Solving Multi-Step Equations

1. Clear parentheses using the distributive property.
2. Combine like terms within each side of the equal sign.
3. Add/subtract terms to both sides of the equation to get the terms with variables on one side and constant terms on the other side.
4. Isolate the variable by multiplying/dividing both sides of the equation by the number with the variable.

Ex: $3(2x - 5) - 3 = 2x + 8 + 6x$

$$6x - 15 - 3 = 2x + 8 + 6x$$

$$6x - 18 = 8x + 8$$

$$-8 \qquad -8$$

$$6x - 26 = 8x$$

$$-6x \qquad -6x$$

$$\frac{-26}{2} = \frac{2x}{2}$$

$$-13 = x \rightarrow \boxed{x = -13}$$

Solving Absolute Value Equations

1. Isolate the absolute value.
2. Break the absolute value equation into two separate equations. For the first equation, set the expression inside the absolute value notation equal to the opposite side of the equation. For the second equation, make the number on the opposite side negative.
3. Solve each equation.

Ex: $-3|3x+2| - 2 = -8$

$$-3|3x+2| - 2 = -8$$

$$+2 \qquad +2$$

$$\frac{-3|3x+2|}{-3} = \frac{-6}{-3}$$

$$|3x+2| = 2$$

$$3x + 2 = 2 \qquad 3x + 2 = -2$$

$$x = 0 \qquad x = -\frac{4}{3}$$

$$\boxed{x = \{0, -\tfrac{4}{3}\}}$$

Solving Word Problems Algebraically

1. Define a variable.
2. Write an equation.
3. Solve the equation.
4. Label your answer with the appropriate units.

Ex: Bobby is 4 years younger than twice Jimmy's age. If Bobby is 26 years old, how old is Jimmy?

Let j = Jimmy's age

$$2j - 4 = 26$$

$$j = 15$$

→ **Jimmy is 15 years old**

Solve each equation.

1. $-3x - 9 = -27$	2. $25 + 2(n + 2) = 30$	3. $-9b - 6 = -3b + 48$
4. $5 - (m - 4) = 2m + 3(m - 1)$	5. $-24 - 10k = -8(k + 4) - 2k$	6. $f - (-19) = 11f + 23 - 20f$
7. $\frac{3}{4}d - \frac{1}{2} = \frac{3}{8} + \frac{1}{2}d$	8. $-0.5g + 13 = 3g$	9. $-5(h + 12) - (4h - 2) = h - 8$
10. $\lvert 3x + 4 \rvert = 16$	11. $3\lvert x - 5 \rvert = 27$	12. $-8\lvert 2x - 6 \rvert + 4 = -60$

Solve each word problem algebraically.

13. The sum of two consecutive integers is one less than three times the smaller integer. Find the two integers.	14. The length of a rectangular picture is 5 inches more than three times the width. Find the dimensions of the picture if its perimeter is 74 inches.

Solving & Graphing Inequalities

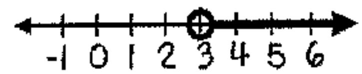
1. Solve the inequality as if it is an equation.
2. If you multiply or divide both sides of the inequality by a negative number, flip the inequality sign.
3. Write your answer with the variable on the left of the inequality sign.
4. Graph the solution on a number line. Make an open circle on the number if the number is not included in the solution (< or >) and make a closed circle if the number is included (≤ or ≥). Shade to the left for less than (< or ≤) and shade to the right for greater than (> or ≥).

Ex: $-24 > 3x - 6 - 9x$

$-24 > -6x - 6$
$+6 \qquad\quad +6$

$\frac{-18}{-6} > \frac{-6x}{-6}$

$3 < x \rightarrow \boxed{x > 3}$



Compound Inequalities

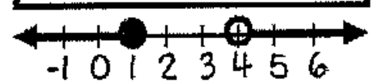
"Or" Inequalities

1. Solve each inequality separately and graph the solution to each on one number line.

Ex: $x + 2 > 6$ or $-2x \geq -2$

$x + 2 > 6$ or $\frac{-2x}{-2} \geq \frac{-2}{-2}$
$\;\;\; -2 \;\; -2$

$\boxed{x > 4 \quad \text{or} \quad x \leq 1}$



"And" Inequalities:

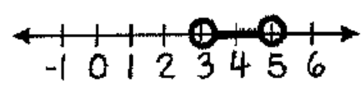
1. Isolate the variable, making sure to do the same thing to all 3 parts of the inequality.
2. Graph the solution to each part of the compound inequality and see where those graphs overlap. The overlapping part is the solution.

Ex: $3 < 2x - 3 < 7$

$3 < 2x - 3 < 7$
$+3 \qquad +3 \;\; +3$

$\frac{6}{2} < \frac{2x}{2} < \frac{10}{2}$

$\boxed{3 < x < 5}$



Absolute Value Inequalities

1. Isolate the absolute value.
2. Change the absolute value inequality into a compound inequality. For > or ≥, turn it into an "or" inequality. For < or ≤, turn it into an "and" inequality. For the first inequality, keep everything the same, except eliminate the absolute value symbols. For the second inequality, make the number on the opposite side negative and flip the inequality sign.
3. Solve and graph the compound inequality.

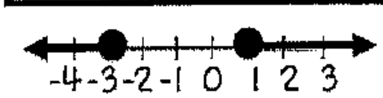
Ex: $|x + 1| - 3 \geq -1$

$|x + 1| - 3 \geq -1$
$\qquad\quad +3 \quad +3$

$|x + 1| \geq 2$

$x + 1 \geq 2$ or $x + 1 \leq -2$
$\;\;\; -1 \;\; -1 \qquad\quad -1 \;\; -1$

$\boxed{x \geq 1 \quad \text{or} \quad x \leq -3}$



Solve each inequality. Graph the solution on a number line.

15. $-6x + 3 > -39$	16. $25 - 3(n - 2) \geq -8n + 6$
17. $8g - 6(g + 1) < 4(2g - 9)$	18. $7k + 1 \leq 8$ or $-7 < k - 10$
19. $-4 < 3b + 2 \leq 20$	20. $9 < -3m < 24$
21. $y + (-6) \geq -13$ or $-3y + 8 > -7$	22. $\|2x + 5\| < 13$
23. $7\|w - 6\| \geq 21$	24. $-2\|3m\| + 3 < -51$

Finding Slope from 2 Points

Slope Formula: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Special Cases:

$\frac{0}{\#} \rightarrow$ slope = 0 $\qquad \frac{\#}{0} \rightarrow$ slope is undefined

Ex: Find the slope of the line that passes through the points (-9, -3) and (7, -7)

$$m = \frac{-7-(-3)}{7-(-9)} = \frac{-4}{16} = \boxed{-\frac{1}{4}}$$

Slope-Intercept Form

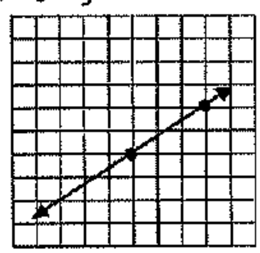
$y = mx + b$

m = slope & b = y-intercept

Graphing from Slope-Intercept Form:

1. Make a point at the y-intercept.
2. Use the slope $\left(\frac{\text{rise}}{\text{run}}\right)$ to make more points.
3. Connect the points to form a line.

Ex: Graph $y = \frac{2}{3}x - 1$



y-intercept is -1
slope = $\frac{2}{3}$, (so from the y-intercept go up 2 & right 3)

Standard Form

$Ax + By = C$

A, B, & C are integers & A is not negative

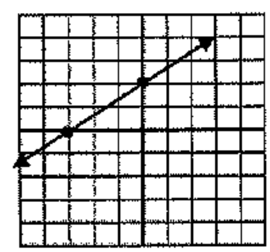
Graphing Using Intercepts:

1. Find the x-intercept by substituting 0 for y.
2. Find the y-intercept by substituting 0 for x.
3. Make a point at each intercept and then connect the points to form a line.

Ex: Graph $2x - 3y = -6$

x-intercept: $2x - 3(0) = -6$
$2x = -6 \rightarrow x = -3$
(-3, 0)

y-intercept: $2(0) - 3y = -6$
$-3y = -6 \rightarrow y = 2$
(0, 2)



Point-Slope Form

$y - y_1 = m(x - x_1)$

m = slope & (x_1, y_1) is a point on the graph

Converting Point-Slope Form to Slope-Intercept Form:

1. Distribute m.
2. Move y_1 to the other side of the equation.

Converting Slope-Intercept Form to Standard Form:

1. Bring the x term to the left.
2. If there are fractions in the equation, multiply everything through by the least common denominator.
3. If A is negative, multiply everything through by -1.

Ex: Write the equation of the line passing through the points (-1, 2) and (3, 4) in point-slope form. Then convert it to slope-intercept and standard form.

$$m = \frac{4-2}{3-(-1)} = \frac{2}{4} = \frac{1}{2}$$

Point-Slope Form: $\boxed{y - 2 = \frac{1}{2}(x + 1)}$

Convert to Slope-Intercept Form:

$\rightarrow y - 2 = \frac{1}{2}x + \frac{1}{2} \rightarrow \boxed{y = \frac{1}{2}x + \frac{5}{2}}$

Convert to Standard Form:

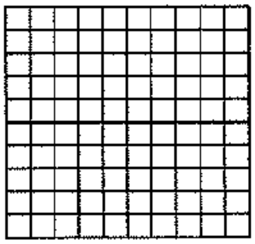
$\rightarrow -2\left(-\frac{1}{2}x + y = \frac{5}{2}\right) \rightarrow \boxed{x - 2y = -5}$

Find the slope of the line that passes through the pair of points.

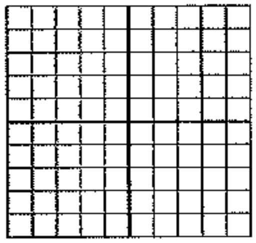
25. (9, -3) and (9, -8)	26. (-8, 5) and (3, -6)	27. (7, -1) and (15, 9)

Graph each line.

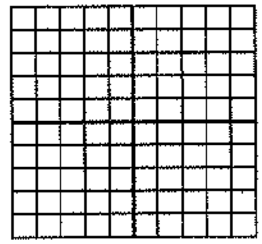
28. $y = -\frac{3}{2}x + 2$



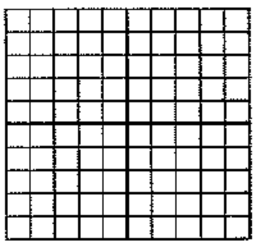
29. $y = x - 3$



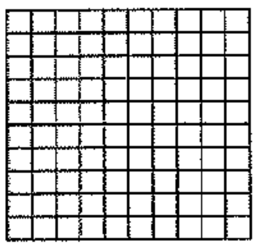
30. $y = \frac{1}{3}x + 5$



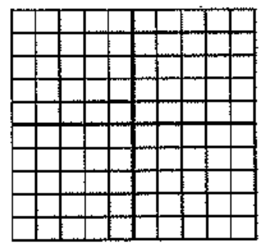
31. $2x - y = -2$



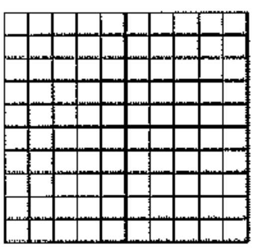
32. $x + y = 4$



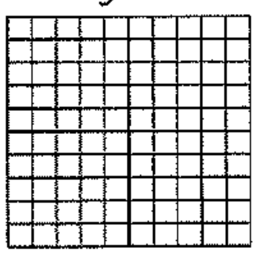
33. $3x + 4y = -12$



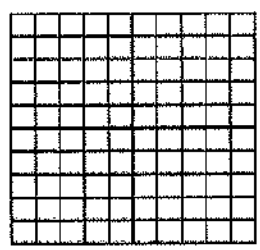
34. $y + 3 = \frac{1}{2}(x + 2)$



35. $y - 1 = \frac{2}{3}(x - 3)$



36. $y - 2 = 0$



Write the equation of the line in point-slope, slope-intercept, and standard form.

37. Line passing through point (3, 5) with a slope of 1	38. Line passing through points (-4, 2) and (0, 3)	39. Line passing through points (1, 3) and (2, 5)

Parallel & Perpendicular Lines

Parallel Lines have the *same slope* but different y-intercepts.

Perpendicular Lines have *opposite reciprocal slopes.*

Writing Equations of Parallel Lines:

1. Find the slope of the original line by first converting it to slope-intercept form if it is in Standard Form. The slope of the line parallel will have that same slope.
2. Use the given point along with the slope you just found to write the equation of the line in point-slope form.
3. Convert the point-slope form equation to slope-intercept form.

Writing Equations of Perpendicular Lines:

1. Find the slope of the original line. The slope of the line perpendicular will have the opposite (negative) reciprocal slope.
2. Use the given point along with the slope you just found to write the equation of the line in point-slope form.
3. Convert the point-slope form equation to slope-intercept form.

Ex: Write the equation of the line that is parallel to the line $y = 3x - 5$ and passes through the point $(-2, 4)$.

$y = 3x - 5$

$m = 3$, so slope of parallel line is 3, too

→ $y - 4 = 3(x + 2)$

→ $y - 4 = 3x + 6$

→ $\boxed{y = 3x + 10}$

Ex: Write the equation of the line that is perpendicular to the line $x - 3y = -6$ and passes through the point $(-1, 1)$.

$x - 3y = -6$ → $-3y = -x - 6$

→ $y = \frac{1}{3}x + 2$

$m = \frac{1}{3}$, so slope of perpendicular line is -3

→ $y - 1 = -3(x + 1)$

→ $y - 1 = -3x - 3$

→ $\boxed{y = -3x - 2}$

Linear Inequalities

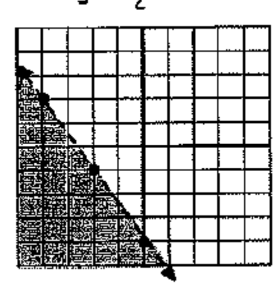
1. Convert the linear inequality in slope-intercept form. Be sure the y is on the left and remember to flip the inequality sign if you multiply or divide by a negative.
2. Graph the line as if it is an equation, except use a dotted line if the inequality sign is < or >. If the sign is ≤ or ≥, use a regular solid line.
3. Shade above the line for a "greater than" inequality (> or ≥). Shade below the line for a "less than" inequality (< or ≤). (For vertical lines, shade to the right for greater than and to the left for less than).

Ex: $-3x - 2y > 8$

$$\begin{array}{r} -3x - 2y > 8 \\ +3x \qquad\quad +3x \\ \hline \end{array}$$

$$\frac{-2y}{-2} > \frac{3x}{-2} + \frac{8}{-2}$$

$$y < -\frac{3}{2}x - 4$$



Determine whether the lines are parallel, perpendicular, or neither. Justify your answer.

40. $y = 2x - 8$
$y = \frac{1}{2}x + 6$

41. $y = x$
$x + y = -2$

42. $3x + 2y = 18$
$y + 4 = -\frac{3}{2}(x - 4)$

Write the equation of the line parallel to the given line that passes through the given point in slope-intercept form.

43. $y = -4x - 2$; $(0, -1)$

44. $2x - y = -4$; $(2, 5)$

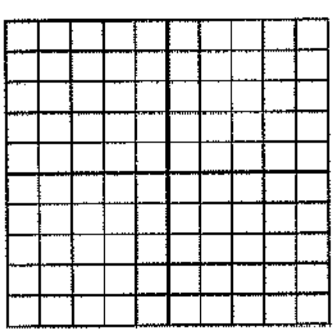
Write the equation of the line perpendicular to the given line that passes through the given point in slope-intercept form.

45. $y = \frac{2}{3}x - 9$; $(-6, -2)$

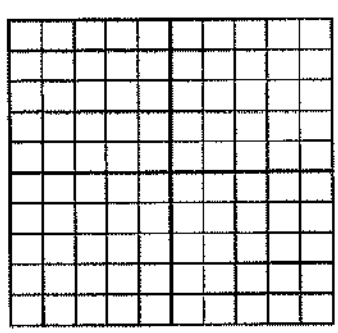
46. $4x + y = -6$; $(4, 5)$

Graph the solution to each linear inequality.

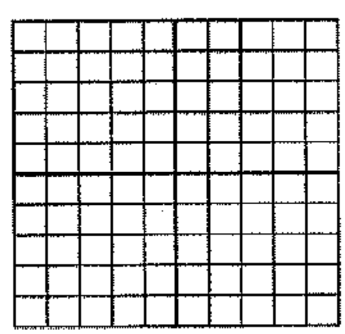
47. $y \leq -4x - 3$



48. $2x - y < 1$



49. $x + 3y > 3$

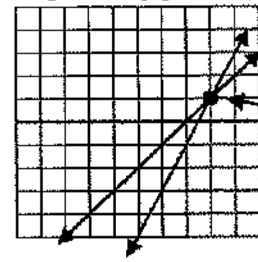


Solving Systems of Equations by Graphing

1. Graph both lines on the same coordinate plane.
2. Find the point where the lines meet, and write that solution as an ordered pair.

<u>Special Cases:</u>
- parallel lines: no solution
- coincident lines (lines that are the same): infinitely many solutions

Ex: Solve the system by graphing: $\begin{cases} y = x - 2 \\ y = 2x - 5 \end{cases}$



solution: **(3, 1)**

Solving Systems of Equations Using Substitution

1. Solve one of the equations for x or y.
2. Replace the x or y in the other equation with the expression you found in step 1 that equals that variable.
3. Solve the equation.
4. Substitute the solution you found in step 3 with the variable in your step 1 equation to solve for the other variable.
5. Write your solution as an ordered pair.

Ex: Solve the system by substitution: $\begin{cases} x + 3y = 4 \\ 2x - 3y = -1 \end{cases}$

$x + 3y = 4 \rightarrow x = -3y + 4$

$2x - 3y = -1 \rightarrow 2(-3y + 4) - 3y = -1$

$\rightarrow -6y + 8 - 3y = -1$

$\rightarrow -9y + 8 = -1$

$\rightarrow -9y = -9 \rightarrow \boxed{y = 1}$

$\rightarrow x = -3y + 4 \rightarrow x = -3(1) + 4 \rightarrow \boxed{x = 1}$

solution: **(1, 1)**

Solving Systems of Equations Using Elimination

1. Write both equations in Standard Form.
2. Multiply neither, one, or both of the equations by constants so that either the x coefficients or the y coefficients are opposites (i.e. 2 and -2).
3. Add the two equations. The terms with the opposite coefficients will cancel out.
4. Solve the equation for the variable that didn't cancel out.
5. Substitute the solution you found in step 4 for the variable in any of the equations, and solve to find the other variable.
6. Write your solution as an ordered pair.

Ex: Solve the system by elimination: $\begin{cases} 3x + 4y = 2 \\ -2x + 2y = -6 \end{cases}$

Multiplying by -2 will give the y-terms opposite coefficents

$3x + 4y = 2$

$-2(-2x + 2y = -6)$

$\rightarrow$ $3x + 4y = 2$
$+ \; 4x - 4y = 12$
$7x \quad = 14 \rightarrow \boxed{x = 2}$

$\rightarrow 3x + 4y = 2 \rightarrow 3(2) + 4y = 2$

$\rightarrow 6 + 4y = 2 \rightarrow 4y = -4 \rightarrow \boxed{y = -1}$

solution: **(2, -1)**

Systems of Equations Word Problems

1. Define 2 variables.
2. Write 2 equations.
3. Solve the system of equations using the method of your choice.
4. Label your solution with the appropriate units.

Ex: A 24 question test contains some 3 point questions and some 5 point questions. If the test is worth 100 points, how many of each type of questions are there?

Let x = # of 3 point questions
y = # of 5 point questions

$x + y = 24$

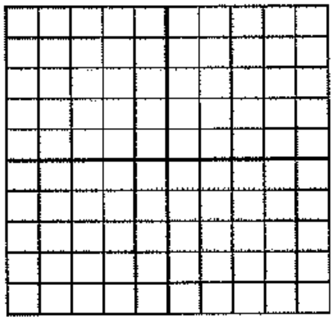
$3x + 5y = 100$

solve using substitution or elimination → solution: (10, 14)

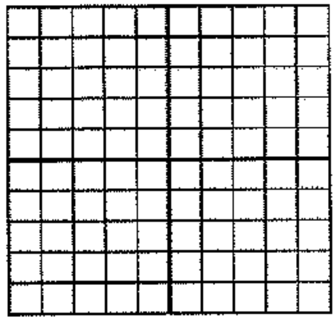
→ **There were 10 3-point questions and 14 5-point questions.**

Solve each system of equations by graphing.

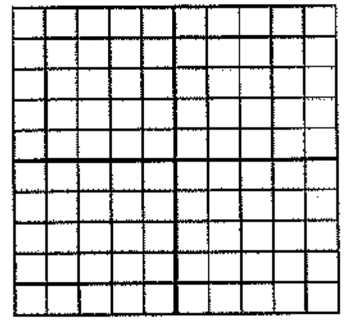
50. $\begin{cases} y = \frac{1}{2}x - 4 \\ y = -x - 1 \end{cases}$



51. $\begin{cases} y = 2x + 1 \\ -y = -2x + 1 \end{cases}$



52. $\begin{cases} x - 2y = 4 \\ -3x + 2y = -8 \end{cases}$



Solve each system of equations using substitution.

53. $\begin{cases} y = 2x + 3 \\ 5x - 2y = -6 \end{cases}$

54. $\begin{cases} x + 4y = 5 \\ -2x + 5y = 16 \end{cases}$

55. $\begin{cases} 9y - 7x = -13 \\ -9x + y = 15 \end{cases}$

Solve each system of equations using elimination.

56. $\begin{cases} 3x - 7y = -29 \\ -4x + 7y = 27 \end{cases}$

57. $\begin{cases} -4x - 8y = -48 \\ 8x + 3y = -34 \end{cases}$

58. $\begin{cases} 3x - 7y = 21 \\ 6x = 14y + 42 \end{cases}$

Solve each word problem using a system of equations.

59. Joe bought 5 apples and 4 bananas for \$6. Dawn bought 3 apples and 6 bananas for \$6.30. How much does each apple and each banana cost?

60. Wesley and Brian have a total of 87 baseball cards. Wesley has 30 less than twice as many cards as Brian. How many baseball cards do they each own?

Exponent Rules

Zero Exponent: Any base raised to the zero power equals 1. Ex: $(-9)^0 = \boxed{1}$

Negative Exponent: Move the base to the opposite side of the fraction bar and make the exponent positive. Ex: $3^{-4} = \frac{1}{3^4} = \boxed{\frac{1}{81}}$

Monomial x Monomial: Multiply the coefficients and add the exponents of like bases. Ex: $(-2x^3)(8x^{-5}) = -16x^{-2} = \boxed{\frac{-16}{x^2}}$

Monomial ÷ Monomial: Divide the coefficients and subtract the exponents of like bases. Ex: $\frac{4ab^3}{4a^2b^2} = 1a^{-1}b^1 = \boxed{\frac{b}{a}}$

Power of a Monomial: Raise each base (including the coefficient) to that power. If a base already has an exponent, multiply the two exponents. Ex: $(3x^3y^2)^3 = 3^3x^9y^6 = \boxed{27x^9y^6}$

Power of a Quotient: Raise each base (including the coefficients) to that power. If a base already has an exponent, multiply the two exponents. Ex: $\left(\frac{5a^3b}{2c^{-1}}\right)^2 = \frac{5^2a^6b^2}{2^2c^{-2}} = \boxed{\frac{25a^6b^2c^2}{4}}$

Multiplying & Dividing Numbers in Scientific Notation

Multiplying Numbers in Scientific Notation:
Multiply the coefficients and add the exponents. If necessary, "fix" the answer to put it in Scientific Notation.

Ex: $(3 \times 10^4)(5.8 \times 10^7)$

$= (3 \times 5.8) \times 10^{4+7}$

$= 17.4 \times 10^{11}$

$= (1.74 \times 10^1) \times 10^{11} = \boxed{1.74 \times 10^{12}}$

Dividing Numbers in Scientific Notation:
Divide the coefficients and subtract the exponents. If necessary, "fix" the answer to put it in Scientific Notation.

Ex: $\frac{3.6 \times 10^5}{7.2 \times 10^2}$

$= (3.6 \div 7.2) \times 10^{5-2}$

$= 0.5 \times 10^3$

$= (5 \times 10^{-1}) \times 10^3 = \boxed{5 \times 10^2}$

Exponential Growth & Decay

Exponential Growth: $y = a(1 + r)^t$

Exponential Decay: $y = a(1 - r)^t$

y = new amount, a = initial amount, r = rate of change (as a decimal), t = time

Ex: You bought a new car for $25,000. If the car's value depreciates at a rate of 12% per year, how much will the car be worth in 5 years?

use exponential decay formula

$y = 25{,}000(1 - 0.12)^5$

$= 25{,}000(0.88)^5$

$= \boxed{\$13{,}193.30}$

Compound Interest: $A = P(1 + \frac{r}{n})^{nt}$

A = new balance, P = principal (starting value), r = interest rate (as a decimal), n = number of times the interest is compounded annually, t = time (in years)

Ex: You invest $5,000 in an account with a 2.5% interest rate, compounded monthly. How much money will be in the account after 20 years?

$A = 5{,}000(1 + \frac{0.025}{12})^{12 \cdot 20}$

$= 5{,}000(1 + \frac{0.025}{12})^{240}$

$= \boxed{\$8{,}239.32}$

Simplify each expression completely. Write your answer using only positive exponents.

61. $x^6 \cdot x^4$	62. $(5^3)^2$	63. $-6a^2b^{-4}c \cdot 4ab^2$
64. $\frac{a^3b^{-6}}{c^{-2}}$	65. $\left(\frac{-2x^6y}{3z^5}\right)^3$	66. $\left(8w^3q^{-5}\right)^0$
67. $\frac{24d^5f^{-5}g^8}{36d^{-3}f^9g^2}$	68. $\left(2b^{-3}d^6\right)^4 \cdot 3b^7d$	69. $\left(\frac{-4a^4b^2c^{-1}}{6a^9}\right)^{-1}$

Find each product or quotient. Write your answer in Scientific Notation.

70. $\left(9.8 \times 10^3\right)\left(2.4 \times 10^7\right)$	71. $\frac{9.3 \times 10^3}{3 \times 10^9}$	72. $\frac{4.5 \times 10^{13}}{9.0 \times 10^7}$

Find the new amount.

73. The population of Watesville decreases at a rate of 1.6% per year. If the population was 62,500 in 2014, what will it be in 2020?	74. A population of 30 bunnies is increasing at a rate of 40% per year. How many bunnies will there be in 5 years?	75. If you $15,000 in an account with a 4.5% interest rate, compounded quarterly, how much money will you have in 25 years?

Classifying Polynomials

- Term: each part of a polynomial separated by addition or subtraction
- Degree of a Term: the sum of the exponents of the variables in a term
- Degree of Polynomial: the highest degree of all the terms in a polynomial

Classifying By Number of Terms:

1 term: monomial

2 terms: binomial

3 terms: trinomial

≥ 4 terms: n-term polynomial

Classifying Polynomials By Degree:

0: constant	4: quartic
1: linear	5: quintic
2: quadratic	≥6: nth degree
3: cubic	

Ex: Classify $3x^3 - 9x + 7$.

It is a trinomial because there are 3 terms separated by - and +

The degree of the 1st term is 3, the degree of the 2nd term is 1, and the degree of the 3rd term is 0. So, the degree of the polynomial is 3 since that is the highest degree of all the terms.

→ **Is it a cubic trinomial**

Adding & Subtracting Polynomials

Adding Polynomials:

1. Add like terms together.
2. Write your answer in Standard Form (decreasing order of degree).

Ex: $(4x^2 - 9) + (7x - 9x^2 + 8)$

$(4x^2 - 9) + (7x - 9x^2 + 8)$

$= -5x^2 - 1 + 7x \rightarrow \boxed{-5x^2 + 7x - 1}$

Subtracting Polynomials:

1. Turn into an addition problem by changing the – to + between the two polynomials and reversing the sign of each term in the second polynomial.
2. Add like terms together.
3. Write your answer in Standard Form.

Ex: $(3x^2 - 6x - 9) - (2x^2 + 8x - 3)$

$\rightarrow (3x^2 - 6x - 9) + (-2x^2 - 8x + 3)$

$= \boxed{x^2 - 14x - 6}$

Multiplying Polynomials

Monomial x Polynomial:

1. Use the Distributive Property to multiply the monomial by each term.
2. Write your answer in Standard Form.

Ex: $4x^2(3x^2 - 8x - 5)$

$4x^2(3x^2 - 8x - 5)$

$= \boxed{12x^4 - 32x^3 - 20x^2}$

Binomial x Binomial:

1. FOIL (multiply the two first terms, the two outer terms, the two inner terms, and the two last terms).
2. Combine like terms and write your answer in Standard Form.

Ex: $(x + 3)(2x - 1)$

$(x + 3)(2x - 1)$

F: $2x^2$ O: $-1x$ I: $6x$ L: -3

$= \boxed{2x^2 + 5x - 3}$

Any Polynomial x Any Polynomial:

1. Multiply each term from the first polynomial by each term in the second polynomial.
2. Combine like terms and write your answer in Standard Form.

Ex: $(x + 2)(x^2 - 3x - 8)$

$(x + 2)(x^2 - 3x - 8)$

$= x^3 - 3x^2 - 8x + 2x^2 - 6x - 16$

$= \boxed{x^3 - x^2 - 14x - 16}$

Classify each polynomial by its degree and number of terms.

76. $8x^3 - 9x$	77. $-2 - 4x^2 + 7x$	78. $8x^2y^2$	79. $6x + 5$

Find each sum or difference. Write your answer in Standard Form.

80. $(2h^3 + 6h) + (3h^3 - 7h - 3)$	81. $(8x - 4x^2 + 3) - (7x^2 - 9)$	82. $(-14a^2 - 5) - (5a^2 + 6a - 7)$

Find each product. Write your answer in Standard Form.

83. $5x^3(9x^2 + 4x - 5)$	84. $(x + 4)(x - 3)$	85. $(3n - 8)(4n - 7)$
86. $(2x + 3)(x^2 + x + 3)$	87. $(6x + 1)^2$	88. $4g(2g - 9)(2g + 9)$

Simplify each expression completely. Write your answer in Standard Form.

89. $(x + 2)(x + 8) + (4x^2 + 8x - 3)$	90. $(x + 5)(x - 5) - 6x(x + 1)$

Factoring Out a GCF

1. Find the largest monomial that is a factor of each term in the polynomial, and pull it out in front of parentheses.
2. Divide each term by the GCF and write the resulting polynomial in the parentheses.

Ex: Factor $25x^4y - 30x^3y^2 + 10x^2y^3$

GCF = $5x^2y$, so divide each term by $5x^2y$

→ $\boxed{5x^2y(5x^2 - 6xy + 2y^2)}$

Factoring 4-Term Polynomials

First factor out a GCF if there is one. Then factor by grouping as described below.

<u>Factor by Grouping</u>

1. Group the first two terms in parentheses and the last two terms in parentheses.
2. Factor out the GCF from each set of parentheses. (The two resulting binomials in parentheses should match).
3. Factor out the common binomial.

Ex: Factor $3x^3 - 6x^2 + 5x - 10$

$(3x^3 - 6x^2) + (5x - 10)$

→ $3x^2(x - 2) + 5(x - 2)$

→ $\boxed{(x - 2)(3x^2 + 5)}$

Factoring Binomials

First factor out a GCF if there is one. Then determine whether it is a difference of squares binomial (in the form $a^2 - b^2$). If it is, use the method below.

<u>Binomials in the form $a^2 - b^2$</u>

1. Find the square root of the first term (a) and the square root of the second term (b). Your answer will be $(a + b)(a - b)$.

Ex: Factor $16x^2 - 25$

The square root of $16x^2 = 4x$ & the squre root of $25 = 5$

→ $\boxed{(4x + 5)(4x - 5)}$

Factoring Trinomials

First factor out a GCF if there is one. Then use the appropriate method below, depending on whether or not the leading coefficient is 1.

<u>Trinomials in the form $x^2 + bx + c$ (leading coefficient = 1)</u>

1. Find two numbers with a product of c and a sum of b.
2. Your answer will be written as the product of two binomials: (x + 1st number)(x + 2nd number).

Ex: Factor $x^2 - 6x + 8$

Need 2 numbers with product of 8 and sum of -6.

→ the 2 numbers are -4 & -2

→ $(x + -4)(x + -2)$ → $\boxed{(x - 4)(x - 2)}$

<u>Trinomials in the form $ax^2 + bx + c$ (leading coefficient > 1)</u>

1. Multiply a and c. Find two numbers with a product of ac and a sum of b.
2. Copy the ax^2 term from the original trinomial, and then split up the bx term into two terms, using the two numbers you found in step 1 as the coefficients of each term. Copy the c term from the original trinomial. (So now you have a 4-term polynomial).
3. Factor by grouping.

Ex: Factor $2x^2 + 7x + 3$

$ac = 2 \cdot 3 = 6$. Need 2 numbers with a product of 6 and a sum of 7.

→ the 2 numbers are 6 & 1

→ $2x^2 + 6x + 1x + 3$

→ $(2x^2 + 6x) + (1x + 3)$

→ $2x(x + 3) + 1(x + 3)$

→ $\boxed{(x + 3)(2x + 1)}$

Factor each polynomial completely.

91. $-18x - 27$	92. $x^2 - 100$	93. $x^2 - 5x + 6$
94. $2x^2 + 7x + 6$	95. $5x^3 + 3x^2 + 10x + 6$	96. $3x^2 - 12$
97. $x^2 + 24x + 144$	98. $9x^3 - 30x^2 - 24x$	99. $8x^3 + 4x^2 - 6x - 3$
100. $5x^2 + 10x - 45$	101. $36x^4 - 121$	102. $5x^2 + 22x + 8$
103. $4x + 16xy + 9y + 36y^2$	104. $x^2 - 3x - 88$	105. $4x^2 - 15x + 9$

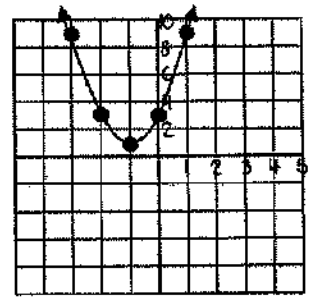
Graphing Quadratic Functions

1. Write the equation in standard form: $y = ax^2 + bx + c$.
2. Find the equation of the axis of symmetry: $x = \frac{-b}{2a}$.
3. Find the vertex of the parabola. The x-coordinate is $\frac{-b}{2a}$. To find the y-coordinate, substitute the x-coordinate for x in the equation and solve for y.
4. Make a table of values by choosing 2 x-values to the left of the axis of symmetry and 2 x-values to the right of the axis of symmetry and substituting them into the equation to find the y-values.
5. Connect the points to form a parabola.

Ex: Graph $y = 2x^2 + 4x + 3$

axis of symmetry: $x = \frac{-4}{2(2)} = -1$

[illegible]	[illegible]
-3	9
-2	3
-1	1
0	3
1	9



Solving Quadratic Equations by Factoring

1. Write the quadratic equation in Standard Form ($ax^2 + bx + c = 0$).
2. Factor the left side of the equation.
3. Use the zero-product property to solve the equation by setting each factor equal to zero and solving for x.

Ex: $x^2 - 6x = 16$

→ $x^2 - 6x - 16 = 0$

→ $(x - 8)(x + 2) = 0$

→ $x - 8 = 0$ $x + 2 = 0$

$x = 8$ or $x = -2$

Solving Quadratic Equations Using Square Roots

*** Only for quadratic equations where $b = 0$. ***

1. Write the equation in the form $ax^2 = c$.
2. Divide both sides of the equation by a.
3. Take the square root of both sides of the equation. Be sure to find both the positive and negative square root!

Ex: $4x^2 - 32 = 0$

$4x^2 = 32$

→ $x^2 = 8$

→ $x = \pm\sqrt{8} \approx$ **± 2.83**

Solving Quadratic Equations Using the Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. Write the quadratic equation in Standard Form ($ax^2 + bx + c = 0$).
2. Substitute a, b, and c into the quadratic formula to find the solution(s) for x.

Ex: $3x^2 + 7x - 8 = 0$

$x = \frac{-7 + \sqrt{7^2 - 4(3)(-8)}}{2(3)}$ → **$x \approx 0.84$**

$x = \frac{-7 - \sqrt{7^2 - 4(3)(-8)}}{2(3)}$ → **$x \approx -3.17$**

Graph each quadratic equation.

106. $y = x^2 - 4x + 1$

x	y

107. $y = -x^2 + 2x + 3$

x	y

Solve each quadratic equation using the method of your choice. Round to the nearest tenth.

108. $x^2 - 3x + 2 = 0$

109. $4x^2 - 40 = 0$

110. $2x^2 + x = 45$

111. $7x^2 + 5x = -2$

112. $x^2 - 9x = 0$

113. $x^2 - 20x = 84$

114. $-15x^2 = -900$

115. $-5x^2 + 17x + 13 = 0$

116. $x^2 - 24x + 22 = -22$

Solve each word problem using a quadratic equation.

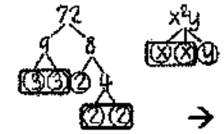
117. The height of an object t seconds after it is thrown from a height of h feet is modelled by the equation $h(t) = -16t^2 + vt + h$. If the ball is thrown from a point 6 feet above ground with an initial velocity, v, of 30 feet per second, how long will it take for the ball to hit the ground?

118. The length of a rectangle is 3mm less than four times the width. If the area of the rectangle is 1,387 mm^2, what are the dimensions of the rectangle?

Simplifying Radicals

1. Find the prime factorization of the radicand, including factoring the variables.
2. Find pairs of the same prime factor or variable.
3. Pull out one from each pair, and multiply everything you pull out. Multiply the remaining prime numbers and variables and write that product under the square root sign.

Ex: Simplify $\sqrt{72x^2y}$



$\rightarrow \boxed{3 \cdot 2 \cdot x}\sqrt{2 \cdot y}$

$\rightarrow \boxed{6x\sqrt{2y}}$

Operations with Radicals

Adding & Subtracting Radicals

1. Simplify the radicals if possible.
2. Add or subtract the numbers in front of like radicals.

Ex: $\sqrt{20} + \sqrt{45}$

$\rightarrow 2\sqrt{5} + 3\sqrt{5} = \boxed{5\sqrt{5}}$

Multiplying Radicals

1. Follow the rules for multiplying polynomials (i.e. distributive property, FOIL, etc.). Multiply the numbers in front of the radicals together and multiply the numbers under the radicals together.
2. Simplify the radical if possible.

Ex: $3\sqrt{6} \cdot 4\sqrt{10}$

$\rightarrow 12\sqrt{60} \rightarrow 12 \cdot 2\sqrt{15} = \boxed{24\sqrt{15}}$

Dividing Radicals

1. Divide the numbers in front of the radicals and the numbers under the radicals.
2. If you have a radical in the denominator, you need to rationalize the denominator.
   - For a single radical term in the denominator, multiply the numerator and denominator by that same radical.
   - For a two-term denominator containing one or two radicals, multiply the numerator and denominator by the conjugate of the denominator. ($a + b$ and $a - b$ are conjugates)

Ex: $\frac{\sqrt{9}}{\sqrt{18}}$

$\rightarrow \frac{\sqrt{1}}{\sqrt{2}} \rightarrow \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$

Ex: $\frac{3}{\sqrt{5}+3}$

$\frac{3}{\sqrt{5}+3} \cdot \frac{(\sqrt{5}-3)}{(\sqrt{5}-3)} = \frac{3\sqrt{5}-9}{5 - \cancel{3\sqrt{5}} + \cancel{3\sqrt{5}} - 9}$

$= \boxed{\frac{3\sqrt{5}-9}{-4}}$

Radical Equations

1. Isolate the radical if possible.
2. Square both sides of the equation.
3. Solve the equation for the variable. (If it is a quadratic equation, you may find two solutions).
4. CHECK YOUR ANSWER(S) by substituting them in for the variable(s) in the original equation. (Some solutions may end up being extraneous).

Ex: $\sqrt{x+6} - x = 0$

$\sqrt{x+6} - x = 0 \rightarrow \sqrt{x+6} = x$

$\rightarrow (\sqrt{x+6})^2 = x^2$

$\rightarrow x + 6 = x^2 \rightarrow x^2 - x - 6 = 0$

$\rightarrow (x-3)(x+2) = 0 \rightarrow x = 3$ or -2

check 3: $\sqrt{3+6} - 3 = 0$ good

check -2: $\sqrt{-2+6} - (-2) \neq 0$ extraneous

$\rightarrow$ solution: $\boxed{x = 3}$

Simplify each radical.

119. $\sqrt{90}$	120. $\sqrt{54a^3b^4}$	121. $\sqrt{600x^2y^2z}$

Simplify each radical expression.

122. $\sqrt{18} - \sqrt{50}$	123. $2\sqrt{5}(\sqrt{3} + 8\sqrt{4})$	124. $\frac{5}{\sqrt{3}}$
125. $7\sqrt{3} + 2\sqrt{12} - 3\sqrt{27}$	126. $(\sqrt{2} + 3\sqrt{3}) \cdot (\sqrt{6} - 4\sqrt{2})$	127. $\frac{2}{\sqrt{5} - \sqrt{3}}$

Solve each radical equation.

128. $\sqrt{3x} - 27 = 0$	129. $\sqrt{2x + 11} = \sqrt{6x - 7}$	130. $\sqrt{4x + 1} = x - 1$