DINWIDDIE COUNTY Public Schools

Algebra I

Math Curriculum Guide

Dinwiddie County Public Schools provides each student the opportunity to become a productive citizen, engaging the entire community in the educational needs of our children.

Algebra I Curriculum Guide

- The DCPS Curriculum Guide contains key concepts and SOL numbers for each week. These skill areas must be cross referenced with the DOE Enhanced Scope and Sequence and DOE Curriculum Framework.
- Grade Level(s): 8-9
- Prerequisite: Math 8
- Course Description:

Virginia Department of Education Mathematics SOL Curriculum Framework

Virginia Department of Education Mathematics SOL Standards

Virginia Department of Education Mathematics 2016 SOL Standards - Effective 2018-2019

http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/2009/stds_math7.pdf

8th Grade - Dinwiddie Middle School Pacing

Nine Weeks	Approximate Number of Days Taught	8th Grade - Dinwiddie Middle School Pacing Topic	Targeted SOL
		Note that students who move from Course 2 (7th grade SOL) to Algebra 1 may not have been exposed to the following Course 3 (8th grade SOL) content: *8.2 Real number system* *8.5b Two consecutive whole numbers* *8.10 Pythagorean Theorem* *8.13b Scatterplots* *8.15 all Multi-step equations/inequalities* *8.17 Domain/Range*	
1	4	Expressions **Basic Vocabulary, Order of Operations, Properties, Substitution, Square/Cube Roots**	A.1
1	7	Solving Equations **Formulas for Given Variable, Justifying Steps using Field Properties and Axioms, Multi-Step Linear Equations (algebraically/graphically), Real World Problems**	A.4abdf
1	3	Solving Inequalities **Multi-Step Linear Inequalities (algebraically/graphically),** **Justifying Steps using Axioms and Properties of Order, Real World Problems**	A.5abc
1	2	Data Analysis **Box and Whisker Plots,** **Collect and Analyze Data, Determine the Equation of the Curve in Order to Make Predictions,** **Solve Real World Problems using Models including Linear and Quadratic Functions**	A.10 A.11
1	4	Standard Deviation **Interpret Variation in Real World Context,** **Calculate and Interpret Mean Absolute Deviation, Standard Deviation, and z-Scores**	A.9
1st Cumulative Assessment **November 1-2**			

Nine Weeks	Approximate Number of Days Taught	**8th Grade - Dinwiddie Middle School Pacing** Topic	Targeted SOL
2	3	Investigate and Analyze Function Families **(linear/graphically)** **Relation/Function, Domain and Range, x- and y-Intercepts, Values of a Function, Making Connections Among Multiple Representations of Functions (Concrete, Verbal, Numeric, Graphic, Algebraic)**	A.7
2	10	Graphing Linear Equations and Linear Inequalities **Determine the Slope of a Line** **Writing the Equation of a Line**	A.6
2	4	Variation **Analyze relation of a Real World Situation (direct/inverse)** **Represent a Direct Variation Algebraically/Graphically** **Represent an Inverse Variation Algebraically**	A.8
2	3	Solving Systems **Two Linear Equations in Two Variables (algebraically/graphically)**	A.4e
2nd Cumulative Assessment **January 22-26**			
		Continued	

Nine Weeks	Approximate Number of Days Taught	8th Grade - Dinwiddie Middle School Pacing Topic	Targeted SOL
3	3	Solving Systems **Two Linear Equation and Inequalities in Two Variables (algebraically/graphically)**	A.5d
3	7	Rules of Exponents	A.2a
3	7	Polynomials	A.2b
3/4	6/3	Factoring **First and Second Degree Binomials/Trinomials (one or two variables)** **(Graphing Calculators will be used)**	A.2c
4	4	Quadratics **(algebraically/graphically)**	A.4c
4	3	Radicals **Square Root and Cube Roots of Whole Numbers and** **Square Root of a Monomial Algebraic Expressions** **(expressed in simplest radical form)**	A.3
Mock SOL Test **April 18-19**			
4		**Review, Remediation, and Extension**	
		Algebra 1 SOL Test	

Algebra 1 - Dinwiddie High School Year Pacing

denotes concepts covered simultaneously with other concepts

Nine Weeks	Approximate Number of Days Taught	Algebra 1 - Dinwiddie High School Year Pacing Topic	Targeted SOL
1	12	Expressions **Basic Vocabulary, Order of Operations, Properties, Substitution, Square/Cube Roots**	A.1
1	5	Rules of Exponents* **Multiplication, Division, Negative Exponents, and Properties of Zero**	A.2ab
1	3	Polynomials* **Use Law of Exponents with Polynomial Expressions**	A.2b
1	11	Solving Equations	A.4
1	6	Solving Inequalities	A.5a-c
1	3	Data Analysis **Box and Whisker**	A.10 A.11

Nine Weeks	Approximate Number of Days Taught	**Algebra 1 - Dinwiddie High School Year Pacing** Topic	Targeted SOL
2	**6**	Standard Deviation **(do as warm-ups after assessments)**	A.9
2	**2**	Rules of Exponents* **Division**	A.2a
2	**6**	Relations and Functions	A.7
2	**20**	Linear Equations/Slope	A.6
2	**4**	Variation	A.8

Nine Weeks	Approximate Number of Days Taught	**Algebra 1 - Dinwiddie High School Year Pacing** Topic	Targeted SOL
3	**9**	Systems **Compare and Contrast Solving for Quads vs. Solving for Systems**	A.4e A.5d
3	**15**	Polynomials **GCF, More Complex Addition, Subtraction, and Distribution Problems**	A.2b
4	**8**	Factoring **Link Factors to Zero**	A.2c
4	**8**	Quadratics **Zeros, Solutions, x-Intercepts, and Roots, y-Intercepts**	A.4c
4	**5**	Radicals	A.3
4	**2**	Data Analysis **Line of Best Fit**	A.10 A.11

Algebra 1 - Dinwiddie High School Semester Pacing

**denotes concepts covered simultaneously with other concepts*

Nine Weeks	Approximate Number of Days Taught	Algebra 1 - Dinwiddie High School Semester Pacing Topic	Targeted SOL
1	7	Expressions **Basic Vocabulary, Order of Operations, Properties, Substitution, Square/Cube Roots**	A.1
1	7	Rules of Exponents* **Multiplication,Division, Negative Exponents, and Properties of Zero**	A.2a
1	3	Polynomials* **Addition and Subtraction of Like Terms**	A.2b
1	2	Data Analysis **(do as warm-ups after assessments)** **Variance, Mean, Box and Whiskers, z-Score, and Mean Absolute Deviation**	A.10
1	3	Standard Deviation	A.9
1	7	Solving Equations	A.4b
1	4	Solving Inequalities	A.5abc

Nine Weeks	Approximate Number of Days Taught	**Algebra 1 - Dinwiddie High School Semester Pacing** Topic	Targeted SOL
1	**4**	Relations and Functions	A.7
1	**9**	Linear Equations **Slope**	A.6
1	**2**	Variation	A.8
2	**8**	Systems **Compare and Contrast Solving for Quads vs. Solving for Systems**	A.4e A.5d
2	**4**	Polynomials **GCF, More Complex Addition, Subtraction, and Distribution Problems**	A.2b
2	**10**	Factoring **Link Factors to Zero**	A.2c
2	**6**	Quadratics **Zeros, Solutions, x-Intercepts, and Roots, y-Intercepts**	A.4c

Nine Weeks	Approximate Number of Days Taught	**Algebra 1 - Dinwiddie High School Semester Pacing** Topic	Targeted SOL
2	**3**	Radicals	A.3
2	**3**	Data Analysis **Line of Best Fit**	A.10 A.11

ALGEBRA I SOL TEST QUESTION BREAKDOWN (50 QUESTIONS)
(Based on 2009 SOL Objectives and Reporting Categories)

Expressions and Operations	12 Questions	24 % of the Test
Equations and Inequalities	18 Questions	36 % of the Test
Functions and Statistics	20 Questions	40 % of the Test

Curriculum Information	Essential Knowledge and Skills Key Vocabulary	Essential Questions and Understandings Teacher Notes and Elaborations
SOL Reporting Category Expressions and Operations **Topic** Expressions and Operations **Virginia SOL A.1** The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representations to:** ● **Translate** verbal quantitative situations into algebraic expressions and vice versa. ● **Model** real-world situations with algebraic expressions in a variety of representations (concrete, pictorial, symbolic, verbal). ● **Evaluate** algebraic expressions for a given replacement set to include rational numbers. ● **Evaluate** expressions that contain absolute value, square roots, and cube roots. **Cognitive Level (Bloom's Taxonomy, Revised)** **Analyze** – model **Evaluate** - Evaluate **Key Vocabulary** absolute value algebraic expression cube root negative root positive root square root variable	**Essential Questions** ● What is Algebra? ● How is a variable used in an algebraic expression? ● How are algebraic expressions modeled? ● How is order of operations applied when simplifying and evaluating expressions? **Essential Understandings** ● Algebra is a tool for reasoning about quantitative situations so that relationships become apparent. ● Algebra is a tool for describing and representing patterns and relationships. ● Mathematical modeling involves creating algebraic representations of quantitative real-world situations. ● The numerical values of an expression are dependent upon the values of the replacement set for the variables. ● There is a variety of ways to compute the value of a numerical expression and evaluate an algebraic expression. ● The operations and the magnitude of the numbers in an expression impact the choice of an appropriate computational technique. ● An appropriate computational technique could be mental mathematics, calculator, or paper and pencil. **Teacher Notes and Elaborations** A *variable* is a symbol, usually a letter, used to represent a quantity. This quantity represents an element of any subset of the real numbers. An element, or member, of a set is any one of the distinct objects that make up that set. An *algebraic expression* may contain numbers, variables, operations, and grouping symbols. An algebraic expression may be evaluated by substituting values for the variables in the expression. The numerical values of an expression are dependent upon the values of the replacement set for the variables. The *absolute value* of a number is the distance from 0 on the number line regardless of direction $\left(\text{e.g., } \left\|-\frac{1}{2}\right\| = \frac{1}{2}, \left\|\frac{-1}{2}\right\| = \frac{1}{2}, \left\|\frac{1}{-2}\right\| = \frac{1}{2}, \text{ and } \left\|\frac{1}{2}\right\| = \frac{1}{2}\right)$.

Curriculum Information	Essential Questions and Understandings Teacher Notes and Elaborations
SOL Reporting Category Expressions and Operations **Topic** Expressions and Operations **Virginia SOL A.1** The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables.	**Teacher Notes and Elaborations** (*continued*) In evaluating algebraic expressions, the laws of the order of operations must be followed to find the value of an expression. The *square root* of a number is any number which when multiplied by itself equals the number. Whole numbers have both positive and negative roots. For example, the square root of 25 is 5 and –5 (written as ± 5), where 5 is the *positive root* and –5 is the *negative root*. The inclusion of square roots when evaluating expressions requires students to add, subtract, multiply, and divide radicals. The following are examples of evaluating expressions containing square roots. Example 1: $-11\sqrt{8}-5\sqrt{23}=-16\sqrt{23}$ Example 2: $2\sqrt{2}+5\sqrt{50}=27\sqrt{2}$ Example 3: $2\sqrt{6}\cdot 3\sqrt{7}=6\sqrt{42}$ Example 4: $\frac{\sqrt{20}}{\sqrt{2}}=\sqrt{10}$ The *cube root* of a number, *n*, is a number whose cube is that number. For example, the cube root of 125 is 5 ($\sqrt[3]{125}=5$) because $5^3 = 125$. In general, $\sqrt[3]{n}=a$ if $a^3=n$. (In grade 8 students worked only with perfect squares.) The following are examples of evaluating expressions containing cube roots. Example 1: (Note: Incorrectly, students often cube the 8 and take the square root of *x*.) $8\sqrt[3]{x}-\sqrt{y}$ where $x=64$ and $y=81$. $8\sqrt[3]{64}-\sqrt{81}$ $8\cdot 4-9$ $32-9$ 23 Example 2: (Note: Incorrectly, students divide 125 by 3 instead of taking the cube root. Also students often fail to not close the radicand and take the cube root of $x+y$.) $\sqrt[3]{x}+y$ where $x=125$ and $y=-12$. $\sqrt[3]{125}+(-12)$ $5+(-12)$ -7 Example 3: $(y\sqrt[3]{x})^2$ where $x=512$ and $y=3$. $(3\sqrt[3]{512})^2$ $(3\cdot 8)^2$ $(24)^2$ 576

	(continued)

Curriculum Information	Essential Questions and Understandings Teacher Notes and Elaborations
<u>SOL Reporting Category</u> Expressions and Operations **<u>Topic</u>** Expressions and Operations **<u>Virginia SOL A.1</u>** The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables.	**<u>Teacher Notes and Elaborations</u>** (*continued*) Word phrases, which describe characteristics of given conditions, can be translated into algebraic expressions for the purpose of evaluation. There are multiple ways to translate an algebraic expression into a verbal expression. Experiences should include activities where students write multiple verbal expressions for a given algebraic expression. Example: $5\sqrt[3]{4x}-\sqrt{y}$ Sample responses may include ● Five times the cube root of the product of 4 and *x*, less the square root of *y*. ● The square root of *y*, subtracted from the product of 5 and the cube root of the product of 4 and *x*. There is a variety of methods to compute the value of an algebraic or numerical expression, such as mental math, calculator or paper and pencil methods. Real world situations are problems expressed in words from day to day life. These problems can be understood and represented using manipulatives, pictures, equations/expressions, and in written and spoken language.

Curriculum Information	Resources	Sample Instructional Strategies and Activities
SOL Reporting Category Expressions and Operations **Topic** Expressions and Operations **Virginia SOL A.1** **Foundational Objectives** **8.1** The student will a. simplify numerical expressions involving positive exponents, using rational numbers, order of operations and properties of operations with real numbers; and b. compare and order decimals, fractions, percents, and numbers written in scientific notation. **8.4** The student will apply the order of operations to evaluate algebraic expressions for given replacement values of the variables. **8.5** The student will determine whether a given number is a perfect square. **7.13** The student will a. write verbal expressions as algebraic expressions and sentences as equations and vice versa; and b. evaluate algebraic expressions for given replacement values of the variables. **6.8** The student will evaluate whole number numerical expressions, using the order of operations.	Text: Virginia Algebra I, ©2012, Pearson Education VDOE Enhanced Scope and Sequence Sample Lesson Plans http://www.doe.virginia.gov/testing/sol/scope_sequence/mathematics_2009/index.php Virginia Department of Education Website http://www.doe.virginia.gov/instruction/mathematics/index.shtml VDOE Project Graduation www.doe.virginia.gov/instruction/graduation/project_graduation/index.shtml	• Using groups of three, have one student write a mathematical expression. Have another student write the expression in words. Next, have a third student translate the words back to the expression. Compare the initial and final expressions. If they differ, verbalize each step to determine what was done incorrectly. • Ask students to evaluate a list of algebraic expressions or give values using a calculator, pencil, or mental mathematics. They will then make up four to five expressions of their own to share with other groups. • Have students evaluate expressions using the graphing calculator. Show students how to enter values into the calculator. • Use algeblocks or algebra tiles to physically model substituting values into a variable expression by replacing the variable blocks with the appropriate number of ones. • Write an algebraic expression for students to see. Roll a die to determine the replacement values for each variable in the expression. Students determine the value of the expression.

Curriculum Information	Essential Knowledge and Skills Key Vocabulary	Essential Questions and Understandings Teacher Notes and Elaborations
<u>SOL Reporting Category</u> Expressions and Operations **<u>Topic</u>** Expressions and Operations **<u>Virginia SOL A.2</u>** The student will perform operations on polynomials, including a. applying the laws of exponents to perform operations on expressions; b. adding, subtracting, multiplying, and dividing polynomials; and c. factoring completely first- and second-degree binomials and trinomials in one or two variables. Graphing calculators will be used as a tool for factoring and for confirming algebraic factorizations. *(continued)*	**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representations to:** ● **Simplify** monomial expressions and ratios of monomial expressions, in which the exponents are integers, using the laws of exponents. ● **Model** sums, differences, products, and quotients of polynomials with concrete objects and their related pictorial representations. ● **Relate** concrete and pictorial manipulations that model polynomial operations to their corresponding symbolic representations. ● **Find** sums and differences of polynomials. ● **Find** products of polynomials. The factors will have no more than five total terms. (i.e. $(4x + 2)(3x + 5)$ represents four terms and $(x + 1)(2x^2 + x + 3)$ represents five terms) ● **Find** the quotient of polynomials using a monomial or binomial divisor, or a completely factored divisor. ● **Factor** completely first- and second-degree polynomials with integral coefficients. ● **Identify** prime polynomials. ● **Use** the x-intercepts from the graphical representation of the polynomial to determine and confirm its factors. ● **Express** numbers, using scientific notation, and perform operations, using the laws of exponents. **<u>Cognitive Level</u>** **<u>(Bloom's Taxonomy, Revised)</u>** **Remember** – Find, Simplify, Factor **Understand** – Identify, Use, Express **Apply** – Relate *(continued)*	**<u>Essential Questions</u>** ● What is inductive reasoning? ● What are the laws of exponents? ● How are numbers written in scientific notation computed? ● What is the difference between a monomial and a polynomial? ● How are polynomials added, subtracted, multiplied, and divided? ● How are manipulatives used to model operations of polynomials? ● What methods are used to factor polynomials? ● What is the relationship between the factors of a polynomial and the x-intercepts of the related function? **<u>Essential Understandings</u>** ● The laws of exponents can be investigated using inductive reasoning. ● A relationship exists between the laws of exponents and scientific notation. ● Operations with polynomials can be represented concretely, pictorially, and symbolically. ● Polynomial expressions can be used to model real-world situations. ● The distributive property is the unifying concept for polynomial operations. ● Factoring reverses polynomial multiplication. ● There is a relationship between the factors of any polynomial and the x-intercepts of the graph of its related function. ● Some polynomials are prime polynomials and cannot be factored over the set of real numbers. ● Polynomial expressions can be used to define functions and these functions can be represented graphically. **<u>Teacher Notes and Elaborations</u>** *Inductive reasoning* is a process of reaching a conclusion based on a number of observations that form a pattern. Repeated multiplication can be represented with exponents. The laws of exponents can be used to evaluate algebraic exponential expressions. Each exponential expression (b^n) is made up of a base (b) and an exponent (n). A positive exponent indicates the number of times the base occurs as a factor. a^{-n} is the reciprocal of a^n. Using this definition, the laws of exponents may be developed.

		(continued)

Curriculum Information	Essential Knowledge and Skills Key Vocabulary	Essential Questions and Understandings Teacher Notes and Elaborations
SOL Reporting Category Expressions and Operations **Topic** Expressions and Operations **Virginia SOL A.2** The student will perform operations on polynomials, including a. applying the laws of exponents to perform operations on expressions; b. adding, subtracting, multiplying, and dividing polynomials; and c. factoring completely first- and second-degree binomials and trinomials in one or two variables. Graphing calculators will be used as a tool for factoring and for confirming algebraic factorizations.	**Extension for Algebra I** ● Simplify expressions with fractional exponents. ● Find the quotient of polynomials using a binomial divisor that is not a factor of the dividend. ● Factor third-degree polynomials with at least one monomial as a factor. ● Factor third-degree polynomials with four terms by grouping. ● Factor first and second degree polynomials with rational number coefficients. ● Simplify rational expressions with polynomials in the numerator and or denominator. **Key Vocabulary** binomial inductive reasoning monomial polynomial prime polynomial trinomial x-intercept	**Teacher Notes and Elaborations** The laws of exponents can be investigated using patterns. **Multiplication Rules** **Product of Powers** For any number a and all integers m and n, $a^m a^n = a^{m+n}$ **Power of a Power** For any number a and all integers m and n, $(a^m)^n = a^{mn}$ **Power of a Product** For all numbers a and b, and any integers m, $(ab)^m = a^m b^m$ **Power of a Monomial** For all numbers a and b, and all integers m, p, and n, $(a^m b^n)^p = a^{mp} b^{np}$ **Division Rules** **Quotient of Powers** For all integers m and n, and any nonzero number a, $\frac{a^m}{a^n} = a^{m-n}$ **Power of a Quotient** For all integers m and any nonzero number b, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ **Other Rules** **Zero Exponent** For any nonzero number a, $a^0 = 1$ **Exponential Identity** For any value a, $a^1 = a$ **Negative Exponents** For any nonzero number a and any integer n, $a^{-n} = \frac{1}{a^n}$

Curriculum Information	Essential Questions and Understandings Teacher Notes and Elaborations
SOL Reporting Category Expressions and Operations **Topic** Expressions and Operations **Virginia SOL A.2** The student will perform operations on polynomials, including a. applying the laws of exponents to perform operations on expressions; b. adding, subtracting, multiplying, and dividing polynomials; and c. factoring completely first- and second-degree binomials and trinomials in one or two variables. Graphing calculators will be used as a tool for factoring and for confirming algebraic factorizations.	**Teacher Elaborations** The following are examples of applying the laws of exponents: $\frac{10b^{-4}}{5b^{-6}} = \frac{10}{5} \cdot \frac{b^{-4}}{b^{-6}} = 2 \cdot b^{-4-(-6)} = 2b^2$ $\frac{6a^{-3} \cdot 3a^{10}}{2a^{-4}} = \frac{6 \cdot 3}{2} \cdot \frac{a^{-3} \cdot a^{10}}{a^{-4}} = \frac{18}{2} \cdot \frac{a^{-3+10}}{a^{-4}} = 9 \cdot a^{7-(-4)} = 9a^{11}$ $\left(\frac{x^3y^4}{x^{-2}y^{-4}}\right)^{-1} = \left((x^{3-(-2)})(y^{4-(-4)})\right)^{-1} = \left(x^5y^8\right)^{-1} = \frac{1}{x^5y^8}$

Curriculum Information	Essential Questions and Understandings Teacher Notes and Elaborations
	Polynomials can be represented in a variety of forms. Physical representations such as Algeblocks should be used to support understanding of these concepts. A *monomial* is a constant, a variable, or the product of a constant and one or more variables. A *polynomial* is an expression of two or more terms. A *binomial* is a polynomial of two terms. A *trinomial* is a polynomial of three terms. Polynomials can be added and subtracted by combining like terms.

The following are examples of adding and subtracting polynomials:

$$(y^2 - 7y - 2) + (3y^2 + 8)$$

$$\begin{array}{r} y^2 - 7y - 2 \\ \underline{+\ 3y^2 \qquad + 8} \\ 4y^2 - 7y + 6 \end{array}$$

$$\begin{aligned}(5x - 3y - 2) - (4x - 5) &= 5x - 3y - 2 - 4x + 5 \\ &= (5x - 4x) - 3y + ((-2) + 5) \\ &= x - 3y + 3\end{aligned}$$

Teacher Extension

When adding or subtracting rational expressions with polynomials factors in denominators, a common denominator must be found. To simplify a rational expression, divide out any common factors of the numerator and denominator.

Example 1:

$$\frac{x+1}{x} + \frac{x-3}{3x}$$

$$\frac{3(x+1)}{3x} + \frac{x-3}{3x}$$

$$\frac{3x+3}{3x} + \frac{x-3}{3x}$$

$$\frac{3x+3+x-3}{3x}$$

$$\frac{4x}{3x}$$

$$\frac{4}{3}$$

Example 2:

$$\frac{4}{x^2-16} + \frac{3}{x^2+8x+16}$$

$$\frac{4}{(x+4)(x-4)} + \frac{3}{(x+4)(x+4)}$$

$$\frac{4(x+4)}{(x+4)^2(x-4)} + \frac{3(x-4)}{(x+4)^2(x-4)}$$

$$\frac{4x+16}{(x+4)^2(x-4)} + \frac{3x-12}{(x+4)^2(x-4)}$$

$$\frac{7x+4}{(x+4)^2(x-4)}$$

Example 3

$$\frac{x^2-3x}{x^2-5x+6} \cdot \frac{(x-2)^2}{2x}$$

$$\frac{x(x-3)}{(x-2)(x-3)} \cdot \frac{(x-2)(x-2)}{2x}$$

$$\frac{(x-2)}{2}$$

Curriculum Information	Essential Questions and Understandings Teacher Notes and Elaborations
<u>SOL Reporting Category</u> Expressions and Operations **<u>Topic</u>** Expressions and Operations **<u>Virginia SOL A.2</u>** The student will perform operations on polynomials, including a. applying the laws of exponents to perform operations on expressions; b. adding, subtracting, multiplying, and dividing polynomials; and c. factoring completely first- and second-degree binomials and trinomials in one or two variables. Graphing calculators will be used as a tool for factoring and for confirming algebraic factorizations.	**<u>Teacher Notes and Elaborations</u>** *(continued)* Polynomial multiplication requires that each term in the first expression will be multiplied by each term in the second expression using the distributive property. The distributive property is the unifying concept for polynomial operations. This property is better understood if students can use a physical model to help them develop understanding. The area model of multiplication should be demonstrated and used by students. Physical models to use include Algeblocks or Algebra Tiles. Students should be able to sketch the physical models, and record the process as they progress.

Curriculum Information	Essential Questions and Understandings Teacher Notes and Elaborations

SOL Reporting Category
Expressions and Operations

Topic
Expressions and Operations

Virginia SOL A.2
The student will perform operations on polynomials, including
a. applying the laws of exponents to perform operations on expressions;
b. adding, subtracting, multiplying, and dividing polynomials; and
c. factoring completely first- and second-degree binomials and trinomials in one or two variables. Graphing calculators will be used as a tool for factoring and for confirming algebraic factorizations.

Teacher Notes and Elaborations *(continued)*
The following are examples of multiplying polynomials.

$$\begin{array}{r} 5a^2-3a-7 \\ \underline{3a+2} \\ 10a^2-6a-14 \\ \underline{15a^3-9a^2-21a} \\ 15a^3+a^2-27a-14 \end{array}$$

$$\begin{aligned}(x-4)(2x^2-x+3) &= x(2x^2-x+3)-4(2x^2-x+3) \\ &= 2x^3-x^2+3x-8x^2+4x-12 \\ &= 2x^3-9x^2+7x-12\end{aligned}$$

Area Model of $(2y+7)(-3y^2+4y-8)$

	$-3y^2$	$4y$	-8
$2y$			
$+7$			

→

	$-3y^2$	$4y$	-8
$2y$	$-6y^3$	$8y^2$	$-16y$
$+7$	$-21y^2$	$28y$	-56

→

	$-3y^2$	$4y$	-8
$2y$	$-6y^3$	$8y^2$	$-16y$
$+7$	$-21y^2$	$28y$	-56
$-6y^3$	$-13y^2$	$+12y$	-56

Division of a polynomial by a monomial requires each term of the polynomial be divided by the monomial.

$$\begin{aligned}\frac{4x^4+8x^3y-12x^2y^2}{4x^2} &= \frac{4x^4}{4x^2}+\frac{8x^3y}{4x^2}-\frac{12x^2y^2}{4x^2} \\ &= x^2+2xy-3y^2\end{aligned}$$

To divide a polynomial by a binomial several methods may be used such as factoring, long division, or using an area model. Factoring and simplifying is the preferred method but does not always work.
Factoring Example:

$$\begin{aligned}\frac{x^2+8x+15}{x+3} &= \frac{(x+5)(x+3)}{(x+3)} \\ &= \frac{(x+5)\cancel{(x+3)}}{\cancel{(x+3)}} \\ &= (x+5)\end{aligned}$$

Curriculum Information	Essential Questions and Understandings Teacher Notes and Elaborations

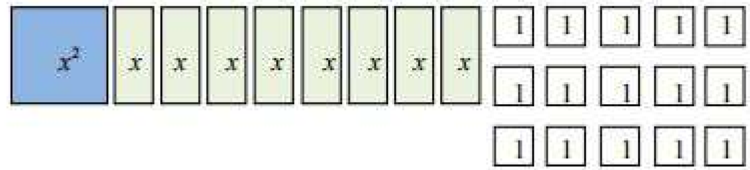
SOL Reporting Category
Expressions and Operations

Topic
Expressions and Operations

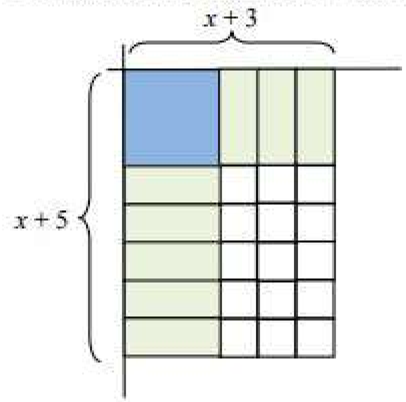
Virginia SOL A.2
The student will perform operations on polynomials, including
a. applying the laws of exponents to perform operations on expressions;
b. adding, subtracting, multiplying, and dividing polynomials; and
c. factoring completely first- and second-degree binomials and trinomials in one or two variables. Graphing calculators will be used as a tool for factoring and for confirming algebraic factorizations.

Teacher Notes and Elaborations *(continued)*

Area Model Example: $\frac{x^2+8x+15}{x+3}$ → Step 1: Model the polynomial $x^2+8x+15$ (the dividend).



Step 2: Place the x^2 tile at the corner of the product mat. Using all the tiles, make a rectangle with a length of $x+3$ (the divisor). The width of the array $x+5$, is the quotient.



Long Division Example:

$$\frac{x^2+8x+15}{x+3} \rightarrow x+3\overline{)x^2+8x+15} \rightarrow x+5$$

$$\begin{array}{r} x+5 \\ x+3\,\overline{)\,x^2+8x+15} \\ \underline{-(x^2+3x)\quad\;\;} \\ 5x+15 \\ \underline{-(5x+15)} \\ 0 \end{array}$$

Curriculum Information	Essential Questions and Understandings Teacher Notes and Elaborations
	<u>Extension for Algebra I</u> When finding the quotient of polynomials, if the polynomial cannot be factored or if there are no common factors by which to divide, long division must be used. **<u>Teacher Notes and Elaborations</u>** *(continued)* Factoring is the reverse of polynomial multiplication. The same models for multiplication can be used to factor. A factor of an algebraic polynomial is one of two or more polynomials whose product is the given polynomial. Some polynomials cannot be factored over the set of real numbers and these are called *prime polynomials*. If the graph of a quadratic function does not cross the x-axis it is a prime polynomial. There is a relationship between the factors of a polynomial and the x-intercepts of its related graph. The *x-intercept* is the point at which a graph intersects the x-axis. Polynomial expressions in a variable x and their factors can be used to define functions by setting y equal to the polynomial expression or y equal to a factor, and these functions can be represented graphically. **Guidelines for Factoring** 1. Factor out the greatest monomial factor first. 2. Look for a difference of squares. 3. Look for a trinomial square. 4. If a trinomial is not a square, look for a pair of binomial factors. 5. If a polynomial has four or more terms, look for a way to group the terms in pairs or in a group of three terms that is a binomial square. 6. Make sure that each factor is prime. Check the work by multiplying the factors. Using a graphical representation of a polynomial, it is possible to determine the apparent factors and to identify the zeros of the function. (Note: Students sometimes confuse the y-intercept and turning point as the zeros of the function.)

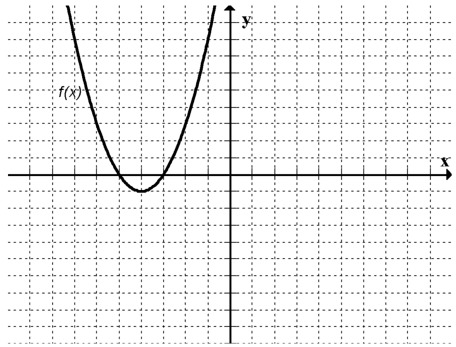
Curriculum Information	Essential Questions and Understandings Teacher Notes and Elaborations

SOL Reporting Category
Expressions and Operations

Topic
Expressions and Operations

Virginia SOL A.2
The student will perform operations on polynomials, including
a. applying the laws of exponents to perform operations on expressions;
b. adding, subtracting, multiplying, and dividing polynomials; and
c. factoring completely first- and second-degree binomials and trinomials in one or two variables. Graphing calculators will be used as a tool for factoring and for confirming algebraic factorizations.

Teacher Notes and Elaborations *(continued)*



The apparent factors of *f(x)* are $(x+3)$ and $(x+5)$.

Factor third-degree polynomials with at least one monomial as a factor such as:

$$3x^3-12x^2+9x$$
$$3x(x^2-4x+3)$$
$$3x(x-3)(x-1)$$

Factor four terms by grouping such as:

$$ay+by+3a+3b$$
$$y(a+b)+3(a+b)$$
$$(a+b)(y+3)$$

Factor completely first and second degree polynomials with rational number coefficients.

$$\frac{1}{4}p^2-2p+4$$
$$\frac{1}{4}\left(p^2-8p+16\right)$$
$$\frac{1}{4}(p-4)^2$$

$$\frac{1}{9}n^2-\frac{1}{25}$$
$$\left(\frac{1}{3}n+\frac{1}{5}\right)\left(\frac{1}{3}n-\frac{1}{5}\right)$$

Curriculum Information	Resources	Sample Instructional Strategies and Activities
SOL Reporting Category Expressions and Operations **Topic** Expressions and Operations **Virginia SOL A.2** **Foundational Objectives** Prime factorization is introduced in elementary school and is applied in middle school. **8.1a** The student will simplify numerical expressions involving positive exponents, using rational numbers, order of operations and properties of operations with real numbers.	Text: Virginia Algebra I, ©2012, Pearson Education VDOE Enhanced Scope and Sequence Sample Lesson Plans http://www.doe.virginia.gov/testing/sol/scope_sequence/mathematics_2009/index.php Virginia Department of Education Website http://www.doe.virginia.gov/instruction/mathematics/index.shtml VDOE Project Graduation www.doe.virginia.gov/instruction/graduation/project_graduation/index.shtml	• Students compare and contrast the different methods for operating on polynomials. • Have each group of students model a polynomial with algeblocks or self-constructed tiles. Allow groups to exchange models and determine the polynomial represented by the other group. • Physical models such as algeblocks or algebra tiles should be used to model factoring. • Graphing calculators can be used demonstrate the connection between *x*-intercepts and factors.

Curriculum Information	Essential Knowledge and Skills Key Vocabulary	Essential Questions and Understandings Teacher Notes and Elaborations
SOL Reporting Category Expressions and Operations **Topic** Expressions and Operations **Virginia SOL A.3** The student will express the square roots and cube roots of whole numbers and the square root of a monomial algebraic expression in simplest radical form.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representations to:** ● **Express** square roots of a whole number in simplest form. ● **Express** the cube root of a whole number in simplest form. ● **Express** the principal square root of a monomial algebraic expression in simplest form where variables are assumed to have positive values. ● **Simplify** the addition, subtraction, and multiplication (not to include the distributive property) of expressions that contain radicals. **Cognitive Level (Bloom's Taxonomy, Revised)** **Remember** – Express, Simplify **Extension for Algebra I** **Post SOL Testing Extension** ● Simplify multiplication (including the distributive property) of expressions that contain radicals. ● Simplify expressions by rationalizing monomial denominators. ● Simplify a radical by rationalizing the denominator by using conjugates. ● Express the cube root of an integer in simplest form. Integers are limited to perfect cubes. **Key Vocabulary** cube root principal square root radical radical expression radicand	**Essential Questions** ● What is a radical? ● How are radical expressions simplified? ● What are the restrictions on the radicands for both square roots and cube roots? **Essential Understandings** ● A square root in simplest form is one in which the radicand (argument) has no perfect square factors other than one. ● A cube root in simplest form is one in which the argument has no perfect cube factors other than one. ● The cube root of a perfect cube is an integer. ● The cube root of a non-perfect cube lies between two consecutive integers. ● The inverse of cubing a number is determining the cube root. ● In the real number system, the argument of a square root must be nonnegative while the argument of a cube root may be any real number. **Teacher Notes and Elaborations** The square root of a number is any number which when multiplied by itself equals the number. Whole numbers have both positive and negative roots. For example, the square root of 25 is 5 and –5 (written as ± 5), where 5 is the positive root and –5 is the negative root. The non-negative square root of a number is called the *principal square root*. Most frequently used perfect squares must be committed to memory to allow for reasonable approximations of non-perfect squares. In Algebra I when finding the principal square root of an expression containing variables the result should not be negative. (All values for variables should be greater than zero.) A root of a number is a *radical* (e.g., $\sqrt{5}$ is called a radical and 5 is the *radicand*). The square root of a whole number is in simplest form when the radicand has no perfect square factors other than one. The *cube root* of a number, n, is a number whose cube is that number. For example, the cube root of 125 is 5 ($\sqrt[3]{125} = 5$) because $5^3 = 125$. 125 is a perfect cube of 5. In general, $\sqrt[3]{n} = a$ if $a^3 = n$. Sometimes the cube root is not a perfect cube. For example, the cube root of 16 is $2\sqrt[3]{2}$ $\left(\sqrt[3]{16} = \sqrt[3]{2^3 \cdot 2} = 2\sqrt[3]{2}\right)$. *(continued)*

Curriculum Information	Essential Questions and Understandings Teacher Notes and Elaborations

SOL Reporting Category
Expressions and Operations

Topic
Expressions and Operations

Virginia SOL A.3
The student will express the square roots and cube roots of whole numbers and the square root of a monomial algebraic expression in simplest radical form.

Teacher Notes and Elaborations *(continued)*

Extension for Algebra I

Negative numbers can also be cubed. If -5 is cubed the result is -125 ($-5\cdot-5\cdot-5=-125$). The cube root of -125 is -5 ($\sqrt[3]{-125}=-5$).

A *radical expression* is in simplest form when all three statements are true.

1. The expression under the radical sign has no perfect square factors other than one.
2. The expression under the radical sign does not contain a fraction.
3. The denominator does not contain a radical expression.

Examples:

$\sqrt{54}+\sqrt{24}+\sqrt{48}$

$\sqrt{9\cdot 6}+\sqrt{4\cdot 6}+\sqrt{16\cdot 3}$

$3\sqrt{6}+2\sqrt{6}+4\sqrt{3}$

$5\sqrt{6}+4\sqrt{3}$

$\left(\sqrt{3x}\right)^4$

$\sqrt{3x}\cdot\sqrt{3x}\cdot\sqrt{3x}\cdot\sqrt{3x}$

$\left(\sqrt{3x}\cdot\sqrt{3x}\right)\cdot\left(\sqrt{3x}\cdot\sqrt{3x}\right)$

$3x\cdot 3x$

$9x^2$

$\sqrt{120x^3y^5z^4}$

$\sqrt{3\cdot 2^2\cdot 2\cdot 5\cdot x^2\cdot x\cdot y^4\cdot y\cdot z^4}$

$\sqrt{3}\cdot\sqrt{2^2}\cdot\sqrt{2}\cdot\sqrt{5}\cdot\sqrt{x^2}\cdot\sqrt{x}\cdot\sqrt{y^4}\cdot\sqrt{y}\cdot\sqrt{z^4}$

$\sqrt{3}\cdot 2\cdot\sqrt{2}\cdot\sqrt{5}\cdot x\cdot\sqrt{x}\cdot y^2\cdot\sqrt{y}\cdot z^2$

$2\cdot x\cdot y^2\cdot z^2\cdot\sqrt{3}\cdot\sqrt{2}\cdot\sqrt{5}\cdot\sqrt{x}\cdot\sqrt{y}$

$2xy^2z^2\sqrt{30xy}$

Extension for Algebra I

Rationalizing a denominator is the process of expressing a fraction with an irrational denominator as an equal fraction with a rational denominator.

For example: $\dfrac{-3}{\sqrt{2}}=\dfrac{-3}{\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}}=-\dfrac{3\sqrt{2}}{2}$

An irrational binomial can sometimes be made rational by multiplying by its conjugate. When rationalizing a denominator, the numerator may remain irrational, though. In order to keep the value of the fraction the same, it is multiplied by the conjugate divided by itself.

<table>
<tr><th>Curriculum Information</th><th>Resources</th><th>Sample Instructional Strategies and Activities</th></tr>
<tr><td>

<u>SOL Reporting Category</u>
Expressions and Operations

<u>Topic</u>
Expressions and Operations

<u>Virginia SOL A.3</u>

<u>Foundational Objectives</u>
8.5
The student will
a. determine whether a given number is a perfect square; and
b. find the two consecutive whole numbers between which a square root lies.
7.1d
The student will determine square roots.
6.5
The student will investigate and describe concepts of positive exponents and perfect squares.

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Text:
<u>Virginia Algebra I</u>, ©2012, Pearson Education

VDOE Enhanced Scope and Sequence Sample Lesson Plans
http://www.doe.virginia.gov/testing/sol/scope_sequence/mathematics_2009/index.php

Virginia Department of Education Website
http://www.doe.virginia.gov/instruction/mathematics/index.shtml

VDOE Project Graduation
www.doe.virginia.gov/instruction/graduation/project_graduation/index.shtml

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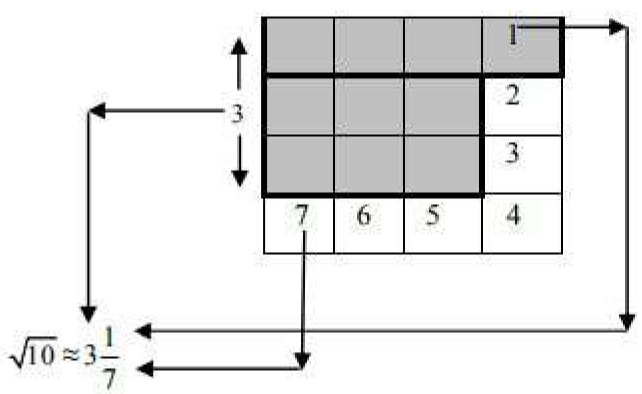
• Make a set of approximately 35 (3×5) index cards with the numbers from 0 to 10 to include $\sqrt{57}$, $3\sqrt{5}$, etc. Stretch a string across the front of the classroom and have students draw a card from the hat and pick a clothespin. (Some students may draw two cards and get two clothespins). Students arrange their numbers on the string from the least to the greatest using a clothespin.
• Students, working in pairs, write numbers on cards and the number's square on other cards. Groups exchange cards and shuffle. Next, students match cards using a format similar to "Concentration".
• Estimate square roots by using 1" tiles
1. Select a number that is not a perfect square.
2. Using paper squares or tiles, students make the largest square possible from the total number of tiles. The length of the side of this square is the whole number part of the solution.
3. The number of paper shapes (tiles) left over when making this square form the numerator of the fraction.
4. The denominator is formed by counting the total number of paper shapes (tiles) necessary to make the next size square.



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Curriculum Information	Essential Knowledge and Skills Key Vocabulary	Essential Questions and Understandings Teacher Notes and Elaborations
<u>SOL Reporting Category</u> Equations and Inequalities **<u>Topic</u>** Equations and Inequalities **<u>Virginia SOL A.4</u>** The student will solve multi-step linear and quadratic equations in two variables, including a. solving literal equations (formulas) for a given variable; b. justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets; c. solving quadratic equations algebraically and graphically; d. solving multi-step linear equations algebraically and graphically; e. solving systems of two linear equations in two variables algebraically and graphically; and f. solving real-world problems involving equations and systems of equations. Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions. *(continued)*	**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representations to:** ● **Solve** a literal equation (formula) for a specified variable. ● **Simplify** expressions and solve equations using the field properties of the real numbers and properties of equality to justify simplification and solution. ● **Solve** multi-step linear equations in one variable. ● **Confirm** algebraic solutions to linear and quadratic equations, using a graphing calculator. ● **Determine** if a linear equation in one variable has one, an infinite number, or no solutions. ● **Solve** quadratic equations. ● **Identify** the roots or zeros of a quadratic function over the real number system as the solution(s) to the quadratic equation that is formed by setting the given quadratic expression equal to zero. ● **Given** a system of two linear equations in two variables that have a unique solution **solve** the system by substitution or elimination to find the ordered pair that satisfies both equations. ● **Given** a system of two linear equations in two variables that has a unique solution, **solve** the system graphically by identifying the point of intersection. ● **Determine** whether a system of two linear equations has one solution, no solution, or infinite solutions. *(continued)*	**<u>Essential Questions</u>** ● How are the field properties and properties of equality of real numbers used to solve equations? ● What is a literal equation? ● How are equations modeled? ● What is a quadratic equation? ● What is the standard form of a quadratic equation? ● What methods are used to solve quadratic equations? ● What is the relationship between the solutions of quadratic equations and the roots of a function? ● What is a system of equations? ● In what instances will there be one solution, no solutions, or an infinite number of solutions for a system of equations? ● What methods are used to solve a system of linear equations? ● How are solutions written in set builder notation? **<u>Essential Understandings</u>** ● A solution to an equation is the value or set of values that can be substituted to make the equation true. ● The solution of an equation in one variable can be found by graphing the expression on each side of the equation separately and finding the x-coordinate of the point of intersection. ● The process of solving linear and quadratic equations can be modeled in a variety of ways, using concrete, pictorial, and symbolic representations. ● Properties of real numbers and properties of equality can be used to justify equation solutions and expression simplification. ● Real-world problems can be interpreted, represented, and solved using linear and quadratic equations. ● Equations and systems of equations can be used as mathematical models for real-world situations. ● Set builder notation may be used to represent solution sets of equations. ● The zeros or the x-intercepts of the quadratic function are the real root(s) or solution(s) of the quadratic equation that is formed by setting the given quadratic expression equal to zero. ● A system of linear equations with exactly one solution is characterized by the graphs of two lines whose intersection is a single point, and the coordinates of this point satisfy both equations. ● A system of two linear equations with no solution is characterized by the graphs of two lines that are parallel. *(continued)*

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<tr><th>Curriculum Information</th><th>Essential Knowledge and Skills
Key Vocabulary</th><th>Essential Questions and Understandings
Teacher Notes and Elaborations</th></tr>
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SOL Reporting Category
Equations and Inequalities

Topic
Equations and Inequalities

Virginia SOL A.4
The student will solve multi-step linear and quadratic equations in two variables, including
a. solving literal equations (formulas) for a given variable;
b. justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets;
c. solving quadratic equations algebraically and graphically;
d. solving multi-step linear equations algebraically and graphically;
e. solving systems of two linear equations in two variables algebraically and graphically; and
f. solving real-world problems involving equations and systems of equations.

Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.

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(continued)
• **Write** a system of two linear equations that models a real-world situation.
• **Interpret** and determine the reasonableness of the algebraic or graphical solution of a system of two linear equations that models a real-world situation.
• **Investigate** and analyze real-world problems to determine the best method to solve a problem.

Cognitive Level (Bloom's Taxonomy, Revised)
Understand - Identify, Interpret
Apply – Solve, Determine, Simplify
Evaluate – Confirm

Extension for Algebra I
• Factor and solve quadratic equations by completing the square.
• Derive the quadratic formula.
• Determine the number of real roots for a quadratic equation using the discriminant.
• Use the equation for the axis of symmetry to graph a quadratic equation.
• Solve a linear system of equations with three or more equations.

Post SOL Testing Extension
• Solve absolute value equations in one variable graphically and algebraically.
• Solve proportions whose elements are monomial and binomial expressions.

Key Vocabulary
coefficient
constant
element
Field Properties: closure, commutative, associative, inverse, identity, and distributive

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Essential Understandings *(continued)*
• A system of two linear equations having infinite solutions is characterized by two graphs that coincide (the graphs will appear to be the graph of one line), and the coordinates of all points on the line satisfy both equations.
• Systems of two linear equations can be used to model two real-world conditions that must be satisfied simultaneously.

Teacher Notes and Elaborations
Real numbers are composed of rational and irrational numbers.

Experiences with solving equations should include simple roots (square and cube) and 2^{nd} and 3^{rd} degree exponents (e.g., $(3x+8)^3-5=22$).

Linear and Literal Equations
In a *linear equation*, the exponent of the variable(s) is one. For example: $x+5=9$ or $y=3x-8$.

A *literal equation* is an equation that shows the relationship between two or more variables. Each variable in the equation "literally" represents an important part of the whole relationship expressed by the equation. To solve a literal equation means to rewrite the equation so a different variable stands alone on one side of the equal sign. That variable must be identified first. Given the literal equation $3x+4y+7z=25-7z$, by applying rules of algebra it is possible to solve for each variable. A *formula* is a special type of literal equation. Experiences should include examples such as the following.

Example 1:
$A=\frac{1}{2}(ah-bh)$ for h
$h=\frac{2A}{(a-b)}$
Note: An upper and lower case letter is used to represent two different numbers.

Example 2:
$Q=3a+5ac$ for a
$a=\frac{Q}{(3+5c)}$
Note: $3a+5ac$ must be factored to isolate a.

The application of solving literal equations will be used when writing equations of lines and solving systems of equations using substitution and elimination.

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</table>

Curriculum Information	Essential Knowledge and Skills Key Vocabulary	Essential Questions and Understandings Teacher Notes and Elaborations
<u>SOL Reporting Category</u> Equations and Inequalities **<u>Topic</u>** Equations and Inequalities **<u>Virginia SOL A.4</u>**	**<u>Key Vocabulary</u>** formula infinite number of solutions linear equation literal equation parabola Properties of Equality: reflexive, symmetric, transitive, substitution, addition, subtraction, multiplication, and division quadratic equation real numbers root(s) set builder notation solution set standard form of a quadratic equation system of equations *x*-intercept zeros of a function	**<u>Teacher Notes and Elaborations</u>** *(continued)* A solution to an equation is the value or set of values (*solution set*) that can be substituted to make the equation true. Experiences should include solving multi-step equations with fractions and variables on both sides of the equal sign. (e.g., $\frac{3}{4}(12x+8)=\frac{2}{3}(6x+9)$). *Set builder notation* is used to represent solutions. For example, if the solution is $y = 10$ then in set notation the answer is written $\{\, y : y = 10 \,\}$ or $\{\, y \mid y = 10 \,\}$. If the solution has no elements, the solution is the empty set (null set) and the set notation is $\varnothing$ or $\{\ \}$. An *element*, or member, of a set is any one of the distinct objects that make up that set. The *coefficient* is the numerical part of a term. A *constant* is a symbol representing a value that does not change. Coefficients and constants as rational numbers will be emphasized. Real-life situations involving literal equations (formulas) will be investigated and solved. The substitution property of equality (*substitution*) states that one name of a number can be substituted for another name of the same number in any expression. Substitution can be used if one statement is replaced with an equivalent one and **<u>no</u>** other property or definition works. In Grade 8 students identified the field properties of real numbers; and addition, subtraction, multiplication, division and substitution properties of equality to solve equations. Each step in the solution of the equation will be justified using the field properties of real numbers and the properties of equality. These properties may be modeled using manipulatives and pictorial representations. - *Properties of Equality*: reflexive, symmetric, transitive, substitution, addition, subtraction, multiplication, and division. - *Field Properties* of Real Numbers: closure, commutative, associative, inverse, identity, and distributive. *(continued)*

Curriculum Information	Essential Questions and Understandings Teacher Notes and Elaborations

SOL Reporting Category
Equations and Inequalities

Topic
Equations and Inequalities

Virginia SOL A.4
The student will solve multi-step linear and quadratic equations in two variables, including
a. solving literal equations (formulas) for a given variable;
b. justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets;
c. solving quadratic equations algebraically and graphically;
d. solving multi-step linear equations algebraically and graphically;
e. solving systems of two linear equations in two variables algebraically and graphically; and
f. solving real-world problems involving equations and systems of equations.

Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.

Teacher Notes and Elaborations *(continued)*

Example:

$\frac{2}{3}x-6=22$	Step 1 – Given equation
$\frac{2}{3}x-6+6=22+6$	Step 2 – Addition property of equality
$\frac{2}{3}x+0=22+6$	Step 3 – Additive inverse
$\frac{2}{3}x=22+6$	Step 4 – Additive identity property
$\frac{2}{3}x=28$	Step 5 – Substitution
$\frac{3}{2}\cdot\frac{2}{3}x=\frac{3}{2}\cdot 28$	Step 6 – Multiplication property of equality
$1x=\frac{84}{2}$	Step 7 – Multiplicative inverse
$x=\frac{84}{2}$	Step 8 – Multiplicative identity
$x=\frac{84}{2}$	Step 9 – Substitution

By transforming given linear equations into simpler forms, the number of solutions can be determined. An example of a linear equation with one solution is $6x-2=x+13$ where $x=3$. An example of a linear equation with no solution is $2x=2x+1$. An example of a linear equation with an *infinite number of solutions* (identity, all real numbers) is $5x+10-2x=3x+10$ where $x=x$, $10=10$, or $0=0$.

Experiences should include translating a real world problem into an equation; determining the graph of the equation and plotting points with integral coordinates; and/or completing a table of values to represent the rule.

Example:
A function is represented by this rule:
One more than one-fourth the square of a number x is y.

Students are asked to plot three points on a grid that are represented by this rule. Each point must have coordinates that are integers.
Sample responses may include $(-6,10)$, $(-2,2)$, $(4,5)$
(Although students are asked for three points, this grid limits the choices.

	(continued)

Curriculum Information	Essential Questions and Understandings Teacher Notes and Elaborations
SOL Reporting Category Equations and Inequalities **Topic** Equations and Inequalities **Virginia SOL A.4** The student will solve multi-step linear and quadratic equations in two variables, including a. solving literal equations (formulas) for a given variable; b. justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets; c. solving quadratic equations algebraically and graphically; d. solving multi-step linear equations algebraically and graphically; e. solving systems of two linear equations in two variables algebraically and graphically; and f. solving real-world problems involving equations and systems of equations. Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.	**Teacher Notes and Elaborations** *(continued)* A *quadratic equation* is an equation that can be written the form $ax^2 + bx + c = 0$ where $a \neq 0$. This form is called the *standard form of a quadratic equation*. The graph of a quadratic equation is a *parabola*. The *zeros of a function* or *the x-intercepts* of the quadratic function are the real *root(s)*/solution(s) of the quadratic equation that is formed by setting the given quadratic expression equal to zero. For example: $f(x) = 5x^2 + 28x - 12$ $f(x) = (x+6)(5x-2)$ if $x+6=0$ or $5x-2=0$, then the roots are -6 and $\frac{2}{5}$ *(continued)*

Curriculum Information	Essential Questions and Understandings Teacher Notes and Elaborations
SOL Reporting Category Equations and Inequalities **Topic** Equations and Inequalities **Virginia SOL A.4** The student will solve multi-step linear and quadratic equations in two variables, including a. solving literal equations (formulas) for a given variable; b. justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets; c. solving quadratic equations algebraically and graphically; d. solving multi-step linear equations algebraically and graphically; e. solving systems of two linear equations in two variables algebraically and graphically; and f. solving real-world problems involving equations and systems of equations. Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions. Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.	**Teacher Notes and Elaborations** *(continued)* **Quadratic Equations** Quadratic equations can be solved in a variety of ways: 1. factoring 2. graphing 3. using the graphing calculator 4. using the quadratic formula If $ax^2 + bx + c = 0$, and $a \neq 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Given the set of real numbers, quadratic equations may have no solutions, one solution, or two solutions. The solution(s) to a quadratic equation make the equation true when the value is substituted into the equation.

	(continued)

Curriculum Information	Essential Questions and Understandings Teacher Notes and Elaborations
SOL Reporting Category Equations and Inequalities **Topic** Equations and Inequalities **Virginia SOL A.4** The student will solve multi-step linear and quadratic equations in two variables, including a. solving literal equations (formulas) for a given variable; b. justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets; c. solving quadratic equations algebraically and graphically; d. solving multi-step linear equations algebraically and graphically; e. solving systems of two linear equations in two variables algebraically and graphically; and f. solving real-world problems involving equations and systems of equations. Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.	**Teacher Notes and Elaborations** **Systems of Equations** A *system of equations* (simultaneous equations) is two or more equations in two or more variables considered together or simultaneously. The equations in the system may or may not have a common solution. A linear system may be solved algebraically by the substitution or elimination methods, or by graphing. Graphing calculators are used to solve, compare, and confirm solutions. A system of linear equations with exactly one solution is characterized by the graphs of two lines whose intersection is a single point, and the coordinates of this point satisfy both equations. A point shared by two intersecting graphs and the ordered pair that satisfies the equations characterizes a system of equations with only one solution. A system of two linear equations with no solution is characterized by the graphs of two lines that do not intersect, they are parallel. A system of two linear equations that has infinite solutions is characterized by two graphs that coincide (the graphs will appear to be the graph of one line), and all the coordinates on this one line satisfy both equations. Systems of two linear equations can be used to represent two conditions that must be satisfied simultaneously. Students will develop an understanding that representations of math ideas (equations, models, etc.) are an essential part of learning, doing, and communicating mathematics. They will engage in extensive problem solving using real-world problems. Instruction should include numerous opportunities to investigate multiple strategies to solve word problems. Students should learn to apply appropriate strategies to find solutions to these problems.

Curriculum Information	Resources	Sample Instructional Strategies and Activities
SOL Reporting Category Equations and Inequalities **Topic** Equations and Inequalities **Virginia SOL A.4** The student will solve multi-step linear and quadratic equations in two variables, including A. solving literal equations (formulas) for a given variable; B. justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets; C. solving quadratic equations algebraically and graphically; D. solving multi-step linear equations algebraically and graphically; E. solving systems of two linear equations in two variables algebraically and graphically; and F. solving real-world problems involving equations and systems of equations. **Foundational Objectives** **8.1a** The student will simplify numerical expressions involving positive exponents, using rational numbers, order of operations and properties of operations with real numbers. **8.2** The student will describe orally and in writing the relationships between the subsets of the real number system. **8.3a** The student will solve practical problems involving rational numbers, percents, ratios, and proportions.	Text: Virginia Algebra I, ©2012, Pearson Education VDOE Enhanced Scope and Sequence Sample Lesson Plans http://www.doe.virginia.gov/testing/sol/scope_sequence/mathematics_2009/index.php Virginia Department of Education Website http://www.doe.virginia.gov/instruction/mathematics/index.shtml VDOE Project Graduation www.doe.virginia.gov/instruction/graduation/project_graduation/index.shtml	• Give students a list of equations to solve graphically. Equations should include multi-step equations, equations with variables on both sides, and equations with distributive property and variables on both sides. Using a graphing calculator, let y_1 equal the left member of the equation and let y_2 equal the right member. Graph in the simultaneous mode. Use calculator functions to determine the point of intersections. Substitute the solution in the equation to check the problem. • Students will use a graphing calculator to find the distance, rate, or time, given two of the unknowns. They are asked to tell how each answer was determined and write the literal equations. Example: $d = rt$; $t = \frac{d}{r}$; $r = \frac{d}{t}$ • Students, working in groups of two or three, will be given a set of cards. The names of the properties will be on one set of colored cards. Several examples of each property will be on cards of a different color. The cards should be shuffled. Students will try to match the examples with the property name. • Groups will be given a list of solved equations and asked to name the property that justifies each step. • Strategy for solving quadratic equations: - Graphing calculators are given to students, who then enter the equation, investigate its graph, and identify the x intercepts as the roots of the equation. - Check the solutions algebraically. Emphasize that quadratic equations may have no real roots, but this does not preclude the equation from having no solution. • Students will be divided into groups. Have each group solve a system of equations by a prescribed method. Make sure that all methods are assigned. Have students display their solutions to the class and discuss the most appropriate method for solving the system. • Given an equation such as $3x + 4y = 12$ find two or more equations that satisfy each of these requirements. 1. The graphs of the given equation and a second equation intersect at a single point. 2. The graphs of the given equation and a second equation intersect at an infinite number of points. •

Curriculum Information	Resources	Sample Instructional Strategies and Activities
<u>SOL Reporting Category</u> Equations and Inequalities **<u>Topic</u>** Equations and Inequalities **<u>Virginia SOL A.4</u>** **<u>Foundational Objectives</u>** **7.14** The student will a. solve one- and two-step linear equations in one variable; and b. solve practical problems requiring the solution of one- and two-step linear equations. **7.16** The student will apply the following properties of operations with real numbers: a. the commutative and associative properties for addition and multiplication; b. the distributive property; c. the additive and multiplicative identity properties; d. the additive and multiplicative inverse properties; and e. the multiplicative property of zero. **6.11** The student will a. identify the coordinates of a point in a coordinate plane; and b. graph ordered pairs in a coordinate plane. **6.18** The student will solve one-step linear equations in one variable involving whole number coefficients and positive rational solutions. **<u>Foundational Objectives</u>** **6.19** The student will investigate and recognize *(continued)* a. the identity properties for addition and multiplication; b. the multiplicative property of zero; and c. the inverse property for multiplication.		

<table>
<tr><th>Curriculum Information</th><th>Essential Knowledge and Skills
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Teacher Notes and Elaborations</th></tr>
<tr><td>
<u>SOL Reporting Category</u>
Equations and Inequalities

<u>Topic</u>
Equations and Inequalities

<u>Virginia SOL A.5</u>

The student will solve multi-step linear inequalities in two variables, including
a. solving multi-step linear inequalities algebraically and graphically;
b. justifying steps used in solving inequalities, using axioms of inequality and properties of order that are valid for the set of real numbers and its subsets;
c. solving real-world problems involving inequalities; and
d. solving systems of inequalities.
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The student will use problem solving, mathematical communication, mathematical reasoning, connections and representations to:
● Solve multi-step-linear inequalities in one variable.
● Justify steps used in solving inequalities, using axioms of inequality and properties of order that are valid for the set of real numbers.
● Solve systems of linear inequalities algebraically and graphically.
● Solve real world problems involving inequalities.

<u>Cognitive Level</u> <u>(Bloom's Taxonomy, Revised)</u>
Apply – Solve
Evaluate - Justify

<u>Extension for Algebra I</u>
● Solve compound inequalities.
● Write solution sets for inequalities in interval notation.

<u>Key Vocabulary</u>
addition property of inequality
division property of inequality
inequality
multiplication property of inequality
set builder notation
subtraction property of inequality
systems of inequalities
</td><td>
<u>Essential Questions</u>
● How are the properties of real numbers used to solve inequalities?
● What is the same about solving equations and solving inequalities and what is different?
● How are the solutions of systems of linear inequalities the same or different from the solutions of systems of equations?
● How are solutions written in set builder notation?

<u>Essential Understandings</u>
● A solution to an inequality is the value or set of values that can be substituted to make the inequality true.
● Real-world problems can be modeled and solved using linear inequalities.
● Properties of inequality and order can be used to solve inequalities.
● Set builder notation may be used to represent solution sets of inequalities.

<u>Teacher Notes and Elaborations</u>
An inequality is a statement that one quantity is less than (or greater than) another. The nature of the inequality is not changed when adding or subtracting any real number or when multiplying or dividing by a positive real number. However, when multiplying or dividing by negative numbers you must reverse the inequality symbol to maintain a true statement. A solution to an inequality is the value or set of values, which can be substituted to make the inequality true. A multi-step inequality will involve the combination of two or more operations. Each step in the solution of the inequality will be justified using the axioms of inequality. An axiom is a statement universally recognized as true without proof.

The addition property of inequalities states that if the same number is added to each side of a true inequality, the resulting inequality is also true.
For any numbers a, b, and c the following are true,
1. if $a > b$, then $a + c > b + c$.
2. if $a < b$, then $a + c < b + c$.
If $x - 2 > 4$, then $x - 2 + 2 > 4 + 2$ and if $x - 2 < 4$, then $x - 2 + 2 < 4 + 2$.

The subtraction property of inequalities states that if the same number is subtracted from each side of a true inequality, the resulting inequality is also true.
For any numbers a, b, and c the following are true.
1. if $a > b$, then $a - c > b - c$.
2. if $a < b$, then $a - c < b - c$.
If $x + 2 > 4$, then $x + 2 - 2 > 4 - 2$ and if $x + 2 < 4$, then $x + 2 - 2 < 4 - 2$.
</td></tr>
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Curriculum Information	Essential Questions and Understandings Teacher Notes and Elaborations
SOL Reporting Category Equations and Inequalities **Topic** Equations and Inequalities **Virginia SOL A.5** The student will solve multi-step linear inequalities in two variables, including a. solving multi-step linear inequalities algebraically and graphically; b. justifying steps used in solving inequalities, using axioms of inequality and properties of order that are valid for the set of real numbers and its subsets; c. solving real-world problems involving inequalities; and d. solving systems of inequalities	**Teacher Notes and Elaborations** *(continued)* The *multiplication property of inequalities* states - If each side of an inequality that is true are multiplied by a positive number, the resulting inequality is also true. For any real numbers a and b and any positive real number c the following is true. 1. if $a > b$, then $ac > bc$. 2. if $a < b$, then $ac < bc$. If $2x > 4$, then $2x\left(\frac{1}{2}\right) > 4\left(\frac{1}{2}\right)$ and if $2x < 4$, then $2x\left(\frac{1}{2}\right) < 4\left(\frac{1}{2}\right)$. - If both sides of an inequality that is true are multiplied by a negative number, the direction of the inequality sign is reversed to make the resulting inequality also true. For any real numbers a and b and any negative real number c the following is true. 1. if $a > b$, then $ac < bc$. 2. if $a < b$, then $ac > bc$. If $2x > 4$, then $2x(-3) < 4(-3)$ and if $2x < 4$, then $2x(-3) > 4(-3)$. The *division property of inequalities* states - If both sides of a true inequality are divided by a positive number, the resulting inequality is also true. For any real numbers a and b and any positive real number c the following is true. 1. if $a > b$, then $\frac{a}{c} > \frac{b}{c}$. 2. if $a < b$, then $\frac{a}{c} < \frac{b}{c}$. If $2x > 4$, then $\frac{2x}{2} > \frac{4}{2}$ and if $2x < 4$, then $\frac{2x}{2} < \frac{4}{2}$. - If both sides of a true inequality are divided by a negative number, the direction of the inequality sign is reversed to make the resulting inequality also true. For any real numbers a and b and any negative real number c the following is true. 1. if $a > b$, then $\frac{a}{c} < \frac{b}{c}$. 2. if $a < b$, then $\frac{a}{c} > \frac{b}{c}$. If $-3x > 12$, then $\frac{-3x}{-3} < \frac{12}{-3}$ and if $-3x < 12$, then $\frac{-3x}{-3} > \frac{12}{-3}$.

Curriculum Information	Essential Questions and Understandings Teacher Notes and Elaborations
SOL Reporting Category Equations and Inequalities **Topic** Equations and Inequalities **Virginia SOL A.5** The student will solve multi-step linear inequalities in two variables, including a. solving multi-step linear inequalities algebraically and graphically; b. justifying steps used in solving inequalities, using axioms of inequality and properties of order that are valid for the set of real numbers and its subsets; c. solving real-world problems involving inequalities; and d. solving systems of inequalities.	*Systems of inequalities* (simultaneous inequalities) are two or more inequalities in two or more variables that are considered together or simultaneously. The system may or may not have common solutions. Practical problems can be interpreted, represented, and solved using linear inequalities. The solution set of a system of linear inequalities can be determined by graphing each inequality in the same coordinate system. The overlapping region is the graph of the solution set. *Set builder notation* is used to represent solutions. For example, if the solution is the set of all real numbers less than 5 then in set notation the answer is written $\{ x : x < 5 \}$ or $\{ x \mid x < 5 \}$. If the solution has no elements, the solution is the empty set (null set) and the set notation is $\varnothing$ or $\{ \}$. An element, or member, of a set is any one of the distinct objects that make up that set. Graphing can be used to demonstrate that both x < 5 and 5 > x, represent the same solution set. Examples should include solutions with variables on either side.

Curriculum Information	Resources	Sample Instructional Strategies and Activities
<u>SOL Reporting Category</u> Equations and Inequalities **<u>Topic</u>** Equations and Inequalities **<u>Virginia SOL A.5</u>** **<u>Foundational Objectives</u>** **8.2** The student will describe orally and in writing the relationships between the subsets of the real number system. **8.15** The student will a. solve two-step linear inequalities and graph the results on a number line; and b. identify properties of operations used to solve an equation. **7.15** The student will a. solve one-step inequalities in one variable and b. graph solutions to inequalities on the number line. **7.16** The student will apply the following properties of operations with real numbers: a. the commutative and associative properties for addition and multiplication; b. the distributive property; c. the additive and multiplicative identity properties; d. the additive and multiplicative inverse properties; and e. the multiplicative property of zero. *(continued)*	Text: <u>Virginia Algebra I</u>, ©2012, Pearson Education VDOE Enhanced Scope and Sequence Sample Lesson Plans http://www.doe.virginia.gov/testing/sol/scope_sequence/mathematics_2009/index.php Virginia Department of Education Website http://www.doe.virginia.gov/instruction/mathematics/index.shtml VDOE Project Graduation www.doe.virginia.gov/instruction/graduation/project_graduation/index.shtml **<u>Foundational Objectives</u>** *(continued)* **6.19** The student will investigate and recognize a. the identity properties for addition and multiplication; b. the multiplicative property of zero; and c. the inverse property for multiplication. **6.20** The student will graph inequalities on a number line.	• Divide the class into four or five small groups. Have each member create an inequality and then graph its solution set. Have groups switch graphs and identify the inequality that each solution set solves. Have groups return papers and verify each other's answers with the use of a graphing calculator. Equations can also be used in this type of activity. Their solution can be checked with the graphing calculator.

Curriculum Information	Essential Knowledge and Skills Key Vocabulary	Essential Questions and Understandings Teacher Notes and Elaborations
<u>SOL Reporting Category</u> Equations and Inequalities **<u>Topic</u>** Equations and Inequalities **<u>Virginia SOL A.6</u>** The student will graph linear equations and linear inequalities in two variables, including a. determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined; and b. writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line. *(continued)*	**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representations to:** ● **Graph** linear equations and inequalities in two variables, including those that arise from a variety of real-world situations. ● **Use the** parent function $y = x$ and describe transformations defined by changes in the slope or y-intercept. ● **Find** the slope of a line given the equation of a linear function. ● **Find** the slope of a line given the coordinates of two points on the line. ● **Find** the slope of a line given the graph of a line. ● **Recognize** and describe a line with a slope that is positive, negative, zero or undefined. ● **Use** transformational graphing to investigate effects of changes in equation parameters on the graph of the equation. ● **Write** an equation of a line when given the graph of a line. ● **Write** an equation of a line when given two points on the line whose coordinates are integers. ● **Write** an equation of a line when given the slope and a point on the line whose coordinates are integers. ● **Write** an equation of a vertical line as $x = a$. ● **Write** an equation of a horizontal line as $y = b$.	**<u>Essential Questions</u>** ● What are the appropriate techniques to graph a linear equation in two variables? ● How can a graph of a linear equation be used to represent a real-world situation? ● What are the appropriate techniques to graph a linear inequality in two variables? ● How can a graph of a linear inequality be used to represent a real-world situation? ● How does the slope of a line relate to real-world situations? ● What is meant by the term rate of change? ● How does slope relate to graphs, equations, and points on a line? ● What are positive, negative, zero, and undefined slopes and why are they important? ● How does changing the coefficient of the independent variable of an equation affect the slope of a line? ● What is the standard form of a linear equation? ● What does the slope intercept form of a linear equation mean? ● What is the parent function of a linear equation? ● What is the role of transformations in graphing linear equations? **<u>Essential Understandings</u>** ● Changes in slope may be described by dilations, reflections, or both. ● Changes in the y-intercept may be described by translations. ● Linear equations can be graphed using slope, x- and y-intercepts, and/or transformations of the parent function. ● The slope of a line represents a constant rate of change in the dependent variable when the independent variable changes by a constant amount. ● The equation of a line defines the relationship between two variables. ● The graph of a line represents the set of points that satisfies the equation of a line. ● A line can be represented by its graph or by an equation. ● The graph of the solutions of a linear inequality is a half-plane bounded by the graph of its related linear equation. Points on the boundary are included unless it is a strict inequality. ● Parallel lines have equal slopes. ● The product of the slopes of perpendicular lines is -1 unless one of the lines has an undefined slope. **<u>Teacher Notes and Elaborations</u>** A graph is a picture of an equation or inequality. The graphs of linear equations are straight lines. The graphs of linear inequalities are regions. *(continued)*

Curriculum Information	Essential Knowledge and Skills Key Vocabulary	Essential Questions and Understandings Teacher Notes and Elaborations
SOL Reporting Category Equations and Inequalities **Topic** Equations and Inequalities **Virginia SOL A.6** The student will graph linear equations and linear inequalities in two variables, including a. determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined; and b. writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.	● **Convert** between alternative forms of linear equations including slope-intercept, standard, and point-slope form. ● **Determine** if two lines are parallel, perpendicular, or neither. **Cognitive Level (Bloom's Taxonomy, Revised)** **Remember** – Write, Find **Understand** – Use, Recognize, Convert **Apply** – Graph, Determine **Extension for Algebra I** ● Find the slope of a line, given two points on the line with rational coordinates. ● Write an equation of a line in slope-intercept form when given two points on the line whose coordinates are rational numbers. **Key Vocabulary** horizontal line form parent function point-slope form rate of change slope slope-intercept form standard form vertical line form *x*-intercept *y*-intercept	**Teacher Notes and Elaborations** *(continued)* The *slope* of a line is the ratio of the change in the *y*-coordinates to the corresponding change in the *x*-coordinates. The slope of a line can be described as $\frac{\text{rise}}{\text{run}}$ or $\frac{\text{change in } y}{\text{change in } x}$ (Given two ordered pairs, the slope may be found by dividing the change in the y-coordinates by the change in the x-coordinates). The slope *m* of a line that passes through the points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$. Students should understand that the subscripts are not interchangeable with exponents. The slope of a linear equation represents a constant rate of change in the dependent variable when the independent variable changes by a fixed amount. The slope of a line determines its relative steepness. Changing the relationship between the rise of the graph (change in the *y*-values) and the run (change in the *x*-values) affects the rate of change or "steepness" of a slope. The slope of a line can be determined in a variety of ways. Changes in slope affect the graph of a line. The slope intercept form of a linear equation is $y = mx + b$ where *m* is the slope and *b* is the *y*-intercept. The slope of a line, *m*, is described as a "*rate of change*," which may be positive, negative, zero, or undefined. A vertical line has an undefined slope and a horizontal line has a slope of zero. Emphasis should be placed on the difference between zero slope and undefined slope. The use of "no slope" instead of "zero slope" should be avoided because it is confusing to students. The graphing calculator is an effective tool to illustrate the effect of changes in the slope on the graph of the line. Slopes and *y*-intercepts are found in every day life where a relationship exists (e.g., motion, temperature, light variations, finance, etc.). *(continued)*

Curriculum Information	Essential Questions and Understandings Teacher Notes and Elaborations
SOL Reporting Category Equations and Inequalities **Topic** Equations and Inequalities **Virginia SOL A.6** The student will graph linear equations and linear inequalities in two variables, including a. determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined; and b. writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.	**Teacher Notes and Elaborations** *(continued)* A line can be represented by its graph or by an equation. The equation of a line defines the relationship between two variables. The graph of a line represents the set of points that satisfies the equation of a line. Linear equations can be written in a variety of forms (Linear inequalities can also be written in a variety of forms.): - *Slope-intercept*: $y = mx + b$, where m is the slope and b is the y-intercept. - *Standard*: $Ax + By = C$, where A, B, and C are integers and A is positive. - *Point-slope*: $y - y_1 = m(x - x_1)$ - *Vertical line*: $x = a$ - *Horizontal line* (constant function): $y = b$ Equivalent equations have the same solution. For example $2x + y = 6$ is equivalent to $4x + 2y = 12$ and $y = -2x + 6$. Experiences should include writing and recognizing equivalent equations. The *parent function* for a linear equation is $y = x$. The *x-intercept* of the line is the value of x when $y = 0$. The *y-intercept* of the line is the value of y when $x = 0$. If the x- and y-intercepts are given, using these two points the equation of the line may be determined just as it is with any two points on the line. Also, given a point and the x- or y-intercept, the equation of the line can be determined. Equations of the line may be written using two ordered pairs (two points on the line), the x- and y-intercepts, or the slope and a point on the line. Using the slope and one of the coordinates, the equation may be written using the point-slope form of the equation, $y - y_1 = m(x - x_1)$. Justification of an appropriate technique for graphing linear equations and inequalities is dependent upon the application of slope, x- and y-intercepts, and graphing by transformations. Appropriate techniques for graphing linear equations and inequalities are determined by the given information and/or the tools available. The solution set for an inequality in two variables contains many ordered pairs when the domain and range are the set of real numbers. The graphs of all of these ordered pairs fill a region on the coordinate plane called a half-plane. An equation defines the boundary or edge for each half-plane. An appropriate technique for graphing a linear inequality is to graph the associated equation, determine whether the line is solid or broken, and then determine the shading by testing points in the region. *(continued)*

Curriculum Information	Essential Questions and Understandings Teacher Notes and Elaborations
<u>SOL Reporting Category</u> Equations and Inequalities **<u>Topic</u>** Equations and Inequalities **<u>Virginia SOL A.6</u>** The student will graph linear equations and linear inequalities in two variables, including a. determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined; and b. writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.	**<u>Teacher Notes and Elaborations</u>** *(continued)* **<u>Extension for Algebra I</u>** Given two points with rational coordinates, students will find the slope of the line. Example: Given $\left(1\frac{1}{2},2\right)$ and $\left(3,6\frac{1}{8}\right)$, the slope equals $\frac{11}{4}$. Given two points with rational coordinates, students will find the equation of the line. Example: Given $\left(1\frac{1}{2},2\right)$ and $\left(3,6\frac{1}{8}\right)$, with a slope of $\frac{11}{4}$, the slope-intercept form is $y=\frac{11}{4}x-\frac{17}{8}$.

Curriculum Information	Resources	Sample Instructional Strategies and Activities
SOL Reporting Category Equations and Inequalities **Topic** Equations and Inequalities **Virginia SOL A.6** **Foundational Objectives** **8.16** The student will graph a linear equation in two variables.	Text: Virginia Algebra I, ©2012, Pearson Education VDOE Enhanced Scope and Sequence Sample Lesson Plans http://www.doe.virginia.gov/testing/sol/scope_sequence/mathematics_2009/index.php Virginia Department of Education Website http://www.doe.virginia.gov/instruction/mathematics/index.shtml VDOE Project Graduation www.doe.virginia.gov/instruction/graduation/project_graduation/index.shtml	• In small groups, students take an equation of a line and describe all the different information that can be determined about it. The students will discuss the most efficient techniques of graphing the equation. • Divide students into groups. Give all groups the same equation, but have students use different graphing techniques. Next, they will compare their results with the class to show that they have the same result. • Use graphing calculators to investigate the changes in the graph caused by changing the value of the constant and coefficient. This allows the student to visually compare several equations at the same time. Describe how changes in the m and b transform the graph from the parent function. • Have students calculate the slope of several staircases in the school. Have students calculate the slope of several delivery ramps and/or handicap ramps. Have students find the equation of the line that would represent the ramps or stairs. Use the equation to draw the graph of the lines on the graphing calculator. Discuss how the changes in slope affect the "steepness" of the line. • Divide students into pairs. One student has a card with a graph or line which he/she describes as accurately and precise as possible to their partner. The other student will write the equation of the line. Students should then look at the graphs to check the answers. The partners then switch positions and repeat. • Students should describe the strategy used to find an equation of the line that passes through two given points. Sketch a flowchart that shows the steps.

Curriculum Information	Essential Knowledge and Skills Key Vocabulary	Essential Questions and Understandings Teacher Notes and Elaborations
<u>SOL Reporting Category</u> Functions and Statistics **<u>Topic</u>** Functions **<u>Virginia SOL A.7</u>** The student will investigate and analyze function (linear and quadratic) families and their characteristics both algebraically and graphically, including a. determining whether a relation is a function; b. domain and range; c. zeros of a function; d. x- and y-intercepts; e. finding the values of a function for elements in its domain; and f. making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representations to:** ● **Determine** whether a relation, represented by a set of ordered pairs, a table or from a graph is a function. ● **Identify** the domain, range, zeros, and intercepts of a function presented algebraically or graphically. ● **Detect** patterns in data and represent arithmetic and geometric patterns algebraically. ● For each x in the domain of f, **find** $f(x)$. ● **Represent** relations and functions using concrete, verbal, numeric, graphic, and algebraic forms. Given one representation, students will be able to represent the relation in another form. **<u>Cognitive Level</u> <u>(Bloom's Taxonomy, Revised)</u>** **Evaluate** - Determine **Understand** - Identify **Analyze** - Represent **Remember** - Find **Create -** Detect **<u>Key Vocabulary</u>** abscissa function family of functions function notation ordinate relation vertical line test zeros of a function	**<u>Essential Questions</u>** ● What is a relation and when does it become a function? ● How are domain/range, abscissa/ordinate, and independent /dependent variables related in a set of ordered pairs, table, or a graph? ● How can the ordered pair (x, y) be represented using function notation? ● What is the zero of a function? ● How are domain and range represented in set builder notation? **<u>Essential Understandings</u>** ● A set of data may be characterized by patterns, and those patterns can be represented in multiple ways. ● Graphs can be used as visual representations to investigate relationships between quantitative data. ● Inductive reasoning may be used to make conjectures about characteristics of function families. ● Each element in the domain of a relation is the abscissa of a point of the graph of the relation. ● Each element in the range of a relation is the ordinate of a point of the graph of the relation. ● A relation is a function if and only if each element in the domain is paired with a unique element of the range. ● The values of $f(x)$ are the ordinates of the points of the graph of f. ● The object $f(x)$ is the unique object in the range of the function f that is associated with the object x in the domain of f. ● For each x in the domain of f, x is a member of the input of the function f, $f(x)$ is a member of the output of f, and the ordered pair $[x, f(x)]$ is a member of f. ● An object x in the domain of f is an x-intercept or a zero of a function f if and only if $f(x) = 0$. ● Set builder notation may be used to represent domain and range of a relation. **<u>Teacher Notes and Elaborations</u>** A set of data may be characterized by patterns and those patterns can be represented in multiple ways. Algebra is a tool for describing patterns, generalizing, and representing a relationship in which output is related to input. Mathematical relationships are readily seen in the translation of quantitative patterns and relations to equations or graphs. Collected data can be organized in a table or visualized in a graph and analyzed for patterns. *(continued)*

Curriculum Information	Essential Questions and Understandings Teacher Notes and Elaborations
SOL Reporting Category Functions and Statistics **Topic** Functions **Virginia SOL A.7**	**Teacher Notes and Elaborations** *(continued)* Pattern recognition and analysis might include: - patterns involving a given sequence of numbers; - a set of ordered pairs from a given pattern; - a pattern using the variable(s) in an algebraic format so that a specific term can be determined; - patterns demonstrated geometrically on the coordinate plane when appropriate; and - patterns that include the use of the ellipsis such as 1, 2, 3 . . . 99, 100. Patterns may be represented as relations and/or functions. A *relation* can be represented by a set of ordered pairs of numbers or paired data values. In an ordered pair, the first number is termed the *abscissa* (*x*-coordinate) and the second number is the *ordinate* (*y*-coordinate). A *function* is a special relation in which each different input value is paired with exactly one output value (a unique output for each input). The set of input values forms the domain and the set of output values forms the range of the function. Sets of ordered pairs that do not represent a function should also be identified. Graphs of functions with similar features are called a *family of functions*. Graphs can be used as visual representations to investigate relationships between quantitative data. Students should have multiple experiences constructing linear and quadratic graphs utilizing **both** paper and pencil and the graphing calculator. Graphically, a function may be determined by applying the *vertical line test* (A graph is a function if there exists no vertical line that intersects the graph in more than one point.). If a sequence does not have a last term, it is called an infinite sequence. Three dots called an ellipsis are used to indicate an omission. If a sequence stops at a particular term, it is a finite sequence. Set builder notation is a method for identifying a set of values. For example, the domain for $y = x^2 - 5$ would be written as $\{x : x \in \Re\}$. This is read, "The set of all *x* such that *x* is an element of the real numbers." The range for this equation would be written as $\{y : y \geq -5\}$. To find the *y*-intercept in a quadratic function, let $x = 0$. In a function, the relationship between the domain and range may be represented by a rule. This rule may be expressed using *function notation*, $f(x)$, which means the value of the function at *x* and is read "*f* of *x*". *Zeros of a function* (roots/solutions) are the *x*-intercepts of the function and are found algebraically by substituting 0 for *y* and solving the subsequent equation (e.g., If $f(x) = 2x + 4$, to find the zero solve $0 = 2x + 4$. The zero is -2, located at $(-2, 0)$). An object *x* in the domain of *f* is an *x*-intercept or a zero of a function *f* if and only if $f(x) = 0$. Domain, range, zeros and intercepts of a function can be presented algebraically and graphically. Experiences determining domain, range, zeros and intercepts should include a variety of graphs.

Curriculum Information	Resources	Sample Instructional Strategies and Activities
<u>SOL Reporting Category</u> Functions and Statistics **<u>Topic</u>** Functions **<u>Foundational Objectives</u>** **8.8** The student will a. apply transformations to plane figures; and b. identify applications of transformations. **8.14** The student will make connections between any two representations (tables, graphs, words, and rules) of a given relationship. **8.16** The student will graph a linear equation in two variables. **8.17** The student will identify the domain, range, independent variable, or dependent variable in a given situation. **7.2** The student will describe and represent, arithmetic and geometric sequences using variable expressions. **7.8** The student, given a polygon in the coordinate plane, will represent transformations (reflections, dilations, rotations, and translations) by graphing in the coordinate plane. **7.12** The student will represent relationships with tables, graphs, rules, and words. **6.17** The student will identify and extend geometric and arithmetic sequences.		● Give students a list of ordered pairs such as ($^-2$, $^-3$), ($^-1$, $^-1$), (0, 1), (1, _), (_, 5), (_, _). Have students identify the rule and complete the pattern. ● Students develop patterns and determine if they are linear, quadratic or neither. ● Draw a coordinate plane on a flat surface outside or on the floor. Students pick a number on the *x*-axis to be used as a domain element. Give them a rule for which they will find the range value for their number. Students will move to this point on the plane. "Connect" the students using yarn or string. Analyze the different types of graphs obtained. ● Divide students into groups. Give each group a folder. Have them write a relation on the outside as well as a list of five numbers to be used as domain elements. They should write the domain and range elements in set notation on a piece of paper and place it in the folder. Folders are passed to each group. When the folder is returned to the group it began with, the results will be analyzed and verified. ● Using a graphing calculator, a series of relations can be graphed. Use a piece of paper to represent a vertical line, and use the vertical line test to test if it is a function. Examples: $y = x^2$ $y = \|x\|$ $y = x$ $y = \frac{1}{x}$ $y = x + 4 - x^2$ $y = 4 - x^2$

<table>
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<u>SOL Reporting Category</u>
Functions and Statistics

<u>Topic</u>
Functions

<u>Virginia SOL A.8</u>

The student, given a situation in a real-world context, will analyze a relation to determine whether a direct or inverse variation exists, and represent a direct variation algebraically and graphically and an inverse variation algebraically.
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The student will use problem solving, mathematical communication, mathematical reasoning, connections and representations to:
● Given a situation, including a real-world situation, determine whether a direct variation exists.
● Given a situation, including a real-world situation, determine whether an inverse variation exists.
● Write an equation for a direct variation, given a set of data.
● Write an equation for an inverse variation, given a set of data.
● Graph an equation representing a direct variation, given a set of data.

<u>Cognitive Level (Bloom’s Taxonomy, Revised)</u>
Remember – Write
Apply – Show
Evaluate - Determine

<u>Key Vocabulary</u>
constant of proportionality (variation)
dependent variable
direct variation
independent variable
inverse variation
</td>
<td>
<u>Essential Questions</u>
● What is direct variation?
● What is inverse variation?
● How is a relation analyzed to determine direct variation?
● How is a relation analyzed to determine inverse variation?
● What is the constant of proportionality in a direct variation?
● What is the constant of proportionality in an inverse variation?
● How do you represent a direct variation algebraically and graphically?
● How do you represent an inverse variation algebraically?

<u>Essential Understandings</u>
● The constant of proportionality in a direct variation is represented by the ratio of the dependent variable to the independent variable.
● The constant of proportionality in an inverse variation is represented by the product of the dependent variable and the independent variable.
● A direct variation can be represented by a line passing through the origin.
● Real-world problems may be modeled using direct and/or inverse variations.

<u>Teacher Notes and Elaborations</u>
Direct variation involves a relationship between two variables. The patterns for direct variation relationships can be observed using equations, tables, and graphs. Direct variation is used to represent a constant rate of change in real-world situations.

Direct variation is defined by $y = kx$, $(k \neq 0)$ where k is the constant of proportionality (variation). The constant, k, in a direct variation is represented by the ratio of y to x, $k = \frac{y}{x}$, where y is the dependent variable and x is the independent variable. Emphasis should be placed on finding the constant of proportionality (k).

A table and graph provide visual confirmation that as x increases, y increases or as x decreases, y decreases.

The value of k is determined by substituting a pair of known values for x and y into the equation and solving for k. The graph and table illustrate a linear pattern where the slope is the constant of variation and the y-intercept is zero.

(continued)
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Curriculum Information	Essential Questions and Understandings Teacher Notes and Elaborations
<u>SOL Reporting Category</u> Functions and Statistics **<u>Topic</u>** Functions **<u>Virginia SOL A.8</u>** The student, given a situation in a real-world context, will analyze a relation to determine whether a direct or inverse variation exists, and represent a direct variation algebraically and graphically and an inverse variation algebraically.	**<u>Teacher Notes and Elaborations</u>** *(continued)* *Inverse variation* is defined by $xy = k$, where $k \neq 0$. This can also be written as $y = \frac{k}{x}$, $(x \neq 0)$ and k is the constant of proportionality (variation). In an inverse variation as the values of x increase the values of y decrease. Algebraically, in a given situation, when determining whether a direct or inverse variation exists, points can be identified to create a specific relation. Example: Given this set of points (1, 10), (3.5, 7), (4, 8), (8, 4), (8, 16), (10, 6) Which points create a relation that is a direct variation? When using $k = \frac{y}{x}$, the points (3.5, 7), (4, 8), and (8, 16) all have $k = 2$. Therefore these points create a relation that is a direct variation. Graphically, the points can be plotted and a line drawn.

Curriculum Information	Resources	Sample Instructional Strategies and Activities

Curriculum Information

SOL Reporting Category
Functions and Statistics

Topic
Functions

Virginia SOL A.8

Foundational Objectives
8.12
The student will determine the probability of independent and dependent events with and without replacement
8.17
The student will identify the domain, range, independent variable, or dependent variable in a given situation.

Resources

Text:
Virginia Algebra I, ©2012, Pearson Education

VDOE Enhanced Scope and Sequence Sample Lesson Plans
http://www.doe.virginia.gov/testing/sol/scope_sequence/mathematics_2009/index.php

Virginia Department of Education Website
http://www.doe.virginia.gov/instruction/mathematics/index.shtml

VDOE Project Graduation
www.doe.virginia.gov/instruction/graduation/project_graduation/index.shtml

Sample Instructional Strategies and Activities

- A bicycle travels at a certain constant rate. The distance the bicycle travels varies with time. Suppose the bicycle travels at a rate of 10 km/h. The distance it travels will vary depending only on time. A bicycle will travel a distance (d) in a time (t) while traveling at 10 km/h.

t	d
1	10
2	20
3	30
4	40

We can say that the distance "varies directly" as time passed or $d = 10t$

If a seesaw is balanced, each person's distance from the fulcrum varies inversely as his weight.

w	d
200	3
180	$3\frac{1}{3}$
150	4
120	5
100	6

$d = \frac{k}{w}$ so that as the person's weight decreases the distance from the fulcrum increases. Solving this equation for k gives the constant of proportionality so k = 600. Using this value, if a person weighs 140 pounds, how far will he sit from the fulcrum?

What other examples can you think of where direct and inverse variations occur?

- Students are given unlabeled graphs. Next, students will make up stories of events that could be happening to describe the situation depicted by the graph or vice-versa. Students will determine whether the situation represents a direct or inverse variation or neither and represent the situation algebraically, if possible.

Curriculum Information	Essential Knowledge and Skills Key Vocabulary	Essential Questions and Understandings Teacher Notes and Elaborations
SOL Reporting Category Functions and Statistics **Topic** Statistics **Virginia SOL A.9** The student, given a set of data, will interpret variation in real-world contexts and calculate and interpret mean absolute deviation, standard deviation, and z-scores.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representations to:** ● **Analyze** descriptive statistics to determine the implications for the real-world situations from which the data derive. ● Given data, including data in a real-world context, **calculate** and **interpret** the mean absolute deviation of a data set. ● Given data, including data in a real-world context, **calculate** variance and standard deviation of a data set and **interpret** the standard deviation. ● Given data, including data in a real-world context, **calculate** and **interpret** z-scores for a data set. ● Explain ways in which standard deviation addresses dispersion by **examining** the formula for standard deviation. ● **Compare** and **contrast** mean absolute deviation and standard deviation in a real-world context. **Cognitive Level (Bloom's Taxonomy, Revised)** **Analyze** – Examine, Contrast, Compare, Analyze, Calculate **Evaluate** – Interpret **Key Vocabulary** dispersion mean mean absolute deviation standard deviation summation notation variance z-score	**Essential Questions** ● What is the importance of statistics? ● What is the mean absolute deviation for a set of data? ● What is the variance and standard deviation for a set of data? ● What is the z-score? Essential Understandings ● Descriptive statistics may include measures of center and dispersion. ● Variance, standard deviation, and mean absolute deviation measure the dispersion of the data. ● The sum of the deviations of data points from the mean of a data set is 0. ● Standard deviation is expressed in the original units of measurement of the data. ● Standard deviation addresses the dispersion of data about the mean. ● Standard deviation is calculated by taking the square root of the variance. ● The greater the value of the standard deviation, the further the data tend to be dispersed from the mean. ● For a data distribution with outliers, the mean absolute deviation may be a better measure of dispersion than the standard deviation or variance. ● A z-score (standard score) is a measure of position derived from the mean and standard deviation of data. ● A z-score derived from a particular data value tells how many standard deviations that data value is above or below the mean of the data set. It is positive if the data value lies above the mean, and negative if the data value lies below the mean. **Teacher Notes and Elaborations** This objective is intended to extend the study of descriptive statistics beyond the measures of center studied during the middle grades. Although calculation is included in this objective, instruction and assessment emphasis should be on understanding and interpreting statistical values associated with a data set including standard deviation, mean absolute deviation, and z-score. While not explicitly included in this objective, the arithmetic mean will be integral to the study of descriptive statistics. The study of statistics includes gathering, displaying, analyzing, interpreting, and making predictions about a larger group of data (population) from a sample of those data. Data can be gathered through scientific experimentation, surveys, and/or observation of groups or phenomena. Numerical data gathered can be displayed numerically or graphically (examples would include line plots, histograms, and stem-and-leaf plots). Methods for organizing and summarizing data make up the branch of statistics called descriptive statistics. *(continued)*

Curriculum Information	Essential Questions and Understandings Teacher Notes and Elaborations
SOL Reporting Category Functions and Statistics **Topic** Statistics **Virginia SOL A.9** The student, given a set of data, will interpret variation in real-world contexts and calculate and interpret mean absolute deviation, standard deviation, and z-scores.	**Teacher Notes and Elaborations** *(continued)* **Sample vs. Population Data** Sample data can be collected from a defined statistical population. Examples of a statistical population might include *SOL scores of all Algebra I students in Virginia, the heights of every U.S. president,* or *the ages of every mathematics teacher in Virginia*. Sample data can be analyzed to make inferences about the population. A data set, whether a sample or population, is comprised of individual data points referred to as elements of the data set. An element of a data set will be represented as x_i, where *i* represents the i^{th} term of the data set.* When beginning to teach this standard, start with small, defined population data sets of approximately 30 items or less to assist in focusing on development of understanding and interpretation of statistical values and how they are related to and affected by the elements of the data set. Related to the discussion of samples versus populations of data are discussions about notation and variable use. In formal statistics, the arithmetic *mean* (average) of a population is represented by the Greek letter μ (mu), while the calculated arithmetic mean of a sample is represented by $\overline{x}$, read "x bar." In general, a bar over any symbol or variable name in statistics denotes finding its mean. The arithmetic mean of a data set will be represented by μ.* On both brands of approved graphing calculators in Virginia, the calculated arithmetic mean of a data set is represented by $\overline{x}$. **Mean Absolute Deviation vs. Variance and Standard Deviation** Statisticians like to measure and analyze the *dispersion* (spread) of the data set about the mean in order to assist in making inferences about the population. One measure of spread would be to find the sum of the deviations between each element and the mean; however, this sum is always zero. There are two methods to overcome this mathematical dilemma: 1) take the absolute value of the deviations before finding the average or 2) square the deviations before finding the average. The mean absolute deviation uses the first method and the variance and standard deviation uses the second. If either of these measures is to be computed by hand, do not require students to use data sets of more than about 10 elements. **NOTE: Students have not been introduced to *summation notation* prior to Algebra I.** An introductory lesson on how to interpret the notation will be necessary. Examples of summation notation: $\sum_{i=1}^{5} i = 1+2+3+4+5$ $\sum_{i=1}^{4} x_i = x_1 + x_2 + x_3 + x_4$ * *(continued)*

*The shaded notation and formulas will be used on the Algebra I SOL assessment and included on the formula sheet for the Algebra EOC SOL.

Curriculum Information	Essential Questions and Understandings Teacher Notes and Elaborations
SOL Reporting Category Functions and Statistics **Topic** Statistics **Virginia SOL A.9** The student, given a set of data, will interpret variation in real-world contexts and calculate and interpret mean absolute deviation, standard deviation, and z-scores.	**Teacher Notes and Elaborations** *(continued)* **Mean Absolute Deviation** Mean absolute deviation is one measure of spread about the mean of a data set, as it is a way to address the dilemma of the sum of the deviations of elements from the mean being equal to zero. The *mean absolute deviation* is the arithmetic mean of the absolute values of the deviations of elements from the mean of a data set. Mean absolute deviation $= \frac{\sum_{i=1}^{n} \lvert x_i - \mu \rvert}{n}$, where μ represents the mean of the data set, n represents the number of elements in the data set, and x_i represents the i^{th} element of the data set.* The mean absolute deviation is less affected by outlier data than the variance and standard deviation. Outliers are elements that fall at least 1.5 times the interquartile range (IQR) below the first quartile (Q_1) or above the third quartile (Q_3). Graphing calculators identify Q_1 and Q_3 in the list of computed 1-varible statistics. Mean absolute deviation cannot be directly computed on the graphing calculator as can the standard deviation. The mean absolute deviation must be computed by hand or by a series of keystrokes using computation with lists of data. More information (keystrokes and screenshots) on using graphing calculators to compute this can be found in the Sample Instructional Strategies and Activities. **Variance** The second way to address the dilemma of the sum of the deviations of elements from the mean being equal to zero is to square the deviations prior to finding the arithmetic mean. The average of the squared deviations from the mean is known as the *variance*, and is another measure of the spread of the elements in a data set. Variance $(\sigma^2) = \frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}$, where μ represents the mean of the data set, n represents the number of elements in the data set, and x_i represents the i^{th} element of the data set.* The differences between the elements and the arithmetic mean are squared so that the differences do not cancel each other out when finding the sum. When squaring the differences, the units of measure are squared and larger differences are "weighted" more heavily than smaller differences. In order to provide a measure of variation in terms of the original units of the data, the square root of the variance is taken, yielding the standard deviation. *(continued)*

*The shaded notation and formulas will be used on the Algebra I SOL assessment and included on the formula sheet for the Algebra EOC SOL.

Curriculum Information	Essential Questions and Understandings Teacher Notes and Elaborations
SOL Reporting Category Functions and Statistics **Topic** Statistics **Virginia SOL A.9** The student, given a set of data, will interpret variation in real-world contexts and calculate and interpret mean absolute deviation, standard deviation, and z-scores.	**Teacher Notes and Elaborations** *(continued)* **Standard Deviation** The *standard deviation* is the positive square root of the variance of the data set. The greater the value of the standard deviation, the more spread out the data are about the mean. The lesser (closer to 0) the value of the standard deviation, the closer the data are clustered about the mean. Standard deviation $(\sigma) = \sqrt{\frac{\sum_{i=1}(x_i - \mu)^2}{n}}$, where μ represents the mean of the data set, *n* represents the number of elements in the data set, and x_i represents the i^{th} element of the data set.* Often, textbooks will use two distinct formulas for standard deviation. In these formulas, the Greek letter "σ", written and read "sigma", represents the standard deviation of a population, and "*s*" represents the sample standard deviation. The population standard deviation can be estimated by calculating the sample standard deviation. The formulas for sample and population standard deviation look very similar except that in the sample standard deviation formula, $n – 1$ is used instead of *n* in the denominator. The reason for this is to account for the possibility of greater variability of data in the population than what is seen in the sample. When $n – 1$ is used in the denominator, the result is a larger number. So, the calculated value of the sample standard deviation will be larger than the population standard deviation. As sample sizes get larger (*n* gets larger), the difference between the sample standard deviation and the population standard deviation gets smaller. The use of $n – 1$ to calculate the sample standard deviation is known as Bessel's correction. Use the formula for standard deviation with *n* in the denominator as noted in the shaded box above. When using Casio or Texas Instruments (TI) graphing calculators to compute the standard deviation for a data set, two computations for the standard deviation are given, one for a population (using *n* in the denominator) and one for a sample (using $n - 1$ in the denominator). Students should be asked to use the computation of standard deviation for population data in instruction and assessments. On a Casio calculator, it is indicated with "$x\sigma n$" and on a TI graphing calculator as "σx". More information (keystrokes and screenshots) on using graphing calculators to compute this can be found in the Sample Instructional Strategies and Activities. *(continued)*

*The shaded notation and formulas will be used on the Algebra I SOL assessment and included on the formula sheet for the Algebra EOC SOL.

Curriculum Information	Essential Questions and Understandings Teacher Notes and Elaborations
<u>SOL Reporting Category</u> Functions and Statistics **<u>Topic</u>** Statistics **<u>Virginia SOL A.9</u>** The student, given a set of data, will interpret variation in real-world contexts and calculate and interpret mean absolute deviation, standard deviation, and z-scores.	**<u>Teacher Notes and Elaborations</u>** *(continued)* **z-Scores** A *z-score*, also called a standard score, is a measure of position derived from the mean and standard deviation of the data set. In Algebra I, the z-score will be used to determine how many standard deviations an element is above or below the mean of the data set. It can also be used to determine the value of the element, given the z-score of an unknown element and the mean and standard deviation of a data set. The z-score has a positive value if the element lies above the mean and a negative value if the element lies below the mean. A z-score associated with an element of a data set is calculated by subtracting the mean of the data set from the element and dividing the result by the standard deviation of the data set. z-score $(z) = \frac{x - \mu}{\sigma}$, where x represents an element of the data set, μ represents the mean of the data set, and σ represents the standard deviation of the data set.* A z-score can be computed for any element of a data set; however, they are most useful in the analysis of data sets that are normally distributed. In Algebra II, z-scores will be used to determine the relative position of elements within a normally distributed data set, to compare two or more distinct data sets that are distributed normally, and to determine percentiles and probabilities associated with occurrence of data values within a normally distributed data set.

*The shaded notation and formulas will be used on the Algebra I SOL assessment and included on the formula sheet for the Algebra EOC SOL.

Curriculum Information	Resources	Sample Instructional Strategies and Activities
SOL Reporting Category Functions and Statistics **Topic** Statistics **Virginia SOL A.9** **Foundational Objectives** **6.15** The student will a. describe mean as balance point; and b. decide which measure of center is appropriate for a given purpose.	Text: Virginia Algebra I, ©2012, Pearson Education VDOE Enhanced Scope and Sequence Sample Lesson Plans http://www.doe.virginia.gov/testing/sol/scope_sequence/mathematics_2009/index.php Virginia Department of Education Website http://www.doe.virginia.gov/instruction/mathematics/index.shtml VDOE Project Graduation www.doe.virginia.gov/instruction/graduation/project_graduation/index.shtml	(see below)

Length of Rivers Feeding into the Ohio River

Monongahela	129 miles
Allegheny	325 miles
Kanawha	97 miles
Kentucky	259 miles
Green	360 miles
Cumberland	694 miles
Tennessee	166 miles

- There are seven navigable rivers that feed into the Ohio River. The lengths of these rivers are shown in the table.

Compare standard deviation and absolute mean deviation of the lengths of the rivers. What conclusions can you reach based on these two measures? The information needed is best organized in a table.

Value (x)	Mean μ	$\lvert x_1 - \mu \rvert$
129	290	161
325	290	35
97	290	193
259	290	31
360	290	70
694	290	404
166	290	124
		$\Sigma = 1018$

$$\mu = \frac{129 + 325 + 97 + 259 + 360 + 694 + 166}{7}$$

$$\mu = \frac{2030}{7}$$

$$\mu = 290$$

The absolute mean deviation = $\dfrac{\sum_{i=1}^{n} \lvert x_i - \mu \rvert}{n}$

$$= \frac{1018}{7}$$

$$= 145$$

(continued)

Value (x)	Mean μ	$(x_i - \mu)^2$
129	290	25,921
325	290	1,225
97	290	37,249
259	290	961
360	290	4,900
694	290	163,216
166	290	15,376
		Σ = 248,848

The standard deviation = $\sqrt{\dfrac{\sum_{i=1}^{n}(x_i - \mu)^2}{n}}$

$= \sqrt{\dfrac{248,848}{7}}$

$= \sqrt{35,550}$

≈ 188.5

The z-score for the Monongahela River = $\dfrac{\text{value} - \text{mean}}{\text{standard deviation}}$

$= \dfrac{129 - 290}{188.5}$

≈ -0.85

(continued)

Curriculum Information	Sample Instructional Strategies and Activities *(continued)*

SOL Reporting Category
Functions and Statistics

Topic
Statistics

Virginia SOL A.9

Example 1: (Computation of descriptive statistics using graphing calculators)

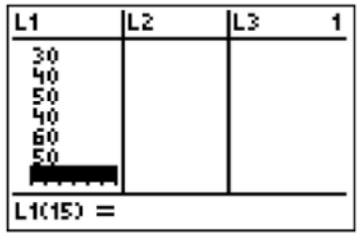
Maya's company produces a special product on 14 days only each year. Her job requires that she report on production at the end of the 14 days. She recorded the number of products produced each day (below) and decided to use descriptive statistics to report on product production.

Number of Products produced daily: 40, 30, 50, 30, 50, 60, 50, 50, 30, 40, 50, 40, 60, 50

Texas Instruments (TI-84)

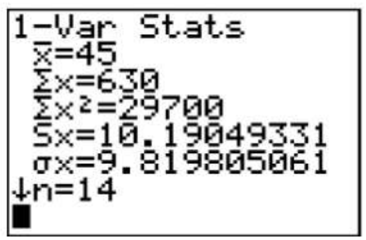
Enter the data into L1

- From the home screen click on STAT.
- To enter data into lists choose option "1: Edit" or ENTER.
- In L1, enter each element of the data set. Enter the first element and press ENTER to move to the next line and continue until all the elements have been entered into L1.



Calculate variance and standard deviation by computing 1-Variable Statistics for L1

- Press STAT ►
- Choose option "1: 1-Var Stats" or ENTER
- Press ENTER to compute the 1-variable statistics (defaults to L1, enter list name after "1-Var Stats" if data are in another list).



$\bar{x}$ = arithmetic mean of the data set
Σx = sum of the x values
Σx^2= sum of the x^2 values
Sx = sample standard deviation
σx = population standard deviation
n = number of data points (elements)

NOTE: "σx" will represent the standard deviation (σ). Squaring σ will yield the variance (σ^2).

(continued)

Curriculum Information	Sample Instructional Strategies and Activities *(continued)*

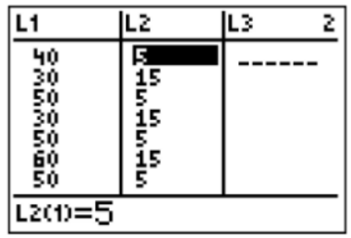
SOL Reporting Category
Functions and Statistics

Topic
Statistics

Virginia SOL A.9

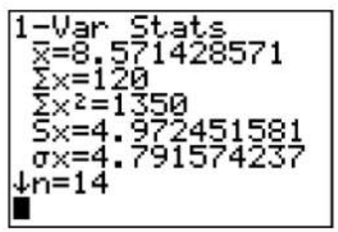
Compute the mean absolute deviation using the data in L1

- From the home screen click on STAT.
- Choose option "1: Edit" or ENTER.
- Press ► to move to L2.
- Press ▲ to highlight L2.
- Press ENTER (to get "L2 =" at the bottom of the screen) MATH ►
- Choose option "1: abs (" or ENTER
- Press 2nd 1 (to get "L1") - VARS choose option "5: Statistics…" and then choose option "2: $\bar{x}$ "(to get " $\bar{x}$ " or type in value)
- Press) ENTER (L2 will automatically fill with data)



L2 now contains the $|x_i - \mu|$ part of the mean absolute deviation formula. In order to complete the calculation of the mean absolute deviation, find the arithmetic mean of L2 by calculating the 1-variable statistics of L2.

- Press STAT ►
- Choose option "1: 1-Var Stats" or ENTER
- Press 2nd 2 (to get "L2") ENTER to compute the 1-variable statistics for L2.



$\bar{x} = 8.571428571$ from the 1-variable statistics of L2 represents the mean absolute deviation of the original data set recorded in L1.

(continued)

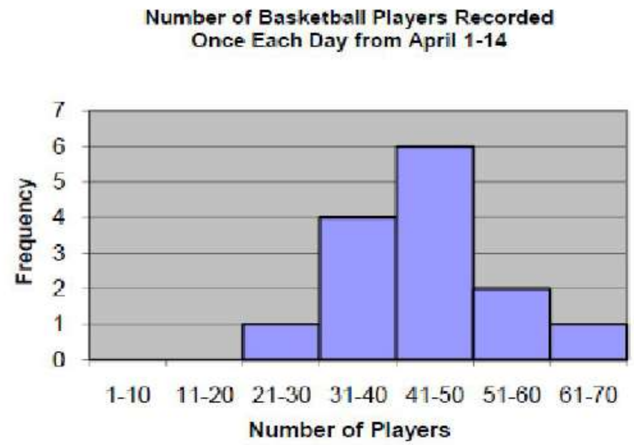
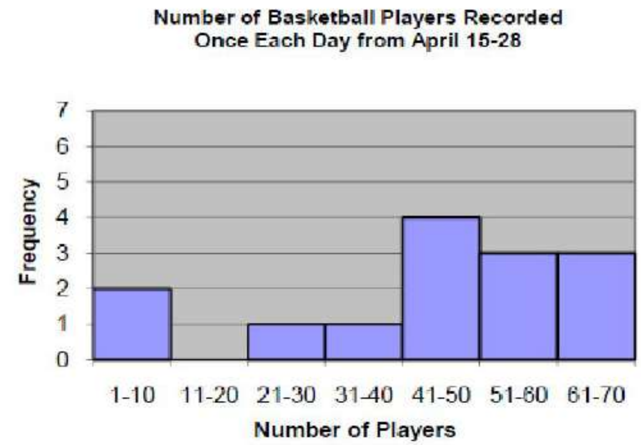
Curriculum Information	Sample Instructional Strategies and Activities *(continued)*

SOL Reporting Category
Functions and Statistics

Topic
Statistics

Virginia SOL A.9

Example 2 - Interpretation of descriptive statistics

Data set 1	Data set 2
	
Mean = 45.0 **Variance = 106.1** **Standard Deviation = 10.3** **Mean Absolute Deviation = 9.1**	**Mean = 45.0** **Variance = 420.3** **Standard Deviation = 20.5** **Mean Absolute Deviation = 16**

What types of inferences can be made about the data set with the given information?

- The standard deviation of data set 1 is less than the standard deviation of data set 2. That tells us that there was less variation (more consistency) in the number of people playing basketball during April 1 – 14 (data set 1).
- When comparing the variance of the two data sets, the difference between the two indicates that data set 2 has much more dispersion of data than data set 1.
- The standard deviation of data set 2 is almost twice the standard deviation of data set 1, indicating that the elements of data set 2 are more spread out with respect to the mean.
- Given a standard deviation and graphical representations of different data sets, the standard deviation could be matched to the appropriate graph by comparing the spread of data in each graph.
- When a data set contains clear outliers (the elements with values of 1 and 2 in data set 2), the outlying elements have a lesser effect on the calculation of the mean absolute deviation than on the standard deviation.

(continued)

Curriculum Information	Sample Instructional Strategies and Activities *(continued)*

SOL Reporting Category
Functions and Statistics

Topic
Statistics

Virginia SOL A.9

How can z-scores be used to make inferences about data sets?

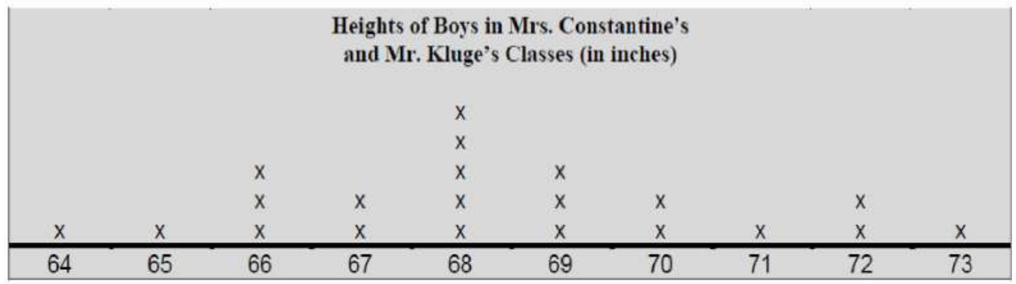
A z-score can be calculated for a specific element's value within the set of data. The z-score for an element with value of 30 can be computed for data set 2.

$$z = \frac{30-45}{20.5} = -0.73$$

The value of -0.73 indicates that the element falls just under one standard deviation below (negative) the mean of the data set. If the mean, standard deviation, and z-score are known, the value of the element associated with the z-score can be determined. For instance, given a standard deviation of 2.0 and a mean of 8.0, what would be the value of the element associated with a z-score of 1.5? Since the z-score is positive, the associated element lies above the mean. A z-score of 1.5 means that the element falls 1.5 standard deviations above the mean. So, the element falls 1.5(2.0) = 3.0 points above the mean of 8. Therefore, the z-score of 1.5 is associated with the element with a value of 11.0.

Example 3 – Interpretation of descriptive statistics

Maya represented the heights of boys in Mrs. Constantine's and Mr. Kluge's classes on a line plot and calculated the mean and standard deviation.



Mean = 68.4 Standard Deviation = 2.3

Note: In this problem, a small, defined population of the boys in Mrs. Constantine's and Mr. Kluge's classes is assumed.

How many elements are above the mean?

There are 9 elements above the mean value of 68.4.

How many elements are below the mean?

There are 12 elements below the mean value of 68.4.

How many elements fall within one standard deviation of the mean?

There are 12 elements that fall within one standard deviation of the mean. The values of the mean plus one standard deviation and the mean minus one standard deviation ($\bar{x}-\sigma = 66.1$ and $\bar{x}+\sigma = 70.7$) determine how many elements fall within one standard deviation of the mean. In other words, all 12 elements between $\bar{x}-\sigma$ and $\bar{x}+\sigma$ (boys that measure 67", 68", 69", or 70") are within one standard deviation of the mean.

(continued)

Curriculum Information	Sample Instructional Strategies and Activities *(continued)*
SOL Reporting Category Functions and Statistics **Topic** Statistics **Virginia SOL A.9**	**Application Scenarios** 1. Dianne oversees production of ball bearings with a diameter of 0.5 inches at three locations in the United States. She collects the standard deviation of a sample of ball bearings each month from each location to compare and monitor production.

Standard deviation of 0.5 inch diameter ball bearing production (in inches)

	July	August	September
Plant location #1	0.01	0.01	0.02
Plant location #2	0.02	0.04	0.05
Plant location #3	0.02	0.01	0.01

Compare and contrast the standard deviations from each plant location, looking for trends or potential issues with production. What conclusions or questions might be raised from the statistical data provided? What other statistical information and/or other data might need to be gathered in order for Dianne to determine next steps?

Sample response: Plant #1 and Plant #3 had standard deviations that seemed steady, but one would be wise to keep an eye on Plant #1 in the coming months, because it had increases in August and September. A potential concern with the standard deviation of Plant #2 exists. The growing standard deviation indicates that there might be an issue with growing variability in the size of the ball bearings. Plant #2 should be asked to take more frequent samples and continue to monitor, to check the calibration of the equipment, and/or to check for an equipment problem.

2. Jim needs to purchase a large number of 20-watt fluorescent light bulbs for his company. He has narrowed his search to two companies offering the 20-watt bulbs for the same price. The Bulb Emporium and Lights-R-Us claim that their 20-watt bulbs last for 10,000 hours. Which descriptive statistic might assist Jim in making the best purchase? Explain why it would assist him.

Sample response: The standard deviation of the lifespan of each company's 20-watt bulbs should be compared. The bulbs with the lowest lifespan standard deviation will have the slightest variation in number of hours that the bulbs last. The bulbs with the slightest variation in the number of hours that they last means that they are more likely to last close to 10,000 hours.

(continued)

Curriculum Information	Sample Instructional Strategies and Activities *(continued)*
SOL Reporting Category Functions and Statistics **Topic** Statistics **Virginia SOL A.9**	See below

3. In a school district, Mr. Mills is in charge of SAT testing. In a meeting, the superintendent asks him how many students scored less than one standard deviation below the mean on the mathematics portion of the SAT in 2009. He looks through his papers and finds that the mean of the scores is 525 and 1653 students took the SAT in 2009. He also found a chart with percentages of z-scores on the SAT in 2009 as follows:

z-score (mathematics)	Percent of students
$z < -3$	0.1
$-3 \leq z < -2$	2.1
$-2 \leq z < -1$	13.6
$-1 \leq z < 0$	34.0
$0 \leq z < 1$	34.0
$1 \leq z < 2$	13.6
$2 \leq z < 3$	2.1
$z > 3$	0.1

How can Mr. Mills determine the number of students that scored less than one standard deviation below the mean on the mathematics portion of the SAT?

Sample response: The z-score tells you how many standard deviations an element (in this case a score) is from the mean. If the z-score of a score is -1 *, then that score is 1 standard deviation below the mean. There are 15.8% (13.6% + 2.1% + 0.1%) of the scores that have a z-score <* -1 *, so there are about 261 (0.158 · 1653) students that scored less than one standard deviation below the mean.*

Questions to explore with students

1. Given a frequency graph, a standard deviation of _______, and a mean of _______, how many elements fall within _______ standard deviation(s) from the mean? Why?
2. Given the standard deviation and mean or mean absolute deviation and mean, which frequency graph would most likely represent the situation and why?
3. Given two data sets with the same mean and different spreads, which one would best match a data set with a standard deviation or mean absolute deviation of _______? How do you know?
4. Given two frequency graphs, explain why one might have a larger standard deviation.
5. Given a data set with a mean of _______, a standard deviation of _______, and a z-score of _______, what is the value of the element associated with the z-score?
6. What do z-scores tell you about position of elements with respect to the mean? How do z-scores relate to their associated element’s value?
7. Given the standard deviation, the mean, and the value of an element of the data set, explain how you would find the associated z-score.

Curriculum Information	Essential Knowledge and Skills Key Vocabulary	Essential Questions and Understandings Teacher Notes and Elaborations
<u>SOL Reporting Category</u> Functions and Statistics **<u>Topic</u>** Statistics **<u>Virginia SOL A.10</u>** The student will compare and contrast multiple univariate data sets, using box-and-whisker plots.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representations to:** ● Construct, compare, contrast, and analyze data, including data from real-world situations displayed in box-and-whisker plots. **<u>Cognitive Level</u> <u>(Bloom's Taxonomy, Revised)</u>** **Analyze** – Compare, Contrast **Create** - Construct **<u>Key Vocabulary</u>** box-and-whisker plot extreme values inter-quartile range median quartiles range	**<u>Essential Questions</u>** ● What is a box-and-whisker plot? ● How is a box-and-whisker plot constructed? ● How is a box-and-whisker plot used in a real-world situation? **<u>Essential Understandings</u>** ● Statistical techniques can be used to organize, display, and compare sets of data. ● Box-and-whisker plots can be used to analyze data. **<u>Teacher Notes and Elaborations</u>** Statistical techniques can be used to organized, display, and compare sets of data. Descriptive and visual forms of numerical data help interpret and analyze data from real-world situations. A univariate data set consists of observations on a single variable. *Box-and-whisker plots* can be used to summarize and analyze data. These plots graphically display the median, quartiles, interquartile range, and *extreme values* (minimum and maximum) in a set of data. They can be drawn vertically or horizontally. A box-and-whisker plot consists of a rectangular box with the ends located at the first and third quartiles. The segments extending from the ends of the box to the extreme values are called whiskers. The *range* of the data is the difference between the greatest and the least values of the set. The median of an odd collection of numbers, arranged in order, is the middle number. The median of an even collection of numbers, arranged in order, is the average of the two middle numbers. The *median* of an ordered collection of numbers roughly partitions the collection into two halves, those below the median and those above. The first quartile is the median of the lower half. The second quartile is the median of the entire collection. The third quartile is the median of the upper half. Box and whisker plots are uniform in their use of the box: the bottom and top of the box are always the 25^{th} and 75^{th} percentile (the lower and upper quartiles, respectively), and the band near the middle of the box is always the 50^{th} percentile (the median). Each quartile represents 25% of the data. *(continued)*

Curriculum Information	Essential Questions and Understandings Teacher Notes and Elaborations

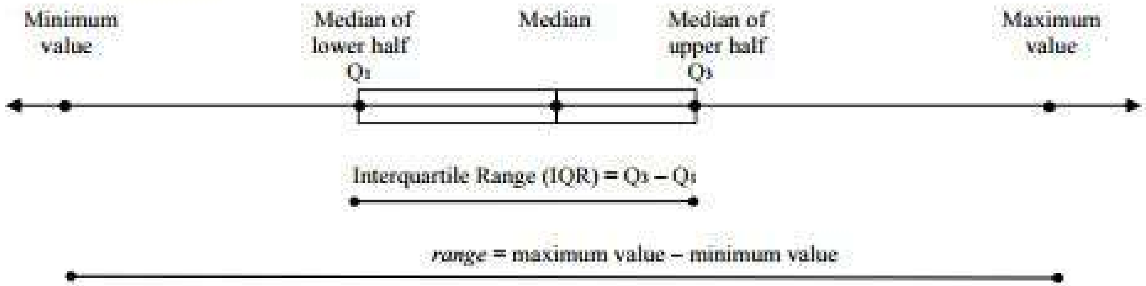
<u>SOL Reporting Category</u>
Functions and Statistics

<u>Topic</u>
Statistics

<u>Virginia SOL A.10</u>
The student will compare and contrast multiple univariate data sets, using box-and-whisker plots.

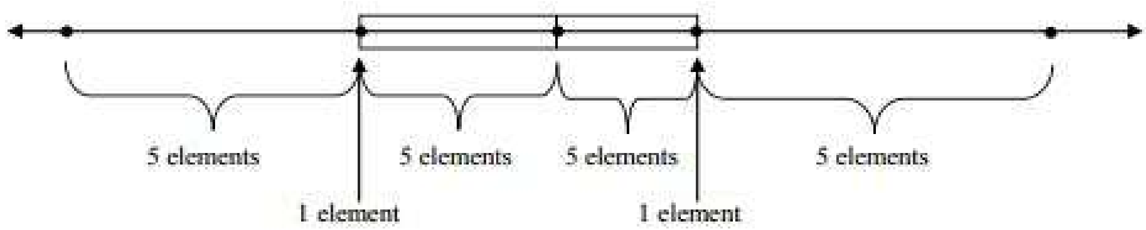
<u>Teacher Notes and Elaborations</u> *(continued)*

Quartiles partition an ordered collection into four quarters. The "whiskers" extend from the minimum to Q_1 and Q_3 to the maximum. The "box" extends from Q_1 to Q_3.



The *interquartile range*, abbreviated IQR, is just the width of the box in the box-and-whisker plot. That is, IQR = $Q_3 - Q_1$. The IQR can be used as a measure of how spread-out the values are. Statistics assumes that values are clustered around some central value. The IQR tells how spread out the "middle" values are; it can also be used to tell when some of the other values are too far from the central value. These "too far away" points are called outliers, because they lie outside the expected range.

In a real world example, given the total number of individual data points (elements) in a set, students determine how many elements are in each quartile. For example, given 22 elements both the upper half and lower half contain 11 elements each. Since the lower half contains 11 elements, the first quartile must contain 5 elements, 1 element lies on the median of the lower half, and the second quartile contains 5 elements. The same is true for the upper half; 5 elements in the third quartile, 1 element lies on the median of the upper half, and 5 elements in the fourth quartile.



A box-and-whisker plot makes it easy to see where the data are spread out and where they are concentrated. Experiences should include adding or removing a data point to determine how the original values change.

Multiple box-and-whisker plots can be used to compare and contrast sets of data. Graphing calculators may be used to compare multiple box-and-whisker plots.

Curriculum Information	Resources	Sample Instructional Strategies and Activities
SOL Reporting Category Functions and Statistics **Topic** Statistics **Virginia SOL A.10** **Foundational Objectives** **6.15** The student will a. describe mean as balance point; and b. decide which measure of center is appropriate for a given purpose.	Text: Virginia Algebra I, ©2012, Pearson Education VDOE Enhanced Scope and Sequence Sample Lesson Plans http://www.doe.virginia.gov/testing/sol/scope_sequence/mathematics_2009/index.php Virginia Department of Education Website http://www.doe.virginia.gov/instruction/mathematics/index.shtml VDOE Project Graduation www.doe.virginia.gov/instruction/graduation/project_graduation/index.shtml	• Students are given a list of their previous class test scores. Use the information obtained to make box-and-whiskers graphs. Discuss how summarizing data is helpful to analyzing data. The students will use their individual scores over a period of time. • Give the students the scores of a high school sport's team for the past four years. From this set of data, ask your students to compare and analyze the team's performance using box and whisker plots. Ask your students to use their plots to check their analysis and make new conclusions.

<table>
<tr><th>Curriculum Information</th><th>Essential Knowledge and Skills
Key Vocabulary</th><th>Essential Questions and Understandings
Teacher Notes and Elaborations</th></tr>
<tr>
<td><u>SOL Reporting Category</u>
Functions and Statistics

<u>Topic</u>
Statistics

<u>Virginia SOL A.11</u>

The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve real-world problems, using mathematical models. Mathematical models will include linear and quadratic functions.</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections and representations to:
● Write an equation for a curve of best fit, given a set of no more than twenty data points in a table, a graph, or a real-world situation.
● Make predictions about unknown outcomes, using the equation of the curve of best fit.
● Design experiments and collect data to address specific, real-world questions.
● Evaluate the reasonableness of a mathematical model of a real-world situation.

<u>Cognitive Level (Bloom's Taxonomy, Revised)</u>
Evaluate – Evaluate
Create – Write, Make, Design

<u>Key Vocabulary</u>
curve of best fit</td>
<td><u>Essential Questions</u>
● What is a curve of best fit?
● How is a curve of best fit used to make predictions in real-world situations?
● How do sample size, randomness, and bias affect the reasonableness of a mathematical model of a real-world situation?

<u>Essential Understandings</u>
● The graphing calculator can be used to determine the equation of a curve of best fit for a set of data.
● The curve of best fit for the relationship among a set of data points can be used to made predictions where appropriate.
● Many problems can be solved by using a mathematical model as an interpretation of a real-world situation. The solution must then refer to the original real-world situation.
● Considerations such as sample size, randomness, and bias should affect experimental design.

<u>Teacher Notes and Elaborations</u>
When real-life data is collected, the data graphed usually does not form a perfectly straight line or a perfect quadratic curve. However, the graph may approximate a linear or quadratic relationship. A curve of best fit is a line that best represents the given data. The line may pass through some of the points, none of the points, or all of the points. When this is the case, a curve of best fit can be drawn, and a prediction equation that models the data can be determined. A curve of best fit for the relationship among a set of data points can be used to make predictions where appropriate. Accuracy of the equation can depend on sample size, randomness, and bias of the collection.

The graphing calculator should be used to determine the equation of the curve of best fit for both linear and quadratic.

A linear curve of best fit (line of best fit) may be determined by drawing a line and connecting any two data points that seem to best represent the data. An equal number of points should be located above and below the line. This line represents the equation to be used to make predictions.

A quadratic curve of best fit may be determined by drawing a graph and connecting several points that seem to best represent the data. Entering these data points into a graphing calculator will result in a quadratic function. The graphing calculator allows for easy entry of larger data sets and therefore more accurate work. Different people may make different judgments for which points should be used, one person's equation may differ slightly from another's.</td>
</tr>
</table>

<table>
<tr><th>Curriculum Information</th><th>Resources</th><th>Sample Instructional Strategies and Activities</th></tr>
<tr><td>
<u>SOL Reporting Category</u>
Functions and Statistics

<u>Topic</u>
Statistics

<u>Virginia SOL A.11</u>

<u>Foundational Objectives</u>
8.13
The student will
a. make comparisons, predictions, and inferences, using information displayed in graphs; and
b. construct and analyze scatterplots.
</td><td>
Text:
<u>Virginia Algebra I</u>, ©2012, Pearson Education

VDOE Enhanced Scope and Sequence Sample Lesson Plans
http://www.doe.virginia.gov/testing/sol/scope_sequence/mathematics_2009/index.php

Virginia Department of Education Website
http://www.doe.virginia.gov/instruction/mathematics/index.shtml

VDOE Project Graduation
www.doe.virginia.gov/instruction/graduation/project_graduation/index.shtml
</td><td>
● The students will measure the height and weight of 10 students in the class. With x representing the height and y representing the weight, the students organize the data in table form. Then they draw a scatter plot and best-fit line on graph paper. After finding the equation of the best- fit line, the students predict the weight of a mystery student based upon his height. After the predictions have been made, the mystery student stands up.
● Dry spaghetti, string, and thread are great for students to use to informally determine where a line of best fit would be. These show up on the overhead projector so work well for demonstration there as well.
● This table shows data for speed and stopping distances of cars. Discuss with students why this will not be a linear curve of best fit. Using a graphing calculator find a quadratic equation that best represents this data. After finding the equation, students make predictions for speeds not in the table.

<table>
<tr><th>Speed</th><th>Stopping Distance</th></tr>
<tr><td>10</td><td>12.5</td></tr>
<tr><td>20</td><td>36</td></tr>
<tr><td>30</td><td>69.5</td></tr>
<tr><td>40</td><td>114</td></tr>
<tr><td>50</td><td>169.5</td></tr>
<tr><td>60</td><td>249</td></tr>
<tr><td>70</td><td>325.5</td></tr>
</table>
</td></tr>
</table>

NOTES

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