring techniques in preparation for solving quadratic equations (in Unit 8).

Student Understandings

Students will understand how to manipulate polynomial expressions by combining like terms when adding and subtracting polynomials and will learn to multiply polynomials. Included in this unit is also a study on factoring polynomial expressions.

Guiding Questions

1. Can students add and subtract polynomials?
2. Can students use the rules of exponents to multiply monomials?
3. Can students multiply polynomials and identify special products?
4. Can students factor expressions using the greatest common factor, and can they factor binomials containing the difference in two perfect squares?
5. Can students factor general trinomials using various factoring techniques?

Unit 7 Common Core Standards and Common Core State Standards (CCSS)

CCSS for Mathematical Content	
CCSS #	CCSS Text
Arithmetic with Polynomials & Rational Expressions	
A-APR.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
Seeing Structure in Expressions	
A-SSE.2 2013-14	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

ELA CCSS	
CCSS #	CCSS Text
Reading Standards for Literacy in Science and Technical Subjects 6-12	
RST.9-10.4	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.

Sample Activities

Activity 1: What is a Polynomial? (CCSS: **A-APR.1**)

Materials List: paper, pencil, a math textbook, Polynomials BLM

In this activity, students should develop an understanding of polynomials. In Algebra I the focus will be on linear and quadratic polynomial expressions. This will be extended beyond quadratic polynomial expressions when students get to Algebra II.

Begin this activity by writing on the board or overhead the following expression:

$5x^2 - 4x + 7$. Thus far, students have dealt with the use of variables and exponents in various applications and have evaluated algebraic expressions. Explain that this is an example of a *polynomial* expression. Polynomials are simply sums or differences of these “variables and exponents.” Explain that in this particular example, the polynomial has three terms: $5x^2$, $4x$, and 7. Polynomial terms have variables which are raised to whole-number exponents, or just plain numbers, such as the “7” in this expression (called constants). Explain that to be a polynomial, the exponents must be positive whole numbers, and no variables in the denominators of any fractions; otherwise, it is not considered a polynomial. Discuss all of these ideas with students. Utilize a math textbook for examples of polynomial expressions, identifying the “coefficient” for a term as well as identifying the “degree of the polynomial” (i.e., constant, linear, quadratic, etc.). Also discuss classifying a polynomial by the number of terms it has, such as monomial (1 term), binomial (2 terms), or trinomial (3 terms). In this particular example, explain that the polynomial has three terms, so it would be considered a trinomial, while the degree of the polynomial is considered to be a 2nd degree expression or a quadratic. Also, discuss the fact that when writing polynomials, the terms of polynomials are typically written in ascending or descending order.

Once a thorough explanation of the properties of polynomials has taken place, provide students copies of the Polynomials BLM. The BLM utilizes a modified form of a *word grid* ([view literacy strategy descriptions](#)). A *word grid* is a learning strategy that allows the user to relate characteristic features among terms or items and helps the learner to compare and contrast similarities and differences between the items in a list. Let students get in groups of three to complete this activity, and then discuss the results as a class.

Based on the student responses to the questions presented on the BLM, clear up any misconceptions students may be having. As a follow-up to the activity, have students create a polynomial that may have certain features. For example, have students come up with an example of a polynomial expression that would be a trinomial but not be linear or quadratic. Use the math textbook as a resource for additional problems if students are still having difficulty with these topics. Once students have corrected their *word grids*, encourage them to use them as a study aid to reinforce their understanding of polynomial properties.

Activity 2: What is Alike? (CCSS: A-APR.1)

Materials List: paper, pencil, a math textbook

Now that students have learned about what a polynomial is and some of the vocabulary associated with polynomials (i.e., terms, degree of polynomial, coefficients, etc.), students need to further understand the concepts of combining “like” terms and the closure property associated with operations with polynomials. Begin by first talking with students about what is meant by “like terms.” Explain that like terms are terms that have the same variable and exponent associated with the variable. For example, in the expression $3x^2 + 2x - 3 + 4x^2$, the terms $3x^2$ and $4x^2$ are considered to be like terms. Point out that like terms can be combined. Relate this concept to combining things that have like names such as cats or dogs, or combining fractions which have like denominators. Also, point out the fact that the $2x$ term is not “like” the other x terms (because the degree of the term is different) and thus cannot be combined with anything; this is true of the constant term as well. Talk about simplifying the expression to $7x^2 + 2x - 3$ by combining like terms. Provide additional examples to ensure students understand. Use the math textbook as a resource as necessary.

Next, relate how when they add, subtract, or multiply integers, they always get an integer (provide several examples to show students this is the case). Explain that when they do the same operations with polynomials, they also get a polynomial. Have students keep this in mind over the rest of this unit, and revisit this idea as the students are taught to add, subtract, and multiply polynomials in the upcoming activities.

Activity 3: Adding and Subtracting Polynomials (CCSS: A-APR.1)

Materials List: paper, pencil, math textbook

Now that students understand what constitutes like terms when dealing with polynomial expressions, expand the discussion to include adding and subtracting polynomials, and simplifying expressions by combining like terms. Demonstrate to students how to do this horizontally, as well as vertically. An example is provided below.

Ex: Find the sum of $(4x^2 - 2x + 3) + (3x^2 + 5x - 9)$.

Vertical Format:

$$\begin{array}{r} 4x^2 - 2x + 3 \\ 3x^2 + 5x - 9 \\ \hline 7x^2 + 3x - 6 \end{array}$$

Horizontal Format:

$$\begin{array}{l} (4x^2 - 2x + 3) + (3x^2 + 5x - 9) \\ 4x^2 - 2x + 3 + 3x^2 + 5x - 9 \\ 7x^2 + 3x - 6 \end{array}$$

Help students to understand how they when we put the problem into a vertical format, they align the terms that are alike. This makes it easier to combine them. In the horizontal format, they get rid of any parentheses first, then combine any like terms. Point out that they added two polynomials, and ended up with a polynomial in their final answer. Also, point out that in this particular problem, no sign changes were needed in the second part of the expression because the two polynomials were being added. However in the next example, this is not the case. Show students the problem below, then discuss the results as a class.

Ex: Find the difference: $(x^2 - 8) - (7x - 4x^2)$

Horizontal Format:

$$\begin{array}{l} (x^2 - 8) - (7x - 4x^2) \\ x^2 - 8 - 7x + 4x^2 \\ 5x^2 - 7x - 8 \end{array}$$

Provide additional examples as needed with increasing complexity. Two examples are provided below:

$$\text{Ex: } 3[2(a + b) + 4a]$$

$$\text{Ex: } 2(4x^2 - 5) - 3(2x^2 - 4)$$

Have students do similar problems as guided practice and homework. Use the math textbook as a resource for additional problems on this skill.

Activity 4: Multiplying Monomials (CCSS: A-APR.1)

Materials List: paper, pencil, math textbook

In previous units, students should have already learned about multiplying integers and using the laws of exponents. In this activity, begin by revisiting the laws of exponents prior to talking about multiplying monomials. Use the discussion about laws of exponents

to introduce its use in multiplying monomials. For example, talk with students about how to multiply $(-7x^5)(5x^3)$. Present the problem to students and have them attempt the problem individually first, then have students get in pairs to share their findings utilizing the *discussion* ([view literacy strategy descriptions](#)) process known as Think Pair Square Share. After each pair of students has shared their thoughts on the solution, have pairs of students share with other pairs, forming small groups of four. Be sure to have students fully defend their answer and explain how they arrived at their answer. Once this process has taken place, gather oral responses to the solution for a full class *discussion* of the problem/solution to take place. This whole class *discussion* will provide the opportunity to clarify any misconceptions. The goal of this form of *discussion* is to provide a deeper processing of content and rehearsal of newly learned content.

Once students have had the opportunity to all talk about their answers and a consensus is reached on the answer $(-35x^8)$, present more examples as both guided practice and then eventually as homework practice. Use the math textbook as a resource for additional problems on this skill. Limit the problems to multiplying monomials at this time. Re-visit the concept that when they multiply polynomials they always get a polynomial as an answer (i.e., the system of operations is closed when they add, subtract, or multiply polynomials).

Activity 5: Multiplying Monomials and Polynomials (CCSS: A-APR.1)

Materials List: paper, pencil, math textbook

Now that the groundwork with multiplying monomials has been set in place, extend this concept to include multiplying monomials with any polynomial. Connect this with what was done in previous grades with the distributive property. In earlier grades, students should have been taught to use the distributive property with numbers such as multiplying 7×535 . The idea of writing the number 535 in expanded form and multiplying each part that is expanded by the single digit factor (7 in this case) should have been the first use of “distribution” by students, as is shown below:

$$\begin{array}{r} 7(500 + 30 + 5) \\ (7 \times 500) + (7 \times 30) + (7 \times 5) \\ 3500 + 210 + 35 \\ 3745 \end{array}$$

After discussing the distributive property in a numerical context, connect this idea with using distribution when multiplying a monomial by a polynomial. An example is provided below:

$$\begin{array}{r} 3x^2(4x^2 - 2x + 1) \\ 12x^4 - 6x^3 + 3x^2 \end{array}$$

Relate how problems of these types work, and provide additional example, guided practice, and homework on these types of problems. Use a math textbook as a resource for additional problems of this type. Limit the work to multiplying polynomials by monomials for now.

Activity 6: Multiplying Binomials by Binomials (CCSS: A-APR.1)

Materials List: paper, pencil, a math textbook

In this activity, students continue to learn how to multiply polynomials to include multiplying binomials by binomials. Just as was done in the previous activity, students should connect what was learned in earlier grades with this new content. Students should have seen/used the distributive property to multiply numbers such as 84×59 by writing both numbers in expanded form. After expanding each number, the distributive property is once again used to multiply the two results. This is shown in the example below:

$$\begin{array}{r} 84 \times 59 \\ (80 + 4) \times (50 + 9) \\ 80(50 + 9) + 4(50 + 9) \\ (80 \times 50) + (80 \times 9) + (4 \times 50) + (4 \times 9) \\ 4000 + 720 + 200 + 36 \\ 4956 \end{array}$$

Emphasize the fact that in this example, both parts of the first expansion ($80 + 4$) each gets multiplied by each element in the second expansion ($50 + 9$). Tie this in with how they multiply a binomial by another binomial, using the example shown below. Present the problem first to students to try on their own, and then fully discuss as a class.

$$\begin{array}{r} (3x + 1)(4x - 2) \\ 3x(4x - 2) + 1(4x - 2) \\ 12x^2 - 6x + 4x - 2 \\ 12x^2 - 2x - 2 \end{array}$$

When discussing the solution with students, point out that at the end, they should always look to see if there were any like terms that can be combined. In this particular case, the two terms $-6x$ and $4x$ can be combined to get $-2x$. It is also important at this point to emphasize the fact that the first two terms resulted in giving the first term in the product ($12x^2$), and the last two terms resulted in the last term of the product (-2). This is important later when they start the factoring process. If these things are emphasized now, the factoring process will make sense later.

Another thing to discuss is the fact that many textbooks refer to the FOIL method for multiplying a binomial by a binomial. This is simply a mnemonic device to remind students to multiply all terms in each binomial. It is much more important for students to understand how to multiply ANY polynomial as the FOIL “method” or “mnemonic” will

only work with multiplying a binomial by a binomial. When students multiply by anything else, the device falls short, so there should instead be a full understanding by students on why and how to multiply polynomials.

Provide additional examples on this topic for students to work, including guided practice and homework. Utilize the math textbook as a resource for additional work. Once the students become proficient at the skill, extend the multiplication process to include a binomial by a trinomial, depending on the level of the students. Students need to be proficient at multiplying binomials to be able to do the upcoming work which deals with factoring.

Activity 7: Application Problems (CCSS: A-APR.1)

Materials List: paper, pencil, a math textbook

To expand upon what students have learned about polynomials, students will write polynomial expressions and simplify them in relation to application problems. Include application problems of varying types such as those shown below:

Example 1: A rectangle has a length that is $(3x + 1)$ yards long and a width that is $(4x - 2)$ yards long. Write expressions that would represent the perimeter and area of the rectangle.

Answer:

$$\text{Perimeter: } 2(3x + 1) + 2(4x - 2) = 14x - 2 \text{ yards}$$

$$\text{Area} = (3x + 1)(4x - 2) = 12x^2 - 2x - 2 \text{ square yards}$$

Example 2: The revenue for a company each month can be expressed by the function $R(x) = 3500x$, where x represents the number of units sold each month. The costs associated with operating the company each month can be expressed by the function $C(x) = 450 + 310x$, where x is the number of units sold during the month. If the profit is the revenue, $R(x)$, minus the cost, $C(x)$, write a profit function, $P(x)$, which would express the profit for each month.

Answer:

$$P(x) = R(x) - C(x) = 3500x - (450 + 310x) = 3190x - 450$$

Utilize the math textbook as a resource for additional work of this type.

2013-14

Activity 8: What is Factoring? (CCSS: A-APR.1, A-SSE.2; RST.9-10.4)

Materials List: paper, pencil, What is Factoring? BLM

Note: This activity addresses some new content based on CCSS and is to be taught in 2013-14. In this activity, students are introduced to the term “factoring” in the context of factoring trinomials. Hand out copies of the What is Factoring? BLM. Have students work in pairs or groups of three on this activity. In the BLM, the term “factoring” is talked about, related to its use in earlier grades, then extended to include its application to trinomials and its relationship to the “undoing” of multiplying binomials (it’s the inverse process). At the end of the reading of the BLM, a trinomial expression is presented to students to try to “factor” and allow students the opportunity to try to figure out what the correct answer is, using the hints provided on the BLM. The teacher should monitor the students’ work and progression over this activity, and use the findings to guide instruction. For now, students should come up with a true understanding of what this process of “factoring” entails and what exactly it is. In the future activities, more sequential processes will be developed.

After students have had the opportunity to discuss in their groups and have come up with a solution for the problem presented, discuss the results as a class. Have students justify their answers and critique the work of others. Once a thorough discussion has taken place for the problem presented, provide additional examples of trinomials (where the x^2 term coefficient is 1) for students to factor. Include in the discussion the fact that some trinomials are not factorable, i.e., they are prime (just as numbers can be prime numbers so can trinomial expressions). One example of a trinomial that is prime would be $x^2 - 2x + 3$. Have students try the problem first, and then discuss it and why it cannot be factored.

2013-14

Activity 9: Special Products of Polynomials (CCSS: A-APR.1, A-SSE.2)

Materials List: paper, pencil, a math textbook

Note: This activity addresses some new content based on CCSS and is to be taught in 2013-14. Now that the concept of factoring trinomials is in place, expand the concept to include those special products of polynomials which show up when factoring trinomials. Discuss the following special product patterns with students and point out the relationship to the products and the factors.

Special Product Patterns:

Sum and Difference Pattern:

$$(a + b)(a - b) = a^2 - b^2$$

$$\text{Example: } (2x - 3)(2x - 3) = 4x^2 - 9$$

Square of a Binomial Pattern:

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$\text{Example: } (2x + 3)^2 = 4x^2 + 12x + 9$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$\text{Example: } (2x - 3)^2 = 4x^2 - 12x + 9$$

Students need to understand that when they see the difference of squares, for example, it can easily be put in factored form using the pattern described. In addition to just learning the pattern, relate this to what students should have learned in the previous activity.

Students need to continue to use these keys when factoring:

- The first two terms in the factors multiply to give the first term in the product
- The last two terms in the factors multiply to give the last term in the product
- The middle term in the product is the result of finding the sum to the multiplication of the two outer and two inner terms of the two factors

All of this information should have been related in the previous activity and should continue to be emphasized when teaching students how to correctly factor trinomials. After fully discussing the special patterns and providing guided practice problems in utilizing these special products to factor, provide additional examples for students using a math textbook as a resource to practice this skill.

Next, have students use and extend what has been learned thus far to teach them how to factor any trinomial of the form $ax^2 + bx + c$ where a , b , and c are integers (with a being a whole number). Build on what has been taught to factor trinomials and apply the concepts to this skill. Some examples are provided below:

Examples of Trinomials to Factor:

$$2x^2 + 11x + 5$$

$$3x^2 - 4x - 7$$

$$6x^2 - 19x + 15$$

Provide additional guided practice and homework problems of this type using a math textbook as a resource. Also teach students how to factor out the greatest common factor (GCF) in those problems where a common factor could be factored out first. For example, to factor $6x^2 - 2x - 8$ since each term in the product is divisible by the factor 2, simply factor out the 2 from each term first, thus giving $2(3x^2 - x - 4)$ and then to completely factor the expression $(3x^2 - x - 4)$ to get the final answer of $2(x + 1)(x - 4)$.

Once students have been fully taught how to factor any quadratic trinomial, have students participate in a version of *professor know-it-all* ([view literacy strategy descriptions](#)). In this particular use of the strategy, form groups of 3 students to come up to the board and act as the professors. Call on groups randomly. Students should quiz the professors on how to correctly factor a trinomial (based upon problems created by the questioners). Students ask the questions, the professors answer the questions and aren't allowed to sit until the class feels its question has been answered satisfactorily. Each "professor" in the group should be required to answer some part of the question. *Professor know-it-all* is a fun way to review a concept and also to see if the students really grasp the material being covered.

Sample Assessments

General Guidelines

Performance assessments can be used to ascertain student achievement. Following are some examples:

General Assessments

- The student will explain the difference between an expression which is a polynomial and one which is not a polynomial.
- The student will create portfolios containing samples of his/her activities.

Activity-Specific Assessments

- Activity 2: Provide the student with the expression $3x^2 - 4x + 5 - 2x^2 + 2x - 3$, then have students simplify the expression by combining like terms. Have students explain in words why the terms they combined are like terms as a formative assessment.
- Activity 3: Have students add and subtract polynomials and simplify them by combining like terms.
- Activity 6: Have students multiply the following two binomials and show their work as a formative assessment.
- Activity 9: Have students factor the trinomial $21n^2 + 8n - 4$