Name____ **Lesson #**_________

Math 455AB – Rolle's Theorem and the Mean Value Theorem

I. Verify that the hypotheses of Rolle's Theorem are satisfied on the given interval, and find all values of *c* in that interval that satisfy the conclusion of the theorem.

1) $f(x) = x^2 - 8x + 12; \quad [2,6]$

$f(2) = 4 - 16 + 12 = 0$ ✓

$f(6) = 36 - 48 + 12 = 0$

$f'(c) = 2c - 8 = 0$

$2c = 8$

$c = 4$

2) $f(x) = 2\sin x; \quad [-\pi, \pi]$

$f(-\pi) = 2\sin(-\pi) = 0$ ✓

$f(\pi) = 2\sin\pi = 0$

$2\cos c = 0$

$\cos c = 0$

$c = -\frac{\pi}{2}, \frac{\pi}{2}$

3) $f(x) = \dfrac{x^2 - 4}{x + 3}; \quad [-2,2]$ cont. on $[-2,2]$ (disc. at $x = -3$)

$f(2) = 0$ ✓

$f(-2) = 0$

$$f'(c) = \frac{(x+3)(2x) - (x^2-4)(1)}{(x+3)^2} = 0$$

$$\frac{2x^2 + 6x - x^2 + 4}{(x+3)^2} = 0$$

(over, please)

$x^2 + 6x + 4 = 0$

$x = -5.236, -.763$

Name________________________________

Chain Rule and Implicit Differentiation Practice

Date________________ Period____

Differentiate each function with respect to *x*.

1) $y = \sqrt{-x+3}$

$\frac{dy}{dx} = \frac{1}{2}(-x+3)^{-\frac{1}{2}}(-1) =$

$\frac{-1}{2\sqrt{-x+3}}$

2) $y = (-x^3+3)^{\frac{1}{4}}$

$\frac{dy}{dx} = \frac{1}{4}(-x^3+3)^{-\frac{3}{4}}(-3x^2)$

$= \frac{-3x^2}{4} \cdot \frac{1}{\sqrt[4]{(-x^3+3)^3}}$

3) $y = \sin x^5$

$\frac{dy}{dx} = \cos(x^5)\, 5x^4 = 5x^4 \cos x^5$

4) $y = \sin(\sec 5x^4)$

$\frac{dy}{dx} = \cos(\sec 5x^4) \cdot \sec 5x^4 \tan 5x^4 \cdot 20x^3$

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of *x* and *y*.

5) $5y^2 + 3xy^2 = x$

$10y\frac{dy}{dx} + 3(x \cdot 2y\frac{dy}{dx} + y^2) = 1$

$10y\frac{dy}{dx} + 6xy\frac{dy}{dx} + 3y^2 = 1$

$\frac{dy}{dx}(10y + 6xy) = 1 - 3y^2$

$\frac{dy}{dx} = \frac{1-3y^2}{10y+6xy}$

6) $2x = -2y^2 + 4y^3$

$2 = -4y\frac{dy}{dx} + 12y^2\frac{dy}{dx}$

$\frac{2}{-4y+12y^2} = \frac{dy}{dx}$

$\frac{1}{-2y+6y^2} = \frac{dy}{dx}$

For each problem, use implicit differentiation to find $\frac{d^2y}{dx^2}$ in terms of *x* and *y*.

7) $x^3 + 4y^2 = 2$

$3x^2 + 8y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-3x^2}{8y}$

$\frac{d^2y}{dx^2} = \frac{(8y)(-6x) - (-3x^2)(8\frac{dy}{dx})}{(8y)^2}$

$= \frac{-48x + \cancel{24}^{3}x^2 \cdot \left(\frac{-3x^2}{\cancel{8}y}\right)}{64y^2} = \frac{-48x - \frac{9x^4}{y}}{64y^2} = \frac{-48xy - 9x^4}{64y^3}$

II. Determine whether or not the premises of the mean value theorem of the given function are satisfied in the given interval. If they are satisfied, then apply the MVT to find *c*.

1) $f(x)=\frac{x^3}{4}+1; \quad [0,2]$

continuous + diff on $[0,2]$

$f'(c)=\frac{3c^2}{4}=\frac{f(2)-f(0)}{2}$

$\frac{3c^2}{4}=\frac{3-1}{2}$

$\frac{3c^2}{4}=1$

$3c^2=4$

$c^2=\frac{4}{3} \quad c=\pm\frac{2}{\sqrt{3}} \quad C=\frac{2}{\sqrt{3}} \quad (-\frac{2}{\sqrt{3}}$ not on $[0,2])$

2) $f(x)=\frac{1}{x^2}+9; \quad [-1,3]$

not cont. at $x=0$
so premises are not satisfied

3) $f(x)=\sqrt{4-x^2}; \quad [-2,1]$

semi ⊙
cont on $[-2,1]$
diff on $(-2,1)$

$\frac{f(1)-f(-2)}{1--2}=\frac{\sqrt{3}-0}{3}$

$f'(c)=\frac{1}{2}(4-c^2)^{-\frac{1}{2}}(-2c)=\frac{\sqrt{3}}{3}$

$\frac{-c}{\sqrt{4-c^2}}=\frac{\sqrt{3}}{3}$

$-3c=\sqrt{12-3c^2}$

$9c^2=12-3c^2$

$12c^2=12$

$c^2=1$

$c=\pm 1$ reject 1 (doesn't work in original)

$C=-1$

4) $f(x)=\sin x; \quad \left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

continuous + differentiable

$\frac{f(\frac{\pi}{2})-f(-\frac{\pi}{2})}{\pi}=\cos c$

$\frac{1--1}{\pi}$

$\frac{2}{\pi}=\cos c$

$c=.8807$

HB Calculus 12/10/15

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AP Mixed Review (after 6.1)
Use a calculator only on those that say it's permitted. Put the CAPITAL letter in the blank for each problem.

_____ 1. (Calculator Permitted)

A particle moves along a straight line with velocity given by $v(t)=7-(1.01)^{-t^2}$ at time $t\geq 0$. What is the acceleration of the particle at time $t=3$?

(A) -0.914 (B) 0.055 (C) 5.486 (D) 6.086 (E) 18.087

B 2.

$\lim_{x\to\infty}\frac{(2x-1)(3-x)}{(x-1)(x+3)}$ is

(A) -3 (B) -2 (C) 2 (D) 3 (E) nonexistent

D 3.

$\int \frac{1}{x^2}dx =$ $\int x^{-2}dx =$

(A) $\ln x^2+C$ (B) $-\ln x^2+C$ (C) $x^{-1}+C$ (D) $-x^{-1}+C$ (E) $-2x^{-3}+C$

D 4.

If $f(x)=(x-1)(x^2+2)^3$, then $f'(x)=$

$(x-1)3(x^2+2)^2(2x)+(x^2+2)^3(1)$
$3(2x^2-2x)(x^2+2)^2+(x^2+2)^3$
$(x^2+2)^2(6x^2-6x+x^2+2)$
$(x^2+2)^2(7x^2-6x+2)$

(A) $6x(x^2+2)^2$

(B) $6x(x-1)(x^2+2)^2$

(C) $(x^2+2)^2(x^2+3x-1)$

(D) $(x^2+2)^2(7x^2-6x+2)$

(E) $-3(x-1)(x^2+2)^2$

A 5.

$$\lim_{x\to 0}\frac{5x^4+8x^2}{3x^4-16x^2} \text{ is}$$

$$\frac{\cancel{x^2}(5x^2+8)}{\cancel{x^2}(3x^2-16)}$$

(A) $-\frac{1}{2}$ (B) 0 (C) 1 (D) $\frac{5}{3}+1$ (E) nonexistent

A 6.

$$f(x)=\begin{cases}\dfrac{x^2-4}{x-2} & \text{if } x\neq 2\\ 1 & \text{if } x=2\end{cases}$$

$x+2$

$$\frac{(x+2)(x-2)}{(x-2)}$$

Let f be the function defined above. Which of the following statements about f are true?

I. f has a limit at $x=2$. ✓

II. f is continuous at $x=2$. ✗

III. f is differentiable at $x=2$. ✗

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

E 7.

If $f(x)=\cos(3x)$, then $f'\left(\frac{\pi}{9}\right)=$

$-\sin(3x)(3)$

$-\sin\left(\frac{\pi}{3}\right)(3)=$

$-\frac{\sqrt{3}}{2}\cdot 3=$

(A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $-\frac{\sqrt{3}}{2}$ (D) $-\frac{3}{2}$ (E) $-\frac{3\sqrt{3}}{2}$

____ 8. skip

If $f(x)=e^{(2/x)}$, then $f'(x)=$

(A) $2e^{(2/x)}\ln x$ (B) $e^{(2/x)}$ (C) $e^{(-2/x^2)}$ (D) $-\frac{2}{x^2}e^{(2/x)}$ (E) $-2x^2e^{(2/x)}$

D 9.

If $\sin(xy)=x$, then $\frac{dy}{dx}=$

$\cos(xy)\cdot(xy'+y)=1$

$xy'+y=\frac{1}{\cos(xy)}-y$

$\frac{1}{x\cos(xy)}-\frac{y}{x}$

(A) $\frac{1}{\cos(xy)}$

(B) $\frac{1}{x\cos(xy)}$

(C) $\frac{1-\cos(xy)}{\cos(xy)}$

(D) $\frac{1-y\cos(xy)}{x\cos(xy)}$

(E) $\frac{y(1-\cos(xy))}{x}$

A 10.

In the xy-plane, the line $x+y=k$, where k is a constant, is tangent to the graph of $y=x^2+3x+1$. What is the value of k?

$-1=2x+3$

$-4=2x$ $x=-2$ $y=4-6+1=-1$

(A) -3 (B) -2 (C) -1 (D) 0 (E) 1

$-2+-1=k$

$-3=k$

B 11.

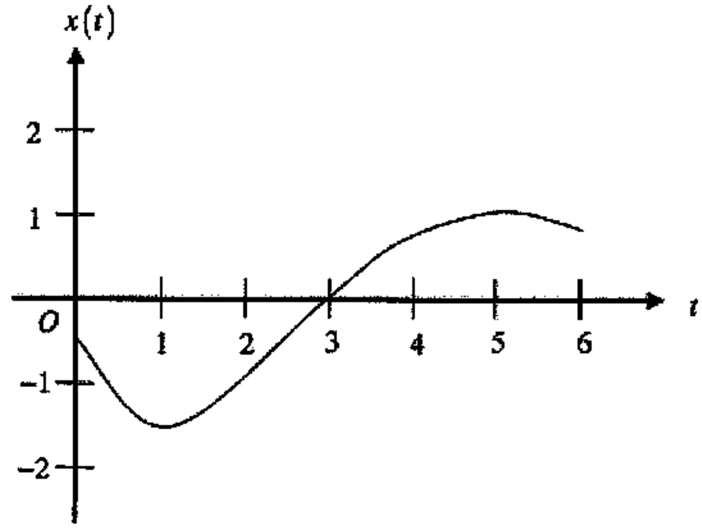
What is the slope of the line tangent to the curve $y=\arctan(4x)$ at the point at which $x=\frac{1}{4}$?

$\frac{1}{1+16x^2}=\frac{1}{2}$

(A) 2 (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) -2

____ 12.

A



A particle moves along a straight line. The graph of the particle's position $x(t)$ at time t is shown above for $0 < t < 6$. The graph has horizontal tangents at $t = 1$ and $t = 5$ and a point of inflection at $t = 2$. For what values of t is the velocity of the particle increasing?

accel > 0
CCU

(A) $0 < t < 2$

(B) $1 < t < 5$

(C) $2 < t < 6$

(D) $3 < t < 5$ only

(E) $1 < t < 2$ and $5 < t < 6$

____ 13.

B

$$f(x) = \begin{cases} cx + d & \text{for } x \le 2 \\ x^2 - cx & \text{for } x > 2 \end{cases}$$

$f'(x) = \begin{cases} c & x \le 2 \\ 2x - c & x > 2 \end{cases}$

$c = 2(2) - c \quad c = 2$

Let f be the function defined above, where c and d are constants. If f is differentiable at $x = 2$, what is the value of $c + d$?

$2x + d = x^2 - 2x$
$a + x = 2$
$4 + d = 4 - 4$
$d = -4$
$c + d = -2$

(A) –4 (B) –2 (C) 0 (D) 2 (E) 4

____ 14.

Let f be a differentiable function such that $f(3)=15$, $f(6)=3$, $f'(3)=-8$, and $f'(6)=-2$. The function g is differentiable and $g(x)=f^{-1}(x)$ for all x. What is the value of $g'(3)$?

(A) $-\frac{1}{2}$

(B) $-\frac{1}{8}$

(C) $\frac{1}{6}$

(D) $\frac{1}{3}$

(E) The value of $g'(3)$ cannot be determined from the information given.

____ 15. (Calculator Permitted)

The first derivative of the function f is defined by $f'(x)=\sin\left(x^3-x\right)$ for $0 \le x \le 2$. On what interval(s) is f increasing?

(A) $1 \le x \le 1.445$

(B) $1 \le x \le 1.691$

(C) $1.445 \le x \le 1.875$

(D) $0.577 \le x \le 1.445$ and $1.875 \le x \le 2$

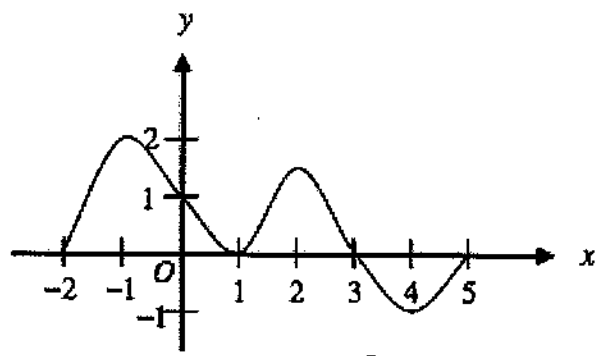
(E) $0 \le x \le 1$ and $1.691 \le x \le 2$

____ 16. (Calculator Permitted)

The derivative of the function f is given by $f'(x)=x^2\cos\left(x^2\right)$. How many points of inflection does the graph of f have on the open interval $(-2, 2)$?

(A) One (B) Two (C) Three (D) Four (E) Five

_____ 17.



Graph of f'

The graph of f', the derivative f, is shown above for $-2 \le x \le 5$. On what intervals is f increasing?

(A) $[-2, 1]$ only

(B) $[-2, 3]$

(C) $[3, 5]$ only

(D) $[0, 1.5]$ and $[3, 5]$

(E) $[-2, -1]$, $[1, 2]$, and $[4, 5]$

_____ 18.

The radius of a sphere is decreasing at a rate of 2 centimeters per second. At the instant when the radius of the sphere is 3 centimeters, what is the rate of change, in square centimeters per second, of the surface area of the sphere? (The surface area S of a sphere with radius r is $S = 4\pi r^2$)

(A) -108π (B) -72π (C) -48π (D) -24π (E) -16π

$\frac{dr}{dt} = -2$

find $\frac{dS}{dt}$ when $r = 3$

$S = 4\pi r^2$

$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$

$8\pi(3)(-2)$

$-48\pi \text{ cm}^2/\text{s}$

_____ 19.

The function f is continuous for $-2 \le x \le 2$ and $f(-2) = f(2) = 0$. If there is no c, where $-2 < c < 2$, for which $f'(c) = 0$, which of the following statements must be true?

(A) For $-2 < k < 2$, $f'(k) > 0$.

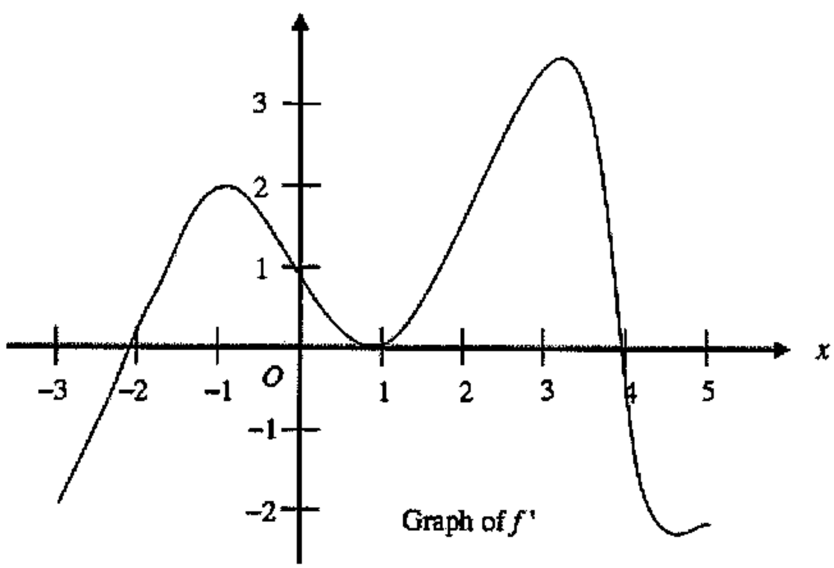
(B) For $-2 < k < 2$, $f'(k) < 0$.

(C) For $-2 < k < 2$, $f'(k)$ exists.

(D) For $-2 < k < 2$, $f'(k)$ exists, but f' is not continuous.

(E) For some k, where $-2 < k < 2$, $f'(k)$ does not exist.

_____ 20.



The graph of the derivative of a function f is shown in the figure above. The graph has horizontal tangent lines at $x = -1$, $x = 1$, and $x = 3$. At which of the following values of x does f have a relative maximum?

SIGP

(A) -2 only

(B) 1 only

(C) 4 only

(D) -1 and 3 only

(E) $-2, 1$, and 4