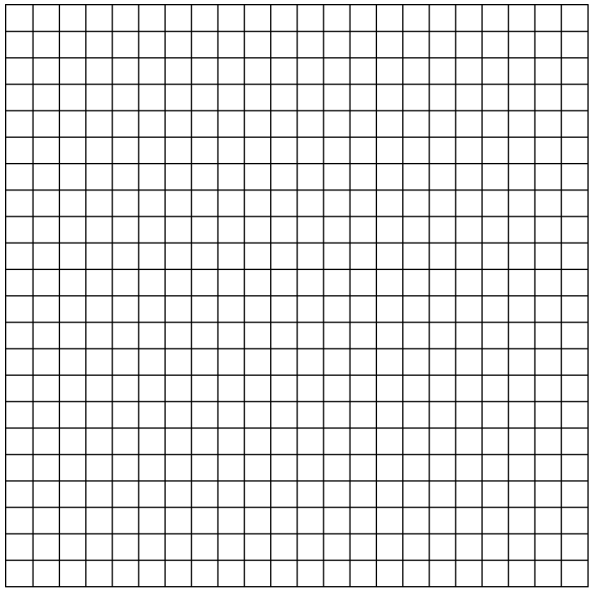
Name _____ Period _____ Date _____

Algebra II Unit 4 Model Curriculum Assessment

1. Use the definition of a parabola as the set of points in the *xy*-plane that are the same distance from a point *F*, called the focus, and a line *d*, called the directrix, to find the equation of a parabola with focus $(0,1)$ and directrix $y = -1$.

2. Find an equation in vertex form for the parabola with focus $(1,4)$ and directrix $x = 5$. Show your work or explain your reasoning.

3. The cost for a phone call on a cruise ship is 65 cents per minute (or part of a minute). Create a graph that shows the total cost, in dollars, for calls between 0 and 10 minutes in length.

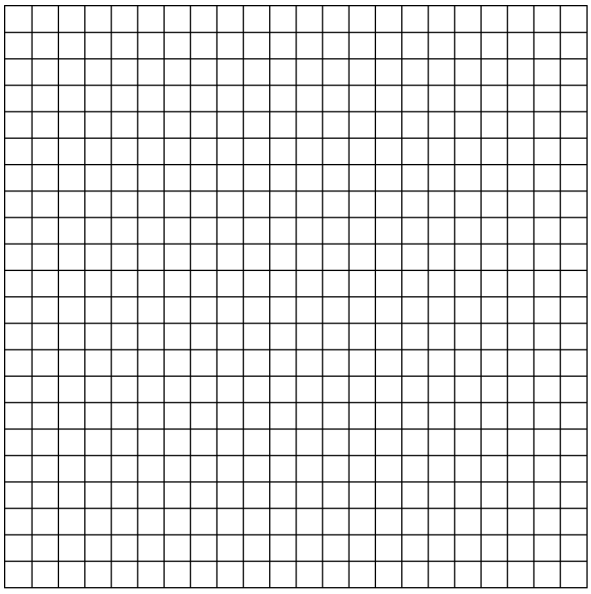


4.

$$h(x)=\begin{cases}3 & 0 \le x < 10\\ \frac{1}{3}(x-1) & 10 \le x < 28\\ 9 & 28 \le x \le 50\end{cases}$$

A person standing at the wall at the shallow end of an empty swimming pool begins walking toward the wall at the deep end of the pool. The height from the bottom to the top of the swimming pool varies depending on the number of feet, x, the person has walked away from the wall at the shallow end. The function $h(x)$ above gives the height, in feet, from the bottom to the top of the pool, where x is measured in feet.

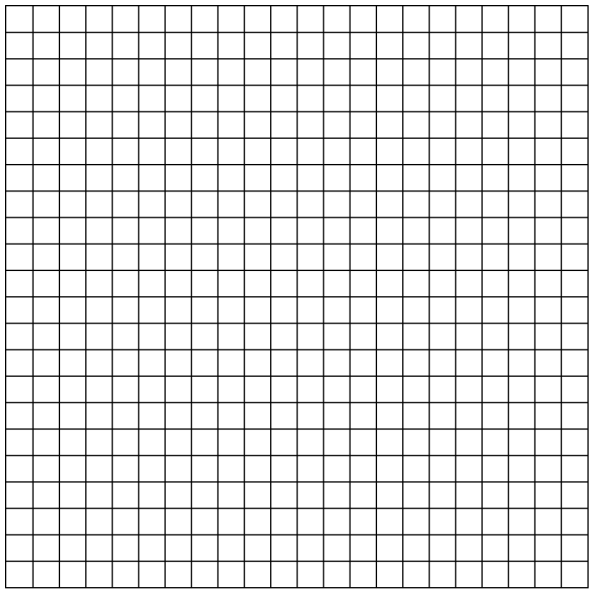
Part A: Graph the function $h(x)$ on the coordinate plane below.



Part B: Describe the change in the height of the pool as the person walks 50 feet from the wall at the shallow end toward the wall at the deep end.

5. A certain bacterial culture grows at a rate that triples the number of bacteria every 2 hours. When the first measurement is made (at time $t = 0$ hours), there are 1,000 bacteria.

Part A: Create a graph to show the number of bacteria present for the 4 hours after the measurement.



Part B: Interpret the intercepts (if any), in context. As t increases, what happens to the number of bacteria?

6. The height above ground, h, of an arrow t seconds after being shot straight up into the air can be modeled by the formula $h(t) = 1.5 + 40t - 4.9t^2$, where t is in seconds and h is in meters. What is the average rate of change of the height, in meters per second, of the arrow over the first three seconds after being shot? Show your work.

7. John placed a container outside during a rainstorm. A gauge on the side of the container shows the height, in millimeters, of the water in the container. The table below lists the height of water the gauge showed along with the corresponding number of hours after the rainstorm started.

Time (hours)	0	2	3	5	7
Height (millimeters)	0	12	16	32	42

What is the average rate of change, in millimeters per hour, of the height of water in the container from time 2 hours to 5 hours?

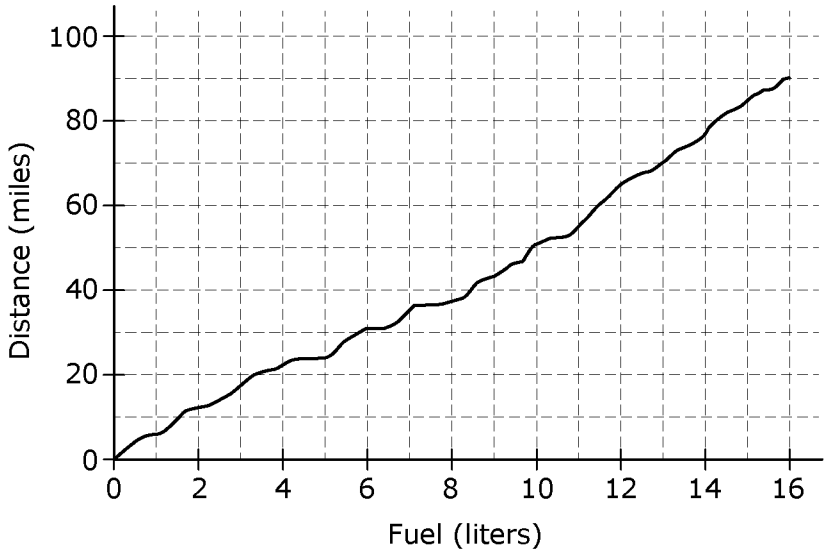
a. $5\frac{1}{3}$

b. 6

c. $6\frac{2}{3}$

d. 8

8.



The graph above shows the distance that a truck travels as a function of the total number of liters of fuel used. The first 6 liters of fuel were used for city driving, while the next 10 liters were used for highway driving. Based on the graph, how does the average miles per liter for city driving compare to the average miles per liter for highway driving?

a. The city average is about 2 miles fewer per liter.
b. The city average is about 1 mile fewer per liter.
c. The city and highway average miles per liter are about the same.
d. The city average is about 1 mile more per liter.

9. A grain silo is in the shape of a right circular cylinder with a hemisphere on top. The volume, V, of the silo is given by $V = \frac{2}{3}\pi r^3 + \pi r^2 h$, where r is the radius of the silo and h is the height of its cylindrical portion. Which of the following is an equivalent form of the volume function that can be used to find a silo's height when its volume and radius are known?

a. $h = \frac{V}{\pi r^2} - \frac{2}{3}r$

b. $h = \frac{V}{\pi r^2} - \frac{2}{3}\pi r^3$

c. $h = \frac{\pi r^2}{V} - \frac{2}{3}r$

d. $h = \frac{\pi r^2}{V} - \frac{2}{3}\pi r^3$

10. The rate, r, of fuel used in a car is related to the efficiency, e, of the engine and the weight, w, of the car by the function $r = \frac{cw}{e}$, where c is a constant.

Part A: Rewrite the function in a form that will allow you to determine the efficiency of an engine, given the car's weight and rate of fuel use.

Part B: Use the result from Part A to fill in the blanks in the statement below to indicate the type of proportionality, direct or inverse, that relates efficiency to weight and rate of fuel use.

Efficiency is _____ proportional to weight, and

_____ proportional to rate of fuel use.

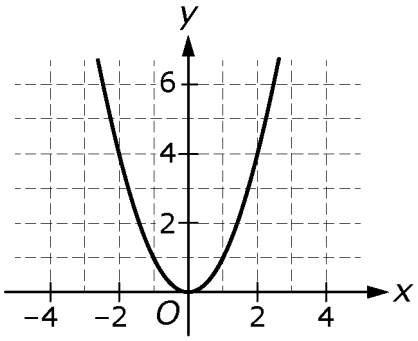
11. Rewrite the equation of the function $f(x) = 3x^2 - 24x + 46$ in a form that emphasizes the location of the vertex of the graph of the function in the coordinate plane. What are the coordinates of the vertex?

12. Compare properties of the following two functions, f and g. Give at least two similarities and two differences.

Function f: The graph of f is a parabola with vertex $(0,0)$ and focus $(4,0)$.

Function g: $y = \frac{1}{8}x^2$

13.



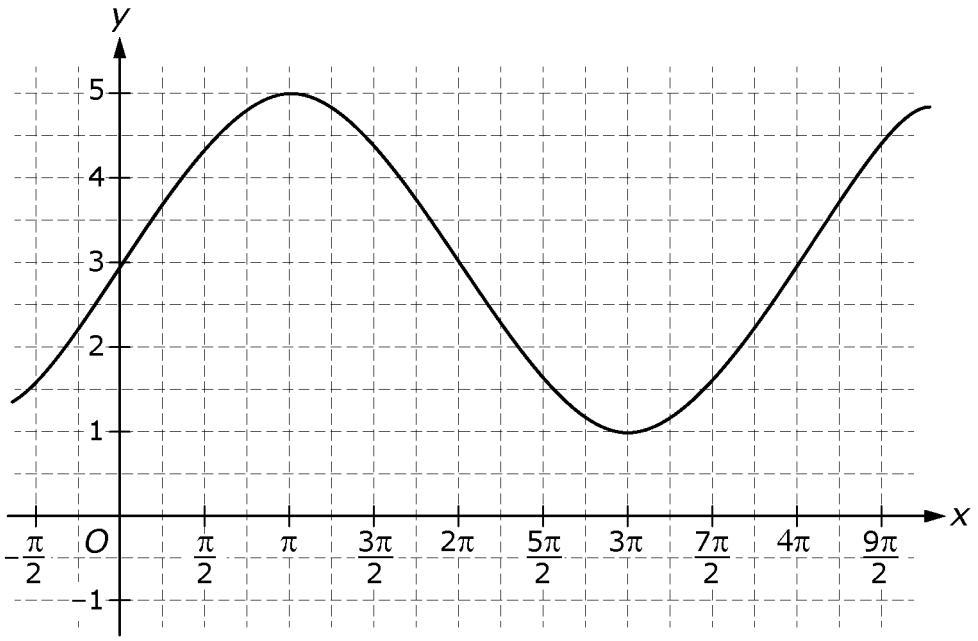
The graph of the quadratic function $g(x)$ is shown in the coordinate plane above, and $f(x) = x^3$. Answer each of the following questions about $f(x)$ and $g(x)$.

Part A: Compute the average rates of change of the two functions on the interval $[0,1]$. Compare the two average rates of change.

Part B: Compute the average rates of change of the two functions on the interval $[1,2]$. Compare the two average rates of change.

Part C: Compare the end behavior of the two functions.

14.



The graph of the function $f(x)$ is shown in the coordinate plane above, and $g(x) = 2\cos(x) + 2$. Answer each of the following questions about $f(x)$ and $g(x)$.

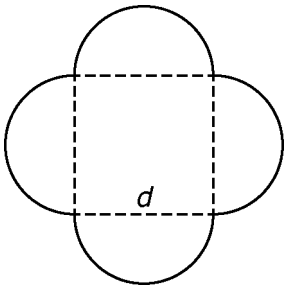
Part A: How do the maximum values of the two functions compare?

Part B: How do the minimum values of the two functions compare?

Part C: How do the amplitudes of the two functions compare?

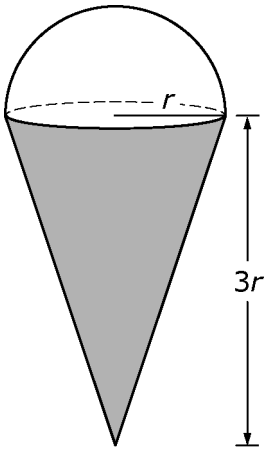
Part D: How do the periods of the two functions compare?

15.



A stained-glass window design is in the shape of a square with semicircles along each side of the square, as shown above. The length of a side of the square is d. Write a function that gives the area of the window design, A, as a function of d.

16.



An ice cream cone is created by packing ice cream into a wafer in the shape of a right circular cone with height $3r$ and base radius r, as shown above. The ice cream forms a hemisphere with radius r that sits on top of the wafer cone. Which of the following functions gives the volume, V, of ice cream both inside and outside the cone as a function of r ?

a. $V(r) = \frac{5}{3}\pi r^3$

b. $V(r) = \frac{7}{3}\pi r^3$

c. $V(r) = \frac{5}{3}\pi^2 r^6$

d. $V(r) = \frac{7}{3}\pi^2 r^6$

17. An object is dropped from a height of 1,000 feet. The distance $s(t)$, in feet, that the object has fallen after t seconds can be modeled by the function $s(t) = 16t^2$.

Part A: Write a function that relates the height of the object above the ground to the time that the object has been falling.

Part B: At what time is the height 0 feet?

18. A cake removed from an oven at 325 degrees Fahrenheit cools off in a 70-degree room. At a time m minutes after being removed, the temperature of the cake is $255e^{-0.2m}$ degrees warmer than the room. Which of the following is a function that gives the temperature, T, of the cake as a function of m ?

a. $T(m) = 325 - 255e^{-0.2m}$

b. $T(m) = 325 + 255e^{-0.2m}$

c. $T(m) = 70 - 255e^{-0.2m}$

d. $T(m) = 70 + 255e^{-0.2m}$

19. The graph of the function $h(x)$ is obtained from the graph of $f(x)$ by shrinking the graph of $f(x)$ vertically by a factor of 5 and reflecting the result over the *y*-axis. Which of the following equations gives $h(x)$ in terms of $f(x)$?

a. $h(x) = 5f(-x)$

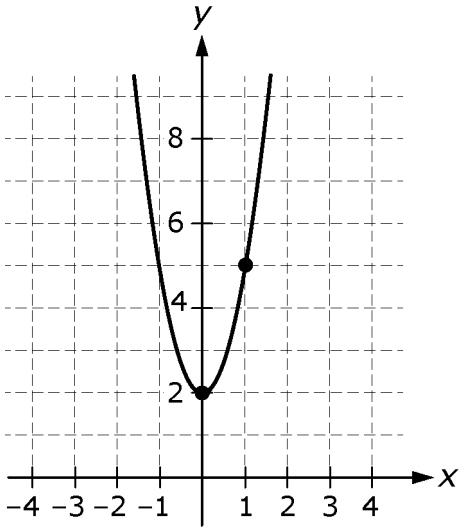
b. $h(x) = \frac{1}{5}f(-x)$

c. $h(x) = -5f(x)$

d. $h(x) = -\frac{1}{5}f(x)$

20. If the graph of $f(x)$ is the top half of a circle with radius 3 centered at the origin, make a graph of $-f(2x)$. Is $-f(2x)$ an even or odd function?

21.



The graph of the function g is shown in the coordinate plane above. If $f(x) = x^2$, and $g(x) = kf(x) + c$, what are the values of k and c ?

a. $k = \frac{1}{3}, c = 2$

b. $k = \frac{1}{3}, c = -2$

c. $k = 3, c = 2$

d. $k = 3, c = -2$

22. What value of x satisfies the equation $2^{3x} = 24$? Write your answer in terms of a logarithm and as a decimal rounded to the nearest thousandth.

23. Which of the following is the solution to the equation $12e^{2x} = 200$?

a. $\ln\left(\frac{100}{3}\right)$

b. $\ln\left(\frac{50}{3}\right)$

c. $\frac{1}{2}\ln\left(\frac{50}{3}\right)$

d. $\ln\left(\frac{25}{3}\right)$

24. Rounded to three decimal places, $\log_2 5$ is approximately equal to 2.322. Use this approximation to solve for x in the equation $40 = 4^x$. Show your work, and round the answer to three decimal places.