Snow Day Packets 11 – 17

Algebra/Algebra Support

Mr. KJ Shaffer's Class

9.8 Solving Equations by Factoring

◆Skill A Using the Zero-Product Property

Recall If a and b are real numbers such that $ab = 0$, then $a = 0$ or $b = 0$.

◆ **Example**

Solve the equation $(x + 3)(x - 1) = 0$.

◆ **Solution**

Use the Zero-Product Property. If the product of two factors is equal to 0, then one of the factors must be 0. Set each factor equal to 0 and solve.

First factor	Second factor
$(x + 3) = 0$	$(x - 1) = 0$
$x = -3$	$x = 1$

Check by substituting in the original equation.
Substitute -3 for x: $(-3 + 3)(-3 - 1) = 0(-4) = 0$
Substitute 1 for x: $(1 + 3)(1 - 1) = 4(0) = 0$

The equation $(x + 3)(x - 1) = 0$ has two solutions. The solutions are -3 and 1.

Solve by factoring.

1. $(x - 3)(x - 2) = 0$

2. $(x + 5)(x - 4) = 0$

3. $(x + 4)(x - 4) = 0$

4. $(x - 6)(x - 6) = 0$

5. $(x - 2.8)(x + 5.2) = 0$

6. $\left(x + \frac{2}{3}\right)(x - 1) = 0$

7. $(2x - 4)(3x - 6) = 0$

8. $(5x + 3)(4x + 7) = 0$

9. $\left(\frac{3}{5}x + 6\right)\left(\frac{7}{8}x - 1\right) = 0$

10. $(4.7x + 14.1)(2.4x - 3.6) = 0$

1. 3 and 2
2. -5 and 4
3. -4 and 4
4. 6

5. 2.8 and -5.2
6. $-\frac{2}{3}$ and 1
7. 2
8. $-\frac{3}{5}$ and $-\frac{7}{4}$
9. -10 and $\frac{8}{7}$
10. -3 and 1.5

Factoring Quadratic Equations

Quadratic Form: ax^2+bx+c

a is the leading Coefficient

Let us look at when $a=1$ so, x^2+bx+c

There are four different operation patterns to look at to make things easier for yourself

1) x^2+bx+c both b & c are positive
So the factors will be $(x+\ )(x+\ )$

2) x^2-bx+c c is positive. Because c is the product of two numbers the only way of getting a positive is both are positive or both are negative
$(+)(+)=(+)$ $(-)(-)=(+)$
And adding two negatives gives you a negative which is what b is.

so the factor are $(x-\ )(x-\ )$

3) $x^2 + bx - c$

This time c is negative and the only way to get a negative product is by having one positive and one negative

$(+)(-) = (-)$ or $(-)(+) = (-)$

but which one is which.

Look at b. b is positive so the larger factor must be positive.

$(x + \underset{\uparrow}{\ \ })(x - \ \ )$

Larger #

4) $x^2 - bx - c$

c is still negative, but so is b this time the larger factor is negative

$(x + \ \ )(x - \underset{\uparrow}{\ \ })$

Larger #

Let us recap and do some examples

x^2+bx+c $\qquad\qquad$ x^2-bx+c

$(x+\ \ )(x+\ \ )$ $\qquad\qquad$ $(x-\ \ )(x-\ \ )$

x^2+bx-c $\qquad\qquad$ x^2-bx-c

$(x+\underset{\uparrow}{\ \ })(x-\ \ )$ $\qquad\qquad$ $(x+\ \ )(x-\underset{\uparrow}{\ \ })$

Larger # $\qquad\qquad$ Larger #

Example 1

$x^2+16x+63$ $\quad$ notice it will be $(x+\ )(x+\ )$

Write down the factors of 63:

63
1, 63
3, 21
7, 9

b is 16 so you want the factors to have a sum of 16

$1+63=64$

$3+21=24$

$\boxed{7+9=16}$

the factors you want are 7 and 9

$(x+7)(x+9)$ or $(x+9)(x+7)$

you can check your work:

$$(x+7)(x+9)$$

$$x(x) + x(9) + 7(x) + 7(9)$$

$$x^2 + 9x + 7x + 63$$

$$x^2 + 16x + 63$$ ✓

<u>Example 2</u>

$r^2 - 10r + 24$ notice it will be $(x- \ )(x- \ )$

<u>factors of 24</u>

1, 24	$1+24=25$
2, 12	$2+12=14$
3, 8	$3+8=4$
4, 6	$\boxed{4+6=10}$

because the factors are both negative you will still add them together to get 10

$(r-4)(r-6)$ or

$(r-6)(r-4)$

Example 3

$b^2 + b - 30$

be careful because the factors are $(x+\ )(x-\ )$

because b is positive the larger factor will be positive

factors of 30

1, 30	$30-1=29$
2, 15	$15-2=13$
3, 10	$10-3=7$
5, 6	$\boxed{6-5=1}$

because the factors are $(+)(-)$ you must subtract the smaller number from the bigger number

$(b+6)(b-5)$ or $(b-5)(b+6)$

Example 4

$m^2 - 4m - 12$

the factors are $(x+\ )(x-\ )$ but the larger number is negative this time because b is negative,

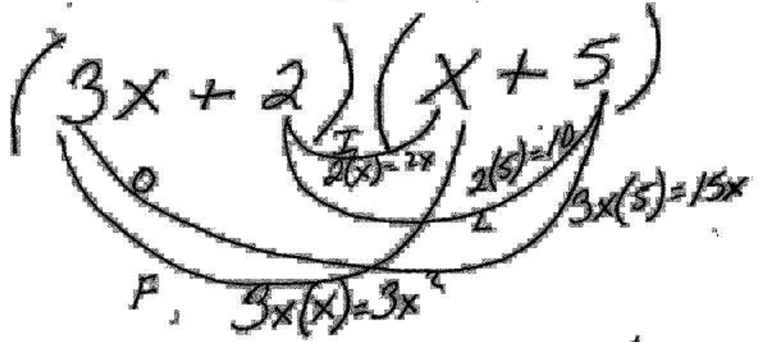
factors of 12

1, 12	$1-12=-11$
2, 6	$\boxed{2-6=-4}$
3, 4	$3-4=-1$

now you must subtract the larger number from the smaller number to get -4

$(m+2)(m-6)$ or $(m-6)(m-2)$

Take a close look at the FOIL Method to see where a, b, & c come from in a quadratic equation.



$$\overset{F}{3x^2} + \underbrace{\overset{O}{15x} + \overset{I}{2x}}_{\text{Combine}} + \overset{L}{10}$$

$$3x^2+17x+10$$

C comes from the factors

a comes from the factors with x

b comes from the products of the constant and leading coefficients

So now what happens when a is not 1?

Let as look at when a is prime.

Example 1

$5x^2 + 34x + 24$ factors are $(x+\ )(x+\ )$

$a = 5$ so its factors are only 1 & 5.

factors of 24

1, 24 $1(5) + 24(1) = 29$

2, 12

3, 8 $1(1) + 24(5) = 121$

4, 6

$$\boxed{2(5) + 24(1) = 34}$$

now we must multiply the 1 and 5 to the factors of 24 and the add to get 34.

(Remember FoIl

$$(5x + 24)(x \quad 2)$$

$24(x) = 24x$ $24x + 10x = 34x$

$2(5x) = 10x$

Looks Like a smiley face with a nose!

Example 2

$2x^2 - 25x + 63$

factors will be $(x-\ )(x-\ )$
you will still add to get 25 for b.

factors of 2 are 2 & 1

factors of 63

1, 63
3, 21
9, 7

start with 9 & 7

$9(1) + 7(2) = 23$

$\boxed{9(2) + 7(1) = 25}$

* You know you are adding so look for a good starting point
63 is over 25
and 21 is almost 25

$(2x - 7)(x - 9)$

$-7x$

$-18x$

$-7x + {}^{-}18x = 25x$

Example 3

$3m^2 - 11m - 4$

factors will be $(x+\ )(x-\ )$
because it is $(+)(-)$ you will subtract to get b

factors of 3 are 3 & 1

factors of 4

1, 4
2, 2

$4(1) - 1(3) = 1$

$4(3) - 1(1) = 11$ correct & wrong sign

so switch it

$1(1) - 4(3) = -11$

$(3x\ \ 1)(x\ \ 4)$

$1x$

$12x$

because $1x - 12x = -11x$
We want the 3x & 4 to have the negative

$(3x + 1)(x - 4)$

Example 4

$7x^2 + 60x - 27$

factors of 7 are 7 & 1

Factors of 27

1, 27
3, 9

$7(3) - 1(9) = 13$

$7(9) - 1(3) = 60$

Correct # and sign

factors $(x+\ )(x-\ )$

you must Subtract

* again, look at b it is 60. Use trial and error but think first some numbers will be way off.

$(7x\ \ 3)(x\ \ 9)$

3x

63x

$63x - 3x = 60x$

So 3 & x make up the negative

$(7x-3)(x+9)$

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* Note if the leading Coefficient is negative, then treat the negative as a common factor and factor it out

[illegible] $-5x^2 - 23x - 12 = -(5x^2 + 23x + 12)$

then factor $5x^2 + 23x + 12$ and bring down negative

$-(5x+3)(x+4)$

$-3x^2 - 14x + 24 = -(3x^2 + 14x - 24)$

$-(3x-4)(x+6)$

Special Factors

Take a look at $9x^2 - 16$

notice two things

- bx is missing, so $b=0$
- 9 & 16 are perfect squares

$4^2=16$

$3^2=9$

C must be negative

for $b=0$ the factor products must be the same but oposite signs.

So the factors are $(x+\ )(x-\ )$

$(3x+4)(3x-4)$

Check answer

$(3x+4)(3x-4)$

$3x(3x)+3x(-4)+3x(4)+4(-4)$

$9x^2-12x+12x-16$

$9x^2-16$ ✓

$25b^2 - 30b + 9$

$(5b-3)(5b-3)$

so $(5b-3)^2$

this one is not as easy to see.

- a & c are perfect squares $5^2 = 25$
  $3^2 = 9$
- factors $(x- \ )(x- \ )$
- $5(3)(2) = 30$
  always 2

try another

$16x^2 - 40x + 25$

So, $(4x-5)^2$

- $4^2 = 16$
  $5^2 = 25$
- $4(5)(2) = 40$
- factors $(x- \ )(x- \ )$

One more

$9x^2 + 12x + 4$

So, $(3x+2)^2$

- $3^2 = 9$
  $2^2 = 4$
- $3(2)(2) = 12$
- factors $(x+ \ )(x+ \ )$

Common Factors

$$6x^2 + 4x + 8$$

6, 4, & 8 all have 2 in common

So $2(3x^2 + 2x + 4)$

$$5x^3 + 25x^2 - 30x$$

all terms have an x
and 5, 25, & 30 have a 5
in Common

$$5x(x^2 + 5x - 6)$$

$$x^3y^4z^2 + x^2y^2z^2 - xz^2$$

all three terms have an x & y
you can only pull out the
smallest degree of each

$$xz^2(x^2y^4 + xy^2 - 1)$$

Snow Day Packet 10

Name________________________________

Solving Polynomials in Factored Form

Date________________

Solve each equation by factoring.

1) $(5a+2)(a-4)=0$

2) $(r+4)(2r-1)=0$

3) $5(n-4)(n+2)=0$

4) $(5n+2)(n-5)=0$

5) $(n+2)^2=0$

6) $(r+1)(4r-5)=0$

7) $2(n-2)^2=0$

8) $(n-4)(n-5)=0$

9) $(k+2)(4k+1)=0$

10) $(x-3)(x+1)=0$

11) $2(n-5)(n-2)=0$

12) $4(3x-2)(5x+2)=0$

13) $(5r-2)(r+4)=0$

14) $(x-3)(x+4)=0$

15) $2(p+2)(p-5)=0$

16) $2(a-1)^2=0$

17) $(v-4)(3v-4)=0$

18) $(5n+1)(n-2)=0$

19) $5(v-5)(3v-2)=0$

20) $(a+4)(a-2)=0$

21) $3(r+2)^2=0$

22) $3(b-4)(2b-5)=0$

23) $2(5k-4)(k-1)=0$

24) $2(x+5)(x+2)=0$

25) $3(3n-2)(n-1)=0$

Snow Day Packet 11 (Algebra I)

Name________________________

Factoring by Common Factor

Date____________

Factor the common factor out of each expression.

1) $-36a^3 - 32a - 16$

2) $-12 - 12p + 24p^3 - 27p^6$

3) $54x^3 + 54x^2 + 42x + 54$

4) $10n^3 - 30n^2$

5) $-6x + 2$

6) $15m^2n^2 + 3m^2n^3 + 15m^3n^4$

7) $-9x^2y + 81x^2$

8) $-12x^2y^2 + 3x^3 + 12y^2$

9) $-9x^3y - 18xy$

10) $3a^2b^2 + 7a^2b - 3ab + 5a^2$

11) $-4p^7q - 4p^6qr$

12) $4pq^5r^2 - 3pq^4r^4 + 10pq^4r^2 + p^3q^5r^2$

13) $14m^5p - 21m^3pq$

14) $7a^2c^2 + 2ac^2 + 8ab^2$

15) $36j^5k^5 + 32j^3k^5h + 16j^3k^4h^2 + 16j^3k^5$

16) $35z^2x - 40zy - 25zx$

17) $30n^4 + 15n^3 + 35n^2 + 30$

18) $-8m^6 - 8m^2 + 24m + 8$

19) $-72x^5 - 27x^3 - 18x^2$

20) $-4n^6 + 2n^3 + 4n + 2$

21) $21x^7 + 15x^4 - 24x^2 - 15x$

22) $4x^3 - 32x^2 + 12x$

23) $-63n + 7nmp^3 + 42n^2$

24) $21mn^4 + 6mn^2 + 15m^3$

25) $-30k - 80 - 50h^3j^6k^2$

Snow Day Packet 12 - Algebra 1

Name________________________________

Factoring when a = 1

Date________________

Factor each completely.

1) $v^2 + v - 90$

2) $p^2 + 2p - 3$

3) $x^2 + 8x + 15$

4) $m^2 - 2m - 3$

5) $x^2 - 16x + 63$

6) $v^2 - 5v + 6$

7) $v^2 - 13v + 40$

8) $n^2 + 6n - 7$

9) $v^2 + 3v - 10$

10) $r^2 - 8r - 9$

11) $n^2 + 3n - 4$

12) $a^2 + 13a + 30$

13) $n^2 - 16n + 60$

14) $b^2 - 5b + 6$

15) $a^2 + 5a - 6$

16) $6k^2 + 42k + 72$

17) $x^2 - 17x + 72$

18) $4x^2 - 36x + 32$

19) $4x^2 + 48x + 140$

20) $5n^2 + 5n - 280$

21) $b^2 - 4b - 45$

22) $a^2 - 7a + 6$

23) $x^2 + 7x - 18$

24) $n^2 + 7n + 6$

25) $5x^2 - 30x + 25$

Snow Day Packet 13

Name____________________________

Review of Factoring Form, Common Factors, and when a=1

Date________________

Solve each equation by factoring.

1) $4(3a+4)(a-2)=0$

2) $(x+2)(3x+5)=0$

3) $(m-1)(m+4)=0$

4) $4(x+2)(x-3)=0$

5) $2(x+5)(x-4)=0$

6) $4(5x-3)(x-1)=0$

7) $2(n-5)(n-4)=0$

Factor the common factor out of each expression.

8) $54x^4+81x^3+36x$

9) $24b^4+6b^2+3b$

10) $-15+40x^4-40x+30x^2$

11) $-56x^2y^2+35x^2y+7x^2-14$

12) $40m^2n^{11} + 30mn^8$

13) $-24x^8y - 42x^3y^2$

14) $2x^2zy + 2x^2z$

15) $-45x^4z^6 - 27x^2z^3y^4 - 45x^3z^3 + 90xz^3y^2$

16) $-32mpq^2 - 72p^2q + 32mq^2 + 72m$

17) $-8b - b^3 + 5b^5 + 10b^6$

18) $2x^4 + 10x^2$

19) $12v^6 + 18v^2u - 4v^3$

Factor each completely.

20) $v^2 + 19v + 90$

21) $k^2 + 13k + 36$

22) $k^2 + k - 12$

23) $n^2 - 2n - 80$

24) $b^2 + 3b - 70$

25) $n^2 - 14n + 48$

Snow Day Packet 14

Name______________________________

Factoring Form, Common Factors, and when a=1 Quiz

Date________________

Solve each equation by factoring.

1) $3(n+5)(3n-2)=0$

2) $4(a-5)^2=0$

3) $(3x+5)(5x+3)=0$

4) $4(3x-4)(x+5)=0$

Factor the common factor out of each expression.

5) $9x^7-5x^6-2x^5+3x^4$

6) $35x+49y$

7) $-36xy^5z^3+4x^2y^2z^3$

8) $-63m^3n^4p^2+27m^4$

Factor each completely.

9) x^2+x-42

10) $4x^2-68x+280$

Snow Day Packet 15 - Algebra 1

Name____________________________

Factoring When Leading Coefficient is Prime

Date______________

Factor each completely.

1) $7r^2 - 15r + 2$

2) $5x^2 - 39x - 54$

3) $5a^2 + 39a - 8$

4) $7a^2 - 60a - 100$

5) $7a^2 + 52a + 60$

6) $7x^2 - 5x - 2$

7) $3x^2 + 19x - 72$

8) $5r^2 - 39r + 54$

9) $3x^2 - 4x + 1$

10) $7n^2 - 34n - 5$

11) $7a^2 + 3a - 10$

12) $3x^2 - 25x + 8$

13) $3m^2 - 20m - 32$

14) $5n^2 + 23n + 12$

15) $3m^2 + m - 24$

16) $2v^2 + 15v + 18$

17) $3k^2 - 17k + 20$

18) $35n^2 - 5n - 40$

19) $7k^2 + 60k - 100$

20) $5b^2 + 31b - 72$

21) $30v^2 - 102v - 72$

22) $10v^2 + 65v - 225$

23) $10x^2 + 65x + 30$

24) $5p^2 + 2p - 7$

25) $30n^2 - 84n - 18$

Snow Day Packet 16 - Algebra 1

Name______________________________

Date_______________

Factoring Specials

Factor each completely.

1) $a^2 - 1$

2) $m^2 - 16$

3) $n^2 - 25$

4) $v^2 - 4$

5) $m^2 - 36$

6) $25v^2 - 81$

7) $4m^2 - 49$

8) $49n^2 - 16$

9) $16r^2 - 49$

10) $9x^2 - 49$

11) $b^2 + 10b + 25$

12) $x^2 - 8x + 16$

13) $b^2 - 6b + 9$

14) $x^2 + 8x + 16$

15) $r^2 + 2r + 1$

16) $4r^2 + 20r + 25$

17) $25x^2 + 40x + 16$

18) $25x^2 - 30x + 9$

19) $16x^2 - 40x + 25$

20) $25x^2 + 20x + 4$

21) $r^2 - 6r + 9$

22) $9n^2 + 30n + 25$

23) $x^2 - 4x + 4$

24) $r^2 - 16$

25) $25k^2 - 9$

Snow Day Packet 17 - Algebra 1

Name______________________________

Review When Leading Coefficient is Prime and Specials

Date________________

Factor each completely.

1) $7n^2 - 39n - 70$

2) $2k^2 - 9k + 7$

3) $2n^2 + 5n + 3$

4) $5b^2 - 36b - 32$

5) $3v^2 - 4v + 1$

6) $14k^2 + 2k - 12$

7) $30x^2 + 6x - 108$

8) $5b^2 - 38b + 21$

9) $6n^2 + 40n + 24$

10) $3v^2 - 26v + 48$

11) $a^2 - 1$

12) $x^2 - 144$

13) $p^2 - 25$

14) $9x^2 - 1$

15) $49b^2 - 1$

16) $16x^2 - 25$

17) $x^2 + 2x + 1$

18) $k^2 + 10k + 25$

19) $b^2 + 6b + 9$

20) $4n^2 + 20n + 25$

21) $m^2 - 2m + 1$

22) $16x^2 + 40x + 25$

23) $9a^2 - 1$

24) $16x^2 + 40x + 25$

25) $4n^2 - 12n + 9$