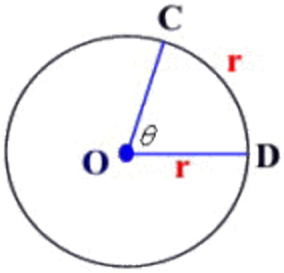
An Introduction to Radians

You've seen that your calculator can be in either radian mode or degree mode. So what are radians? **Basically, radians are another unit with which to measure angles of rotation**. Just as we measure temperature sometimes using the Fahrenheit scale and sometimes the Celsius scale or measure length sometimes using miles and sometimes kilometers, we have two ways to measure angles. Radians are used almost exclusively, rather than degrees, in higher level mathematics and science because radians are easier to work with than degrees (which are from an old base-60 number system.)

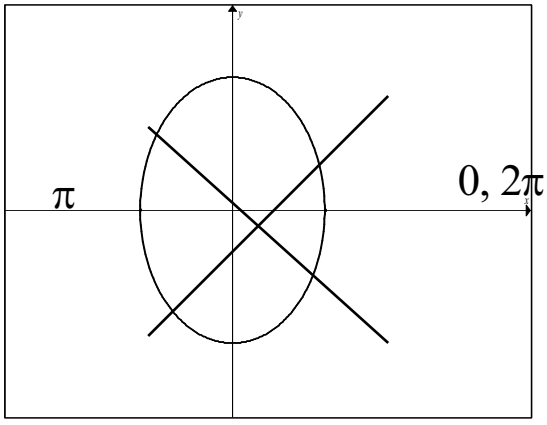


Here's how radians work. **One (1) radian is the measure of the angle that creates an arc the same length as the radius.** (It takes some thinking to figure out what that means, so look at the diagram and think about that for a minute.)

Since the circumference of a circle is $2\pi r$, arc lengths often have π in them. So often (but not always), radian measures have π in them too. Radian measures of angles come from the unit circle, ie. a circle with radius of 1. Radians essentially refer to the length of the intercepted arc on a unit circle. For that reason, $0°$ is the same as 0 radians and $360°$ is the same as 2π radians (since that's all the way around a circle).

Confused? Don't be…It's easy once you get the hang of it. Remember that:

$0°$ is the same as 0 radians
$180°$ is the same as π radians
$360°$ is the same as 2π radians



Going further, we see that the radian measure of $90°$ would be $\frac{\pi}{2}$ and $270°$ would be $\frac{3\pi}{2}$. What would $45°$ be? How about $135°$, $225°$, and $315°$? Fill all these into the diagram above.

It's handy to be able to convert angles back and forth between degrees and radians. See if you can figure out a way to do it! Try these:

Convert $120°$ to radians.

Convert $\frac{7\pi}{6}$ to degrees.

Here's the way to convert back and forth between degrees and radians.

If you know the degrees, multiply by a handy form of 1 that will get rid of the degrees and leave you with radians : $\frac{\pi}{180^\circ}$

If you know the radians, multiply by a handy form of 1 that will get rid of the radians and leave you with degrees : $\frac{180^\circ}{\pi}$

Let's look at some examples:

Convert radians to degrees: $\frac{3\pi}{4}$

$\frac{3\pi}{4} \cdot \frac{180^\circ}{\pi}$

$= \frac{3\pi}{4} \cdot \frac{180^\circ}{\pi}$

$= 135^\circ$ (divide out the π and anything else that simplifies)

Convert degrees to radians: 150°

$150^\circ \cdot \frac{\pi}{180^\circ}$

$= \frac{150\pi}{180}$ (divide the numbers, 150 divided by 180, to see how they simplify. You can do this in your calculator and do math frac to make it easy!)

$= \frac{5\pi}{6}$

Let's try some!

1 Convert each degree measure to radians. (Express radians in terms of π.)

a) 120°

b) 225°

c) 300°

d) 330°

2 Convert each radian measure to degrees. (If necessary, round to nearest tenth of a degree.)

a) $\frac{7\pi}{6}$

b) $\frac{\pi}{4}$

c) $\frac{2\pi}{3}$

d) 6