Algebra I–Part 2
Unit 2: Linear Equations and Graphs

Time Frame: 4 weeks

Unit Description

In this unit, work with linear equations is reviewed with a focus on graphing and interpreting the graphs of such equations from the standpoint of their slopes and intercepts.

Student Understandings

Students develop the concepts of slope, intercepts, graphing techniques, as well relating tabular, algebraic, and graphical models for linear relationships.

Guiding Questions

1. Can students find the slope of a line as well as interpret its meaning in real-life situations and application problems?
2. Can students use the basic methods for writing the equation of a line (using two points, slope-intercept, point-slope, and standard form)?
3. Can students determine if a table of values represents a linear set of data?
4. Can students determine a line of best fit for a set of data?
5. Can students relate a real-life situation and create an algebraic model along with a table and graph to represent a linear situation?

Unit 2 Grade-Level Expectations (GLEs) and Common Core State Standards (CCSS)

Grade-Level Expectations	
GLE #	**GLE Text and Benchmarks**
Grade 9	
Algebra	
8.	Use order of operations to simplify or rewrite variable expressions (A-1-H) (A-2-H)
9.	Model real-life situations using linear expressions, equations, and inequalities (A-1-H) (D-2-H) (P-5-H)
11.	Use equivalent forms of equations and inequalities to solve real-life problems (A-1-H)

13.	Translate between the characteristics defining a line (i.e., slope, intercepts, points) and both its equation and graph (A-2-H) (G-3-H)
15.	Translate among tabular, graphical, and algebraic representations of functions and real-life situations (A-3-H) (P-1-H) (P-2-H)
Geometry	
24.	Graph a line when the slope and a point or when two points are known (G-3-H)
Data Analysis, Probability, and Discrete Math	
29.	Create a scatter plot from a set of data and determine if the relationship is linear or non-linear (D-1-H) (D-5-H) (D-7-H)
Patterns, Relations, and Functions	
38.	Identify and describe the characteristics of families of linear functions, with and without technology (P-3-H)
39.	Compare and contrast linear functions algebraically in terms of their rates of change and intercepts (P-4-H)
CCSS for Mathematical Content	
CCSS #	**CCSS Text**
Reasoning with Equations & Inequalities	
A-REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
Interpreting Categorical & Quantitative Data	
S-ID.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
ELA CCSS	
CCSS #	**CCSS Text**
Writing Standards for Literacy in History/Social Studies, Science and Technical Subjects 6-12	
WHST.9-10.2d	Write informative/explanatory texts, including the narration of historical events, scientific procedures/experiments, or technical processes. Use precise language and domain-specific vocabulary to manage the complexity of the topic and convey a style appropriate to the discipline and context as well as to the expertise of likely readers.

Sample Activities

Activity 1: Graphing Lines from Points (GLEs: 13, 15, 24; CCSS: A-REI.10)

Materials List: paper, pencil, graph paper, math textbook

As a teacher-led activity, write the following equation: $y = 2x - 3$ on the board or overhead. Review how to construct a two-column t-chart by making a list of points which satisfies the given equation. For example, list the following *x* values as shown below:

x	*y*
-2	
-1	
0	
1	
2	

Have students determine the corresponding *y*-values (–7, –5, –3, –1, 1) for each of the *x*-values shown in the chart. On the overhead, and with students following along individually at their desks, plot the points $(-2,-7)$ and $(2,1)$ on a coordinate grid. Connect these two points with a straight line. Have the students identify the locations of the other points in the table on the graph. They should see that the other points are located along the line that was just drawn. Furthermore, students need to see that once any two points for a line are known, the corresponding line represents ALL possible solutions for the equation. This is what a graph is—it is a representation of ALL SOLUTIONS to the equation. Be sure to emphasize this to students.

Use this activity also to remind students how to determine the *x*- and *y*-intercepts for any linear equation or graph (i.e., to determine the *x*-intercept, let $y = 0$ and solve for *x*; to determine the *y*-intercept, let $x = 0$ and solve for *y)*. Students should also see that the *y*- intercept can be thought of as the "initial value," since it is the value of *y* when *x* is zero. Provide students with additional practice in drawing graphs of linear equations by finding ordered pairs that satisfy the equations and by finding *x*- and *y*-intercepts. Use the math textbook as a resource for this work.

Activity 2: Slope of a Line (GLEs: 13, 24)

Materials List: paper, pencil, math textbook, Slope Anticipation Guide BLM, Using Slope BLM

This activity is designed to provide a review on the concept and use of slope in linear equations and graphs. Begin the lesson by making copies of the Slope Anticipation Guide BLM and provide each student with a copy. *Anticipation guides (*view literacy strategy descriptions) are a way to activate prior knowledge of content and help give students a purpose for what they are about to learn by having them respond to statements before and after the lesson being presented.

For these reasons, anticipation guides are especially helpful to struggling and reluctant readers and learners. In this particular use of the strategy, have students respond to the statements presented on the BLM. After students have had the opportunity to give their responses, allow them the opportunity to get in small groups to discuss their responses with their peers. At this point, the students are more prepared to gain information from the activity that is about to be presented, motivated by their desire to find out if their initial responses to the *anticipation guide* statements are correct. When the activity is completed, have students revisit the *anticipation guide* and modify their responses if necessary, including having students explain what caused them to make any modifications.

Find out what students remember about slope. Write down student responses and if there are opposing views on a particular topic, have students debate about the topic in order to help them better understand what they already know and any areas where there may still be misconceptions. Ultimately, make sure students understand the following important ideas relating to slope:

- Slope refers to the steepness of the graph.
- Slope can be calculated by finding the *rise over the run*.
- Slope is the change in *y*-coordinates divided by the change in *x*-coordinates (slope formula).
- Slope can be thought of as a "rate of change" in real-life situations (such as the slope of a distance/time graph), and the rate has both a number and a unit associated with it.

If students don't list the formula for calculating slope, then use guiding questions to assist them in recalling it and remembering its derivation. The formula is directly related to the concept of rise over run. To find the rise, the *y*-coordinates are subtracted. To find the run, the *x*-coordinates are subtracted. The slope is the ratio of these two differences. Do a few examples with students.

After a full discussion on slope has taken place, ask students to find the slope of the line using any two points generated using the equation $y = 2x - 3$ (from Activity 1). Confirm that each group calculated the slope to be 2, and ask students which two points they used to determine this value. Help students realize that no matter which two points are chosen, the slope is the same. This is what makes a linear relationship unique—the slope is CONSTANT. This is a big idea! This is how linear relationships are identified. Regardless of any two points chosen, the slope is the same (emphasize this to students).

Refer students to the original equation $y = 2x - 3$. Ask if they remember the name for the form of the equation and why it is given that name. Students should see that the form is the *slope-intercept form* and that the slope and *y*-intercept are easily identifiable for a linear equation when in this form. Review with students how to take an equation such as 3x + 2y = 6 (standard form) and rewrite it in slope-intercept form. *(Note: Do several examples to make sure all students can do this skill as it will be needed on the Using Slope BLM.)*

Remind students that the format $y = mx + b$ is very useful in graphing equations. Review the meaning of *m* and *b* in the equation. Remind students that when a point and a slope for a line are known, the graph for the line can be produced. Demonstrate on the overhead how to use slope

and *y*-intercept to graph a linear equation. Plot the *y*-intercept of $(0,-3)$ on a coordinate grid. Since the slope is $\frac{2}{1}$, other points can be found on the graph by "rising" 2 units and "running" 1 unit from the *y*-intercept. [Provide additional opportunities for students to become proficient at finding slope, given two points, an equation, or a graph. They should also be able to graph a line given one point and a slope.]

Next, make copies of Using Slope BLM and have students work on it in groups. Fully discuss the worksheet after students complete their work. Clear up any misconceptions students may still be having and provide additional work on these topics using the math textbook as a resource.

Once all of these ideas have been fully discussed and reviewed as a class, have students revisit the Slope *Anticipation Guide* BLM to modify their responses as necessary. Discuss the changes that were made and the reasons for the changes as a class.

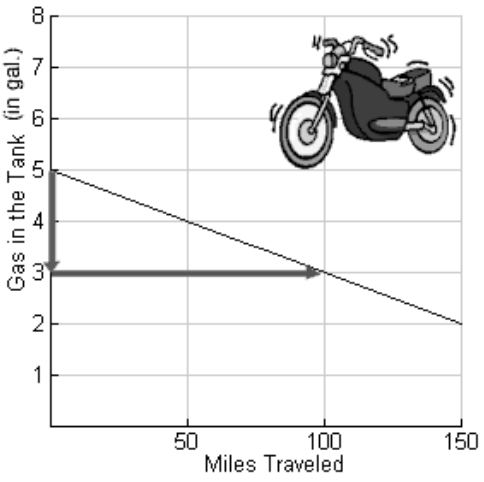
Activity 3: Slope as a Rate of Change (GLEs: 13, 39)

Materials List: paper, pencil, graph paper, textbook, Rate of Change BLM

One aspect of slope that was re-visited in the last activity was slope as a "*rate of change*." When students work with real-life problem situations, slope not only has a number and sign associated with it, but it also has a "unit" associated with it. This is where the "rate" comes into play. For example if there is a distance/time graph, when the information is plotted and look at the "change in y" which has a unit of distance associated with it—i.e., miles, ft, meters, etc. The "*change in x*" then has a unit of time associated with it—i.e., hours, minutes, seconds, etc. Therefore, in a distance/time graph when the slope is looked at, it's a "rate" that might have units such as *miles/hour, meters/sec*, etc.

This activity expands on this concept. To begin the activity, put the following problem on the overhead or smart board and have students think individually about the problem first, then have students share their thoughts/answers.

> *Ex: What does this slope represent in real-life terms? Explain fully what you think it represents.*



Students should see that the slope in this problem has units "gallons/mile." In this case, the numerical value of the slope is "- 2/100" or "- 1/50" or "- 0.02." The "rate of change" for the problem would be "*- 0.02 gallons per mile.*" The interpretation of the real-life meaning of this slope is "the gas tank is losing 0.02 gallons of gas in its tank for each mile traveled." The fact that the slope is negative in this case refers to the tank's losing gas as time goes on. Fully discuss this example with students and clarify any misconceptions students may have.

Next, provide students with copies of the Rate of Change BLM. Have students work in small groups on the activity and then go over the BLM afterwards. Clarify any misconceptions students may still be having. Provide additional work on this topic using the textbook as a resource.

Activity 4: Comparing Lines (GLEs: 13, 24, 38, 39; CCSS: WHST.9-10.2d)

Materials List: paper, pencil, graph paper, graphing calculator

The goal of this activity is to get students to compare and contrast linear functions algebraically and graphically in terms of their rates of change and intercepts. It is also important that students are able to explain how the graph of a linear function changes as the coefficients or constants are changed in a linear equation.

On the same set of axes, have students graph the lines $y = 2x - 2$ and $y = 2x - 3$. When graphing, have students use paper and pencil techniques first, and then use graphing calculator technology while doing this activity. Have students discuss the similarities and differences between the two graphs. The similarity is that both graphs have the same slope; the difference is there are different *y*-intercepts. Remind students that lines with the same slope are parallel lines and will never intersect. Connect the graphs with their respective equations, and point out that since both equations are in slope-intercept form, the same information could be determined by looking at *m* and *b*.

Next, have students graph the equations $y = 2x - 2$ and $y = -2x - 2$. Ask students to discuss the similarities and differences between these two graphs. In this case, the similarities are the graphs have the same *y*-intercept $(0, -2)$; the difference is the graphs have different slopes—one is positive and one is negative. Students need to make the connection that the same *y*-intercept implies a common point. Therefore, the lines will intersect at $(0, -2)$.

Finally, have students start with the equations y = 2x-2 and y = x. Ask students to analyze the two equations and predict the similarities and differences between these two graphs before graphing them. Students should write an explanation for their reasoning in words using a form of *SPAWN writing* (view literacy strategy descriptions). *SPAWN* is a strategy which uses higher-level thinking prompts to elicit student writing. Each letter in SPAWN stands for a particular writing prompt. They are as follows:

S—Special Powers: students are given special powers to do things and then write about how they would use these powers.

P—Problem Solving: students write about the solution of a problem and how they would do this.
A—Alternative Viewpoints: students put themselves in the place of someone or something and write about it.
W—What If?: students are asked to write on what if something happened or changed.
N—Next: students are asked to write on what they think will happen next.

In this particular problem, have students write in response to the following Alternative Viewpoint (the A in the *SPAWN* acronym) prompt:

> *Pretend you are the equation y = x and you are meeting the equation y = 2x – 2 for the first time. Upon meeting the equation, what do you see as similarities and differences between the two of you? Write your impressions in a short paragraph.*

An example of a student *SPAWN* writing might be:

> *Upon meeting the equation y = 2x – 2, the first similarity I noticed was that both of us were linear and had a positive slope, although my slope was less steep than his slope. Both of us were increasing functions. One of the differences I noticed was that while my y-intercept went through the origin, his y-intercept was a negative value and went through the point (0, -2).*

Have select students read their *SPAWN* writings to the class and provide feedback as needed. Afterwards, have student graph the two equations to see if their "predictions" were correct. Pick up all *SPAWN* writings at the end of the activity to check for student understanding as part of ongoing formative assessment. Provide feedback as needed.

Have the students do more comparisons on graphs, and discuss what effect each of the coefficients has on a linear graph by providing equations with varying slopes and the same *y*-intercepts.

Activity 5: Equations of Lines (Three Forms) (GLEs: 8, 11, 13, 15)

Materials List: paper, pencil, math textbook, Equations of Lines BLM

In this activity review writing equations of lines. The activity has two main goals. The first goal is to have students determine a linear equation given a slope and a point or given two points. The second goal is to have students become proficient at translating an equation from one form to the next (i.e., slope-intercept form to standard form; standard form to slope-intercept form). Because of the content involved in this activity, several days of class time can be expected to be utilized in order to fully complete the activity.

Begin by reviewing the three forms used for equations of lines: $y - y_1 = m(x - x_1)$ or point-slope form; $y = mx + b$ or slope-intercept form; and $Ax + By = C$ or standard form. Students may need to be reminded of how to find the equation of a line using a point and a slope. For example, for the point $(4, 2)$ and slope = 3, ask students to generate the point-slope form for the equation.

Allow students the opportunity to collaborate in finding the equation of the line in point-slope form. Once students have come up with an answer, challenge them to write this equation in the other two forms and then demonstrate how all three forms can be developed. This example shows the following:

Point-slope form: $y - 2 = 3(x - 4)$

Slope-intercept form: $y = 3x - 10$

Standard form: $3x - y = 10$

Do several more examples with students, including problems which involve being given two points instead of a point and a slope. Explain that in order to determine the equation of the line, the slope must be found first, and then the slope and one of the points can be used to determine the linear equation.

Next, provide students with copies of the Equations of Lines BLM. Allow students to work in small groups on the BLM, and then discuss the results as a class. Assign additional problems from the textbook you are using on this topic.

Activity 6: More with Writing Equations of Lines (GLEs: 13, 15)

Materials List: paper, pencil, math textbook

In this activity, extend students' understanding of writing equations of lines using a point and a slope or two points to include writing of equations of lines parallel or lines perpendicular to another line that passes through a specific point. Remind students that parallel lines have the same slope while perpendicular lines have the "negative reciprocal" slopes. Three example problems are listed below. Use these examples to discuss this topic, and include additional examples of your own as needed.

Example 1: Write an equation in slope-intercept form of a line which is parallel to $y = 3x - 7$ and has a y-intercept of 4.

Solution: $y = 3x + 4$

Example 2: Write an equation in slope-intercept form for the line that contains the point (4,5) and that is perpendicular to the line $2x + 3y = 7$.

Solution: $y = \frac{3}{2}x - 1$

Example 3: Water freezes at 32°F and 0°C. Water boils at 212°F and 100°C. Write a linear equation which shows the relationship between the Fahrenheit and Celsius temperatures. Let F be the dependent variable and C be the independent variable. Write the equation in slope-intercept form.

Solution: $F = \frac{9}{5}C + 32$

Have students do additional work with this skill by working similar problems found in their math textbooks.

Activity 7: Is the Table Linear or Non-Linear? (GLEs: 15, 29)

Materials List: paper, pencil, calculators, math textbooks, Is the Table Linear BLM

In Activity 2, the following idea was introduced: *What makes a linear relationship unique is that the slope is constant*. In this activity, this concept comes up again. Remind students about this idea before providing each student a copy of the BLM, Is the Table Linear.

For this activity, utilize *discussion* (view literacy strategy descriptions) known as Think-Pair-Square-Share. Students remember best when they participate in dialog about class topics. This strategy is one way to encourage students to think deeper about a topic, and explain their thinking with their peers. In this particular strategy, after giving students the BLM to work on, ask students to think alone about the problems for a short period of time, then have each student pair up with another student to share their thoughts on each of the problems presented on the BLM. After they have had the opportunity for *discussion*, have pairs of students share with other pairs, forming small groups of four. Once this process has taken place, gather oral responses to the questions on the BLM for a full class *discussion* of the problems/solutions to take place. The goal of this *discussion* activity is to provide students an opportunity for deeper processing of linear relationships and rehearsal of the newly learned content about linear relationships.

On the BLM, students are to look at a data table, and determine whether a table of values represents a linear relationship and explain why (or why not). They must make this determination WITHOUT GRAPHING the data. Then, if the data is linear, they will determine the slope associated with the data table along with the equation which represents the relationship (in slope-intercept form).

Once the discussion has taken place, talk about the different answers/viewpoints to the problem presented. There should have been some debate about the answers. In the end, students need to realize that only data that has a constant slope represents a true linear relationship. Utilize the math textbook for additional work on these types of problems.

Activity 8: Line of Best Fit (GLEs: 29; CCSS: S-ID.6)

Materials List: paper, pencil, graphing calculators, grid paper, math textbooks, the Internet

In Activity 7, students were presented data and asked to determine whether it represented a linear relationship by considering the slope. In real-life, although a set of data collected may not be completely linear (meaning, constant slope throughout), in many cases, the data can be modeled by a linear relationship using what is referred to as a "line of best fit." In this activity, the website http://illuminations.nctm.org/ActivityDetail.aspx?ID=146 will be utilized to show students how to use a scatter plot to display the real-life data associated with the number of minutes played by

basketball players and the number of points they scored. When accessing the website, open up the *Instructions* and *Explorations* tabs to access the data that is to be entered. In this activity, the user is allowed to enter a set of data, plot the data on a coordinate grid, and determine the equation for a line of best fit. Also, have students describe in words how the two variables are related in this situation.

After working on this activity via the computer, have students plot the data using graph paper and model the process of determining a line of best fit using paper/pencil methods. Use a straightedge to draw the line so that the line best matches the overall trend. Help students understand how to determine the equation for the line of best fit using the slope and *y*-intercept of the line drawn. Afterwards, demonstrate how to make a scatter plot and find a line of best fit for the data using graphing calculator technology.

Provide students with additional opportunities to work with real data. For example, have students plot the diameter in relationship to the circumference of round objects they measure, and then determine the equation for the line of best fit for the data (and describe the relationship in words). Problems of this type can be found in most math textbooks. Utilize the textbook to provide additional work and learning experiences concerning the use of scatter plots and determine line of best fit for a set of data.

Activity 9: Modeling Real-life Problems (GLEs: 9, 15)

Materials List: paper, pencil, graphing calculators, grid paper, math textbooks

Students need to be able to model real-life linear relationships. In this activity, students are given a verbal situation and are required to model the situation with algebraic, tabular, and graphical representations. Present the following problem to students and have them work in small groups to come up with their answers/solutions.

Problem: Since 1999, the number of music cassettes sold has decreased by an average rate of 27 million per year. There were 124 million music cassettes sold in 1999. Let t = 0 represent the year 1999. Use this information to answer the following questions.

a. Fill in the table of values for this situation for the year listed.

t (years since 1999)	*0*	*1*	*2*
C (Cassettes sold in Millions)			

b. Write a linear equation (algebraic model) to represent the average number of music cassettes sold, C, in any year after 1999 (Let the year t = 0 represent the year 1999). After creating your equation, make sure it produces the same results that you found in the table in part a.

c. Using your equation, determine the approximate number of cassettes sold in 2004. Show your work.

d. Using your linear model, what year would you expect there to be no more cassettes sold. Show your work. What point does this correspond to if you were to create a graph of the data for this problem? Explain.

After students have finished their work, use student solutions/responses to guide instruction. Clear up any misconceptions students may be having and use the information gathered to provide additional work on this topic. Utilize the textbook as a resource for additional work of this type.

The solutions to this problem are as follows:

Part a:

t (years since 1999)	*0*	*1*	*2*
C (Cassettes sold in Millions)	*124*	*97*	*70*

Part b: C = -27t + 124
Part c: C = -27 (4) + 124 = -108 + 124 = 16 (16 million sold in 2003)
Part d: 0 = -27t + 124
27t = 124
t = 124/27 or about 4.6 years (this is essentially the same as the "x-intercept" if you were to graph the data)

Sample Assessments

Performance assessments can be used to ascertain student achievement. Following are some examples:

General Assessments

- The teacher will provide the student with the following information: $P = (2, -1)$ and slope = 3. The student will find three additional points for the graph and graph the line.
- The student will interpret the slope and *y*-intercept of a given real-world linear equation.
- The student will find an example of an application of a linear equation (graph) and prepare a short paragraph on what his/her selected example means.
- The student will create portfolios containing samples of his/her activities.
- The student will take pencil/paper tests created by the teacher on the skills presented throughout this unit.

Activity-Specific Assessments

- Activity 1: The student will graph the equation $y = 3x - 5$ using the point plotting method and a t-chart.

- Activity 2: The student will determine the slope of a line given a graph, two points, an equation in slope-intercept form, or an equation in standard form.

- Activity 5: The student will match the graphs of lines with his/her correct equations using what he/she knows about slope and y-intercepts.

- Activity 6: The student will determine the equation of the line given two points or a slope and a point. He/she will also write the equation of a line which is parallel/perpendicular to another line through a given point when given the equation of the original line.