Algebra II Curriculum Guide
Tier 1 & 2

Unit 3: Rational Functions and Equations
January 20 – February 10

ORANGE PUBLIC SCHOOLS 2016 - 2017

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

Contents

Unit Overview

Unit 3: Rational Functions and Equations

Overview

This course uses Agile Mind as its primary resource, which can be accessed at the following URL:

- www.orange.agilemind.com

Each unit consists of 1-3 topics. Within each topic, there are "Exploring" lessons with accompanying activity sheets, practice, and assessments. The curriculum guide provides an analysis of teach topic, detailing the standards, objectives, skills, and concepts to be covered. In addition, it will provide suggestions for pacing, sequence, and emphasis of the content provided.

Essential Questions

- What is rational function?
- What is the parent function of a rational function?
- What is inverse proportion?
- How can you model with a rational function?
- To simplify the nth root of an expression, what must be true about the expression?
- When you square each side of an equation, is the resulting equation equivalent to the original?
- How are a function and its inverse function related?

Enduring Understandings

- A rational function is a function formed by the quotient of two polynomials.
- Parent function of a rational function is 1/x
- Inverse proportion is when one value decreases at the same rate that the other increases.
- You can model a rational function with a table, equation, and graph.
- Corresponding to every power there is a root
- You can combine like radicals using properties of real numbers
- You can write a radical expression in an equivalent form using a fractional (rational) exponent instead of a radical sign
- Solving a square root equation may require that you square each side of the equation. This process can introduce extraneous solutions

Common Core State Standards ***(NJSLS/CCSS)***

1) A-APR D.6: Rewrite rational expressions 6. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.

2) A.APR.D.7: (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

3) A-CED A.2: equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

4) F-IF A.1: Understand the concept of a function and use function notation 1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). B. Interpret functions that arise in applications in terms of the context

5) F-IF B.4: Interpret functions that arise in applications in terms of the context. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; ~~relative maximums and minimums~~; ~~symmetries;~~ end behavior; and ~~periodicity.~~

6) F-IF B.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

~~7)~~ F-IF.7b: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. b. Graph square root, ~~cube root, and piecewise-defined functions, including step functions and absolute value functions.~~

8) F-BF B.3 Build new functions from existing functions 3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

9) A-CED. A.1: Create equations that describe numbers or relationships 1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

10) A-CED.A3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non- viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. Reasoning with Equations and Inequalities

11) A-REI A.2: Understand solving equations as a process of reasoning and explain the reasoning 2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

12) A.RE.D.11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Major Content
Supporting Content
Additional Content
~~Parts of standard not contained in this unit~~
Algebra I Content

21st Century Career Ready Practice

CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP3. Attend to personal health and financial well-being.
CRP4. Communicate clearly and effectively and with reason.
CRP5. Consider the environmental, social and economic impacts of decisions.
CRP6. Demonstrate creativity and innovation.
CRP7. Employ valid and reliable research strategies.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP10. Plan education and career paths aligned to personal goals.
CRP11. Use technology to enhance productivity.
CRP12. Work productively in teams while using cultural global competence.

Calendar

January 2017						
Sun	Mon	Tue	Wed	Thu	Fri	Sat
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

Algebra II Unit 3

Assessment Framework

Assessment	Assignment Type	Grading	Source	Estimated in-class time	When?
Diagnostic Assessment *Unit 3 Diagnostic*	Test	Traditional (zero weight)	Curriculum Dept. created – see Dropbox	< ½ block	Beginning of unit
Mid-Unit Assessment	Test	Traditional	Teacher created using "Assessments" in Agile Mind	½ to 1 block	Mid unit (optional, must have 3 tests per MP)
End of Unit Assessment *Unit 3 Assessment*	Test	Traditional	Curriculum Dept. created – distributed at end of unit	1 block	End of unit
Performance Task *Unit 3 Performance Task1*	Authentic Assessment	Rubric	Topic constructed response (also see Dropbox)	½ block	In topic 8
Performance Task *Unit 3 Performance Task2*	Authentic Assessment	Rubric	Topic constructed response (also see Dropbox)	½ block	In topic 9
Quizzes	Quiz	Rubric or Traditional	Teacher created or "Practice" in Agile Minds	< ½ block	Varies (must have 3 quizzes per MP)

Overview			
Topic	Name	Agile Mind "Blocks"*	Suggesting Pacing
8	**Rational Functions**	**3**	**2 to 3 days**
9	**Rational Equations**	**1**	**2 days with supplements**
10	**Square Root Function and Equation**	**n/a**	**1 day**

Scope and Sequence

Diagnostic Assessment	**½ day**
Transition lesson	**½ - 1 day**
Mid Unit Assessment	**1 day**
End of Unit Assessment	**1 day**
Performance Task 1	**½ day**
Performance Task 2	**½ day**
Review	**1 day**
Total	**11 days**

*1 Agile Mind Block = 45 minutes

Topic 8: Rational functions

Topic Objectives (Note: these are not in 3-part or SMART objective format)

1. Define rational function and create rational functions.
2. Interpret models of rational functions.
3. Demonstrate transformation of functions on rational functions using parameter changes.

Focused Mathematical Practices

- MP 2: Reason abstractly and quantitatively
- MP4: Model with mathematics
- MP 5: Use appropriate tools strategically
- MP 6: Attend to precision
- MP7: Look for and make sense of structure

Vocabulary
Rational functions, Parent function, Domain, Range.

Fluency

- Computing with percent
- Rates and Ratios
- Writing linear functions with one variables
- Domain and Range

Suggested Topic Structure and Pacing

day	Objective(s) covered	Agile Mind "Blocks" (see Professional Support for further lesson details)	MP	Additional Notes
Day 1	1 & 2	*Block 1 & 2*	2,4,7	Overview: "Rational Function" Pages 1 - 4 Explore: "Modeling with rational functions" page 1 – 5
Day 2	1 & 2 & 3	*Block 3 & 4*	2, 4, 7	Explore: "Modeling with rational functions" pages 6 – 12

CCSS	**Concepts** What students will know	**Skills** What students will be able to do	Material/Resource
~~1)~~ A-CED A.1: Create equations and inequalities in one variable and use them to solve problems. ~~*Include equations arising from linear and quadratic functions, and* simple rational *and exponential functions.*~~ 2) F-BF B.3 Build new functions from existing functions 3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	***Day 1*** ***Review*** • Definition of rates ratio, Definition of linear parent function ***New*** • Definition of Rational function • Definition of vertical asymptote	***Day 1*** ***Review*** • Writing rates and ratios ***New*** • Create rational function • Transform rational functions • Identify vertical asymptote	***Day 1*** Agile Mind Topic 8 *** Overview** **P 1 - 4** *** Exploring** "Modeling with rational function" **P 1 – 5** Suggested assignment: SAS 1 Q7 a – b and 8a – c Guided Practice page 1-5
1) F-BF B.3 Build new functions from existing functions 3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for 2) A-CED.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	***Day 2 (concept)*** ***Review*** • Concept of mixtures • Concept of part to whole • Definition of domain & range ***New*** • Rational function • Definition of the parent rational function	***Day 2 (skills)Review*** • Writing ratios with part to whole • Transformation with any function ***New*** • Writing ration function for a mixture	***Day 2 (Material)*** Agile Mind Topic 8 *** Exploring** ***Modeling with rational functions"** **P 6– 12** More Practice page 1-2 Suggested assignment: SAS 2 Q8 and 9a – c

Topic 9: Rational Equation

Topic Objectives (Note: these are not in 3-part or SMART objective format)

1. write rational equations to model problem situation
2. Solve rational equations using graphs, tables, and analytic strategies
3. Identify extraneous solutions.

Focused Mathematical Practices

- MP 2: Reason abstractly and quantitatively
- MP4: Model with mathematics
- MP7: Look for and make sense of structure

Vocabulary: Rational Equations, Extraneous solutions

Fluency

- Solving proportion
- Solving linear equations with variables on both sides of the equal sign
- Creating linear equations with one variable
- Computing rates in terms of distance and time

Suggested Topic Structure and Pacing

Day	Objective(s) covered	Agile Mind "Blocks" (see Professional Support for further lesson details)	MP	Additional Notes
1	1, 2	*Block 2*	2, 4, 7	Exploring "Solution Methods" Pages 6 – 11

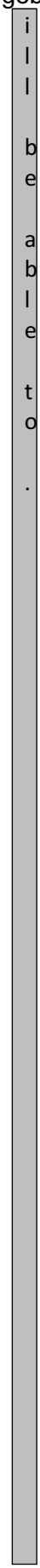
CCSS	**Concepts** What students will know	**Skills** What students will be able to do	Material/Resource
1) A-REI A.2: Understand solving equations as a process of reasoning and explain the reasoning 2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. 2) A-CED. A.1: Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.	***Day1 (Concept)*** ***Review:*** • Definition of rates • Definition of proportion ***New*** • Definition of rational equations • Definition of extraneous solution	***Day 1 (Skills)*** ***Review*** • Computing rates using distance and time • Solving equations with variables on both ides • Writing proportions • Solving proportions ***New*** • Creating rational equations • Solving rational equations	**Day 1 (Material)** Agile Mind Topic 9 *** Exploring** "Rational Equations " P 6-11 SAS 2 Suggested assignment: SAS 2 Q19a-c More Practice 1

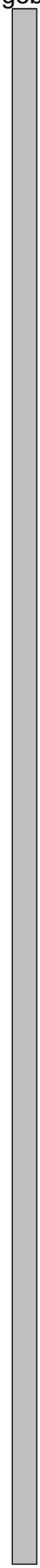
Topic 10: Square root functions and Equa

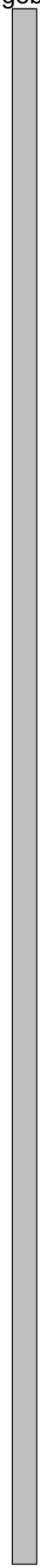
tion
Topic Objectives (Note: these are not in 3-par

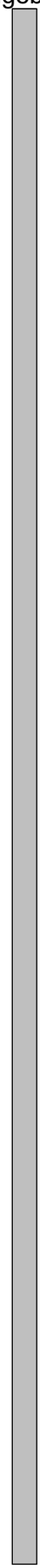
to or SMART objective format) After completing the to

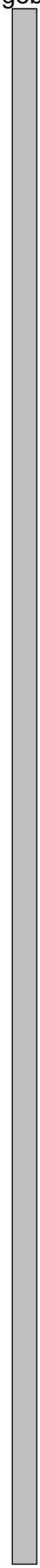
pic square root functions and equations, students w

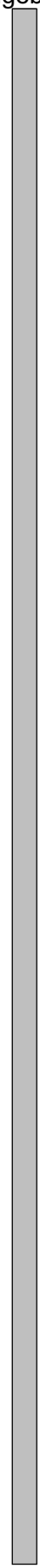

i
l
l
b
e
a
b
l
e
t
o
.

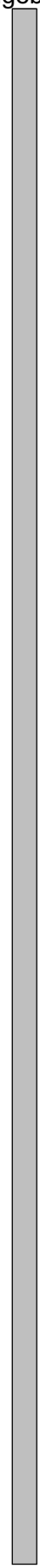


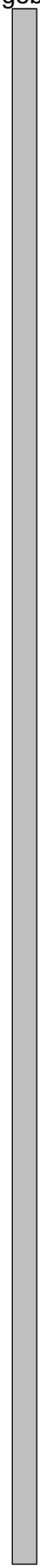


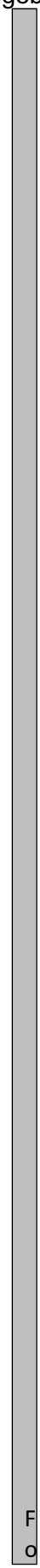











F
o

Used Mathematical Practices

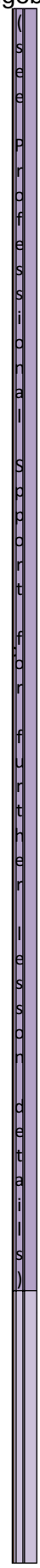
Vocabulary Quadrati

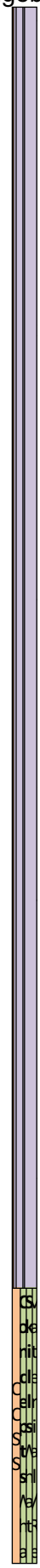
cformula, Imaginary numbers, complex numbers, discr

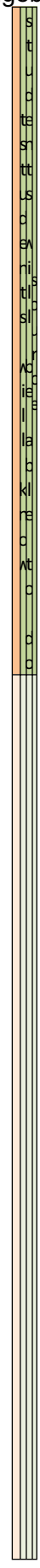
imiinaant, real roots and complex roots Fluency

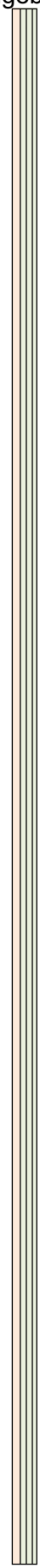
Suggested Topic Structure and Pacing

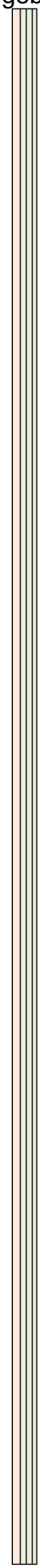
Suggested Topic Structure and Pacing	Additional Notes

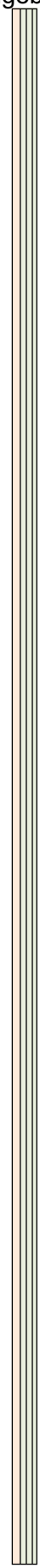

(see professional development for lesson details)

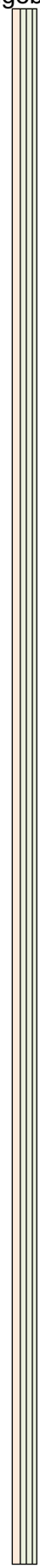


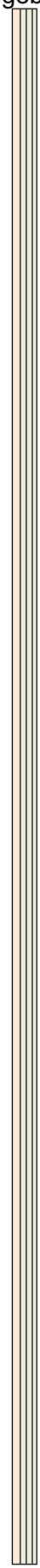


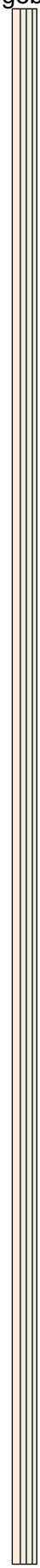


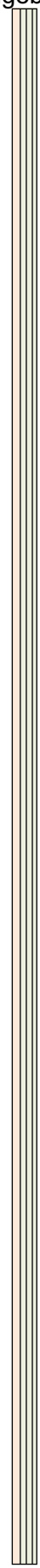


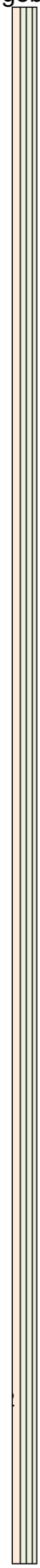


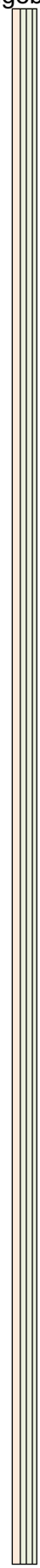


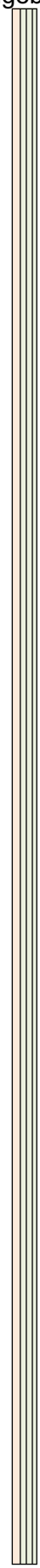


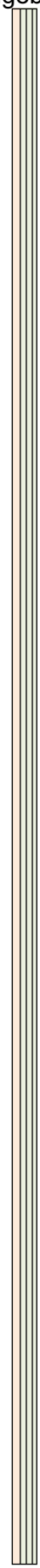


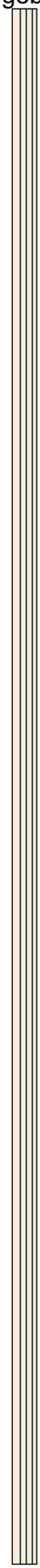


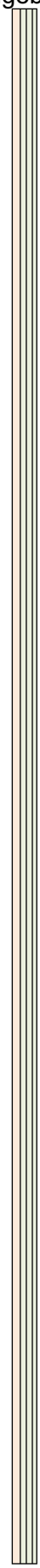


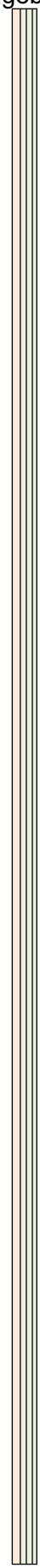


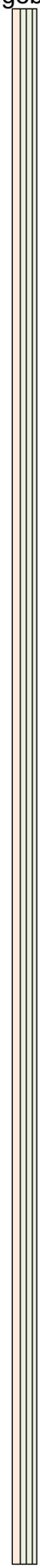


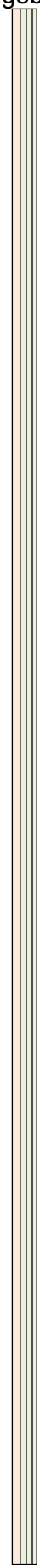


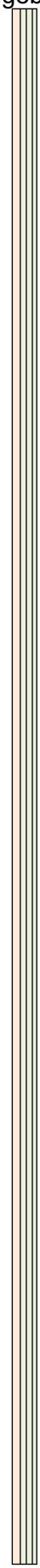


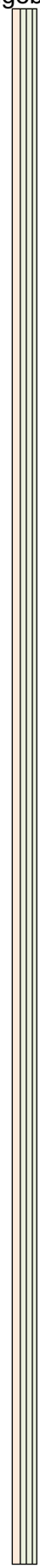


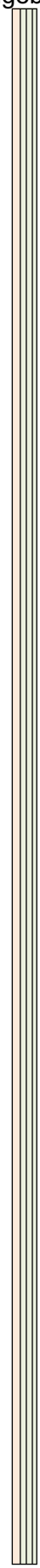


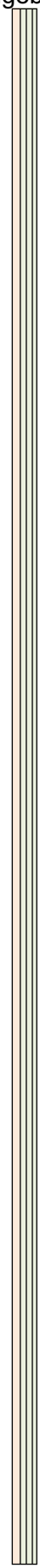












Ideal Math Block

The following outline is the department approved ideal math block for grades 9-12.

1) Fluency Practice (5 min) (see focused fluency skills in each curriculum unit plan)

2) Do Now (7-10 min)
   a. Serves as review from last class' or of prerequisite material
   b. Provides multiple entry points so that it is accessible by all students and quickly scaffolds up

3) Starter/Launch (5 min)
   a. Designed to introduce the lesson
   b. Uses concrete or pictorial examples
   c. Attempts to bridge the gap between grade level deficits and rigorous, on grade level content
   d. Provides multiple entry points so that it is accessible by all students and quickly scaffolds up

4) Mini-Lesson (15-20 min)
   a. Design varies based on content
   b. May include an investigative approach, direct instruction approach, whole class discussion led approach, etc.
   c. Includes CFU's
   d. Anticipates misconceptions and addresses common mistakes

5) Class Activity (25-30 min)
   a. Design varies based on content
   b. May include partner work, group work/project, experiments, investigations, game based activities, etc.

6) Independent Practice (7-10 min)
   a. Provides students an opportunity to work/think independently

7) Closure (5-10 min)
   a. Connects lesson/activities to big ideas
   b. Allows students to reflect and summarize what they have learned
   c. May occur after the activity or independent practice depending on the content and objective

8) DOL (5 min)

a. Exit slip

Sample Lesson Plan (Agile Mind)

Lesson	Topic 8 Rational Functions Exploring "Modeling with Rational Functions"	**Days**	1
Objective	By using the concept of a quiz grade SWBAT • Define rational function and create rational functions. • Interpret models of rational functions.	**CCSS**	A.CED.A1
Learning activities/strategies	**Materials needed:** Computer with projection device, transparency to insert the activity sheets, and activity sheets **Fluency Practice:** (5 minutes) A builder could get 6 sheets of sheetrock for $9. If he bought 12 sheets, how much money would he have spent? **Do Now (5 minutes):** If you scored 16/25, 20/25, 10/25, and 18/25 on 4 different quizzes in your math class, What must you score on the 5th quiz in order to achieve an average grade of 80% **Starter/Launch (3 minutes):** • Students will work on the distance verses time table on the student activity sheet problem 1. After they complete the table ask students the following questions • What kind function is this? • What is the appropriate domain for this context? Discuss the characteristics about the graph to prevent it from being exponential or logarithmic. Introduce today's objective. **Mini lesson and practice (20 minutes):** **Note: Every bullets need to be timed.** • Display page 1 from "Overview" and ask students how can they write 3/20 as percent (Two different way) • Show page 2 and give students time to find the grade after the second quiz. [SAS 1, question 2] Play panel 1 as needed to check student work. • Advance to panel 2 of the animation and give students time to make sense of the table shown there. Be sure students understand that the two additional numbers in the first column of the table reflect the cumulative points earned after the third and fourth quizzes respectively. Students can then complete SAS 1 question 3a [SAS 1, question 3a] • If needed, play panel 2 to allow students to check their entries. Then, give students time to write a function that models the data in the table, as prompted in the final caption. [SAS 1, question 3b] Use these questions if students appear to be struggling: ➢ What was staying constant or same in the process column? ➢ What was varying or changing? • Use panel 3 as needed to confirm students' function rules. Ask student: ➢ What does the final grade depends on? • Ask students to graph the function and determine the number of additional points needed for a grade of 90% [SAS 1, question 4]. To confirm their answer show students Page 3 and ask a student to come up to the smart board to move the slider (animation) to verify their answer • Show page 4 to define rational function. Have students complete student activity sheet 1 questions 5 and 6 Have students do a Read Pair Share of the text on page 4. (Read by themselves, then pair up with someone and share what they read and if there is anything they didn't understand)		

	Ask students based on the example if they can give some examples of rational functions • Have students discuss the similarities and differences between Terrence's graph and the graph on page 4. Encourage them to describe any transformations of the parent function they see in Terrence's graph. **Group work/ Partner work (25 minutes)** Show students page 1. Students can individually read the paragraph about building brick wall then pair up with a partner and complete the questions 1 – 6 from the SAS 2 Debrief question 4, 5, and 6 as a class for 5 minutes **Independent Practice (12 minutes):** • Re-inforce SAS 2 Question 9 • Debrief and check for 2 minutes **Closure (2 minutes):** • Ask what is a rational function? How can you model a rational function? What is the parent rational function? What is domain and range of a rational function? And what are some example of rational functions in real life. **DOL (5 minutes):**

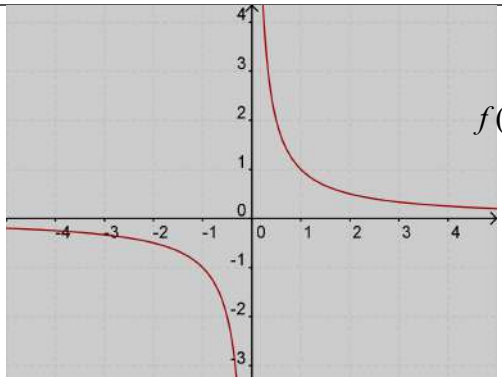
Supplement Materials

Tasks			
CCSS	SMP	Dropbox location and filename	Link (original task and answer key)
		9-12 Dropbox> curriculum algebra 2>Tier1/2 > Unit 1 > Performance Assessment> Task1	

ELL/SWD supplement link

http://nlvm.usu.edu/en/nav/vlibrary.html
http://www.explorelearning.com/index.cfm?method=cResource.dspBrowseCorrelations&v=s&id=USA-000
http://www.thinkingblocks.com/

Multiple Representations

Rational function	
Verbal description	A ***rational function*** is a function formed by the quotient of two **polynomials**.
Function form	$f(x)=\frac{a(x)}{b(x)}$ ***where a(x) and b(x) are polynomial functions***
Parent function (Graph)	 $f(x)=\frac{1}{x}$
Parent function (Table)	x: -3, -2, -1, 0, 1, 2, 3 / y: $-\frac{1}{3}$, $-\frac{1}{2}$, -1, undefined, 1, $\frac{1}{2}$, $\frac{1}{3}$
Function Characteristics	**Domain: {x| x<0} U {x| x>0}**

	Range: {f(x)| f(x) < 0} U {f(x)| f(x) > 0} **Zeros: none** **x-intercept & y-intercept: none** **Decreasing: {x| -∞ < x < 0} U {x| 0 < x < ∞}** **End Behavior: As x approaches - ∞, f(x) approaches 0; as x approaches + ∞, f(x) approaches 0.** **Asymptotes: x = 0, y = 0**
	Rational equation
Verbal description	Equation that contains rational expressions
Equation	$\frac{Distance}{Rate} + \frac{Distance}{Rate} = Time$
Extraneous solution	Extraneous solution is a solution of an equation derived from an original equation that is not a solution of the original equation. When you solve a rational equation or square root equation, it is possible to get extraneous solutions. These values must be eliminated from the solution set. Always check solutions by substituting them into the original equation
Real life example	Jill takes 2 hours to plant 500 flower bulbs. Jack takes 3 hours to plant 450 flower bulbs. Working together, how long should it take them to plant 1500 bulbs $r = \frac{work}{time} = \frac{1500}{time}$ $\frac{500\ flowers}{2\ hr.} + \frac{450\ flowers}{3\ hr.} = \frac{1500}{t}$ $\frac{2400}{6} = \frac{1500}{t}$ t = 3.75 hr.
Square root function and Equation	
Parent function	$f(x) = \sqrt{x}$
Transformed function	$f(x) = a\sqrt{x-h} + k$

Graph of the parent function

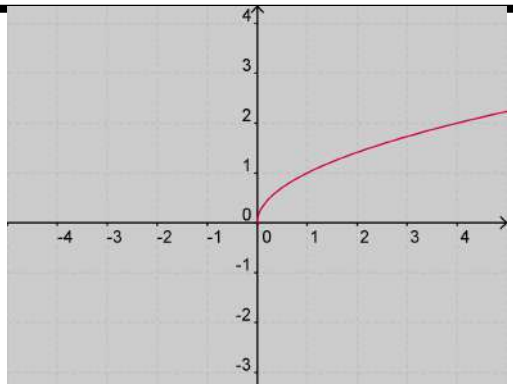


Table of the parent function

x	y
0	0
1	1
2	$\sqrt{2}$
3	$\sqrt{3}$
4	2
5	$\sqrt{5}$

Characteristics

Domain: {x| x ≥ 0 }
Range: {f(x)| f(x) ≥ 0}
Zeros: x=0
x-intercept: (0, 0) **y-intercept:** (0, 0)
Increasing/Decreasing: Increasing on {x| 0 < x < ∞}
End Behavior:
As x approaches + ∞, f(x) approaches + ∞.

Real World Application

The speed of a tsunami is a function of ocean depth:

SPEED = $\sqrt{d \cdot g}$

***g* = acceleration due to gravity (9.81 m/s^2)**

***d* = depth of the ocean in meters**

Understanding the speed of tsunamis is useful in issuing warnings to coastal regions. Knowing the speed can help predict when the tsunami will arrive at a particular location.

PARCC Sample Item

. Solve $\sqrt{a} = a - 6$. What is the extraneous solution?

Enter your answer in the box.

The table shows the steps and explanations that can be used to solve $\sqrt{x} - 5x = -4$.

	Work	Explanation
	$\sqrt{x} - 5x = -4$	Given
Step 1	$\sqrt{x} = 5x - 4$	Addition property of equality
Step 2	$x = 25x^2 - 40x + 16$	Square both sides of the equation
Step 3	$0 = 25x^2 - 41x + 16$	Subtraction property of equality
Step 4	$0 = (25x - 16)(x - 1)$	Factor
Step 5	$(25x - 16) = 0$ or $(x - 1) = 0$	Zero product property
Step 6	$25x = 16$ or $x = 1$	Addition property of equality
Step 7	$x = \frac{16}{25}$ or $x = 1$	Division property of equality