Summit Public Schools
Summit, New Jersey
Grade Level: 10-12/ Content Area: Mathematics

Curriculum:
Algebra 2

Course Description: The course is a continuation of work begun in Algebra l. It emphasizes facility with algebraic expressions and forms, especially linear and quadratic forms, powers and roots, and functions based on these concepts. Students study exponential, logarithmic, polynomial, trigonometric, and other special functions both for their abstract properties and as tools for modeling real-world situations. Geometric ideas are utilized throughout, and applications of statistical analysis are examined. Students will utilize graphing technologies throughout the course.

Revised (2021) by:
Cheryl Adair

<table>
<tr><th colspan="2">Unit 1: Analyzing Equations and Inequalities/Prerequisites</th></tr>
<tr><td colspan="2">Big Ideas: Course Objectives/Content Statement(s)

● Number Types: Rational, Irrational, Integers, Whole, and Natural Numbers
● Solving Equations
● Solving Absolute Value Equations and Inequalities</td></tr>
<tr><td>Essential Questions
What provocative questions will foster inquiry, understanding, and transfer of learning?</td><td>Enduring Understandings
What will students understand about the big ideas?</td></tr>
<tr><td>● How do we apply properties of real numbers to simplify expression and solve equations?
● Why do absolute value equations usually have two solutions?
● What are functions and how do we use them?
● How do transformations affect the parent absolute value function?</td><td>Students will understand that:
● Properties of real numbers help to simplify expressions and make it easier to find the solutions to even the most complicated equations.
● Absolute value represents the distance from zero and thus, x can have two solutions.
● Functions are commonly represented in four ways; verbally, numerically, graphically, and algebraically.
● Depending on the type of transformation, the graph of the absolute value function will move, shift and/or change shape and size.</td></tr>
<tr><td>Areas of Focus: Proficiencies
(New Jersey Student Learning Standards)</td><td>Lessons</td></tr>
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Students will:

A-CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

A-CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.

A-REI.B.3 Solve Linear equations and inequalities in one variable, including equations with coefficients represented by letters.

F-BF.B.3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
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Lesson 1:
Simplify number types using order of operations and then classify that number by the highest order that the number falls into.
Lesson 2:
Rewrite equations and common formulas with more than one variable and use the formula to solve problems.
Lesson 3:
Solve absolute value equations by relating to distance.
Lesson 4:
Determine whether relations between two variables are functions and identify and describe the domain and range of a relation using interval notation.
Lesson 5:
Describe the effects of the parameter changes on the graph of the absolute value function and find the domain and range in interval notation of these graphs.
Write an absolute value function given a graph.
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<tr>
<th>Differentiation</th>
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Interdisciplinary Connections:
Students can research the hot-air balloon ride by James Glaisher and Henry Coxwell in 1862 and calculate the changes in temperature that caused the men to have difficulty.
Technology Integration:
Students can use the STO> button on the TI-83 calculator to help solve complicated substitution
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<td>
Formative Assessments:
Students can complete and create a Venn-diagram to explain and represent number types.
Students can explain the process for solving an equation algebraically and graphically.
Students can use Desmos to describe basic absolute value function transformations.
Students can explain why a vertical line, rather
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problems.

- Students can solve linear equations using the table feature on the II-83 calculator.

Media Literacy Integration:

- Students can use a local news channel to find the high and low temperatures for any day and write compound inequalities representing the temperature range in degrees Fahrenheit, Celsius and Kelvin.

Global Perspectives:

- Students can use to find the day of the week for which an important date took place
  d = day of the month
  m = month

$$w = d + 2m + \left[\frac{3(m+1)}{5}\right] + y + \left[\frac{y}{4}\right] - \left[\frac{y}{100}\right] + \left[\frac{y}{400}\right] + 2$$

d = day of the month
m = month

March (3)	September (9)	*Note: January and February are considered the 13th and 14th months of the previous year. Example: Feb 22, 1996 is (14/22/95)*
April (4)	October (10)	
May (5)	November (11)	
June (6)	December (12)	
July (7)	January (13)	
August (8)	February (14)	

y = the year

Students can research a famous date from another country (suggestion-independence dates, beginning of wars,…) and have students figure out what day of the week these occurred on.

than a horizontal line is used to determine if a graph represents a function.

- Students can use the equation for the volume of a sphere $V(r) = \frac{4}{3}\pi r^3$ to find and explain $V(2)$.

Summative Assessments, Projects, and Celebrations:

- One unit test and one quiz.
- Students can research population and waste production in the US to find linear models to make predictions for future expectations.

Unit 2: Systems of Equations and Inequalities

Big Ideas: *Course Objectives/Content Statement(s)*

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- Solve systems by graphing, elimination, and substitution
- Solve system word problems
- Solve systems of three variables
- Use technology to solve systems

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What provocative questions will foster inquiry, understanding, and transfer of learning?</td><td>Enduring Understandings
What will students understand about the big ideas?</td></tr>
<tr><td>

- How many solutions can exist given a system of equations?
- How can solutions to linear help with cost effectiveness?
- How are systems of equations and inequalities useful?

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Students will understand that:

- A system of equations can have one solution (ie. there is exactly one point that each line shares).
- A system of equations can have an infinite number of solutions (ie. the linear functions share all points because the lines share a common slope and y-intercept).
- A system of equations can have no solution (ie. the lines share no common points because the lines share a common slope but a different y-intercept).
- Real world problems may be represented by the formation and solution of linear equations.
- Linear programming can be used in making real-life economic decisions.

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<tr><td>Areas of Focus: Proficiencies
(New Jersey Student Learning Standards)</td><td>Lessons</td></tr>
</table>

Students will: A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. A-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. A-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). A-REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* A-REI.D.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality) and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	**Lesson 1:** ● Solve a system of equations graphically by hand. ● Solve a system of equations graphically with the use of technology by finding intersections on a graphing calculator. **Lesson 2:** ● Solve a system of linear equations algebraically using substitution and elimination. ● Algebraically determine when a system has no solution or infinitely many solutions. **Lesson 3:** ● Solve word problems involving systems of linear equations by using substitution and elimination **Lesson 4:** ● Solve systems of 2 or more inequalities and determine whether a point lies in its solution set. **Lesson 5:** ● Solve a system of linear equations in three variables algebraically and using matrices (RREF) on the graphing calculator. **Lesson 6:** ● Set up and solve 3X3 word problems involving systems of linear equations and use the graphing calculator to solve such problems. **Lesson 7:** ● Create constraints and graph a system of inequalities to find the feasible region. ● Find the values that minimize or maximize an objective function.
Differentiation	**Assessments**

Interdisciplinary Connections and Integration	Assessments
Interdisciplinary Connections: ● Students can complete the following problem: A portion of the subway in Washington DC heads out of the main part of town in the northwestern direction. It goes under New Hampshire Ave as shown at the right. If the distances are measured in kilometers, the path of the subway can be represented by the equation and the path of New Hampshire Ave can be represented by the equation y = x. What are the coordinates of the point at which the subway goes under New Hampshire Ave. **Technology Integration:** ● Students can use their graphing calculator to find a solution to a system of equations and to solve systems with matrices. **Media Literacy Integration:** ● Students can research real life scenarios for systems of equations. **Global Perspectives:** ● Students can research how a time-saving linear programming method, developed by Narendra Karmarkar, is used in companies today.	**Formative Assessments:** ● Students can complete the problem solving activity below with a partner: ○ One day, while repairing a watch, a watchmaker removed the hour and minute hands. But, she put the hands back on the opposite spindles. When the customer picked up the watch, the time correctly showed 2:00pm. When is the next time the watch showed the correct time? ○ Solve using a table. ○ Solve using a system (Hint: Let (D) represent the distance the minute hand moves on the watch and (d) represent the distance the hour hand moves on the watch. Write two equations, one for the wrong watch and one for a correct watch. ● Students can explain what must occur for an ordered pair to be a solution to a system of equations. ● Students can create systems of equations word problems and have a partner solve it. **Summative Assessments, Projects, and Celebrations:** ● One unit test and one sectional quiz. ● Students can complete the [Life of Trees Project]().

Unit 3: Quadratics	
Big Ideas: *Course Objectives/Content Statement(s)* ● Find the solutions to a quadratic function by graphing, factoring, completing the square and using the quadratic equation ● Perform operations with complex numbers ● Analyze graphs of quadratic functions	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*

• What does it mean to be a root to a quadratic equation? • What are ways we can find roots? • How many roots does a quadratic equation have? • What is an imaginary number and how do they form the set of complex numbers? • What types of real-world situations are modeled by quadratic relationships?	**Students will understand that:** • Roots are solutions to a quadratic equation. They are also known as zeros. • A root can be found where the quadratic function crosses the *x*-axis. • A quadratic function can have 0, 1 or 2 real roots. If a function has zero real roots, then the roots to the function are, in fact, imaginary. • The vertex of the parabola represents the maximum or minimum point of the function. • The characteristics of quadratic functions and their representations are useful in solving real-world problems.
Areas of Focus: Proficiencies (New Jersey Student Learning Standards)	**Lessons**
Students will: A-SSE.A.1 Interpret expressions that represent a quantity in terms of its context.* A-SSE.A.1.A Interpret parts of an expression, such as terms, factors, and coefficients. A-SSE.A.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$. A-SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. * A-SSE.B.3.A Factor a quadratic expression to reveal the zeros of the function it defines A-SSE.B.3.B Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. A-CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. A-REI.4.B. Solve quadratic equations by inspection (e.g.,	**Lesson 1:** • Factor quadratic polynomials completely using GCF, difference of squares, perfect square trinomials and trinomials when $a = 1$ or $a \neq 1$. • Factor four term polynomials using grouping. • Factor using the sum and difference of two cubes. **Lesson 2:** • Solve quadratic equations using factoring and taking square roots. **Lesson 3:** • Solve quadratic equations by completing the square. **Lesson 4:** • Solve quadratic equations using the quadratic formula. • Find the discriminant and use it to describe roots. **Lesson 5:** • Graph quadratic functions in intercept, standard and vertex form. • Identify key characteristics of quadratic functions; maximum or minimum values, *x*-intercepts, *y*-intercepts, domain, and range. **Lesson 6:** • Solve vertical motion problems algebraically and graphically.

for x^2=49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as for real numbers a and b. F-IF.C.7.A Graph linear and quadratic functions and show intercepts, maxima, and minima. F-IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. F-IF.C.8.A Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. F-IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. F-BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. N-CN.A.1 Know there is a complex number such that i^2 = -1, and every complex number has the form a + bi with a and b real. N-CN.A.2 Use the relation i^2 = -1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. N-CN.C.7 Solve quadratic equations with real coefficients that have complex solutions. N-CN.C.8 (+) Extend polynomial identities to the complex numbers. For example, rewrite x2 + 4 as (x + 2i)(x – 2i).	**Lesson 7:** ● Write the equation of a quadratic function given the roots. **Lesson 8:** ● Perform operations (addition, subtraction, multiplication) with complex numbers. ● Use powers of *i* to simplify expressions.

N-CN.C.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	
Differentiation	**Assessments**
Interdisciplinary Connections: ● Students can use concepts discussed in physics to answer projectile motion questions. For example, a ball is thrown up with an initial velocity of 56 feet per sec. The height of the ball t seconds after it is thrown is given by the equation . What is the height of the ball after one second? What is the maximum height? After how many seconds will it return to the ground? **Technology Integration:** ● Students can use a graphing calculator to check solutions, estimate zeros using upper and lower bounds, find maximum/minimum values, and evaluate quadratic functions at a given "time". **Media Literacy Integration:** ● The game Angry Birds has become a nationwide phenomenon that uses parabolas and projectile motion to win the game. Students can research the questions: What is projectile motion? Are there any other games that use parabolas? ● Students can use the aspect ratio of a TV screen to calculate the width and height for a 72 inch TV. **Global Perspectives:** ● Students can research amaranth and create a quadratic model.	**Formative Assessments:** ● Students can complete the sample bell ringer problems below: 1. Graph $f(x) = 2x^2 + 4x - 6$. State the vertex, domain, range and approximate solutions. 2. Solve by factoring: $x^2 + 2x - 35 = 0$ $64x^2 - 169 = 0$ 3. Solve by completing the square: $x^2 - 12x - 10 = 0$ 4. Solve by the quadratic formula: $-x^2 - 6x - 1 = 0$ 5. Write quadratic equations with the following roots: $5, -\frac{7}{2}$ 6. Perform the indicated operation: $2\sqrt{-50} \cdot \frac{1}{8}\sqrt{-2}$ $2\sqrt{-18} + 3\sqrt{-2}$ $(2 - 4i)^2$ **Summative Assessments, Projects, and Celebrations:** ● Two sectional quizzes and one unit test. ● Students can find a real-life image that models a parabolic function and use it to create a quadratic function.

Unit 4: Functions	
Big Ideas: *Course Objectives/Content Statement(s)* • Relations and Functions • Domain and Range of Relations • Operations with Functions • Inverse Functions • Graphing and Evaluating Piecewise Functions • Cubic Function Graphs using Transformations	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
• What is domain and range? Are there ever any values that cannot be in the domain of a function? • What is vertex form and how can it be applied to cubic functions? • How are the properties of functions and functional operations useful? • How can piecewise functions be used in real life situations? • How are a function and its inverse function related?	**Students will understand that:** • Domain refers to x values whereas the range refers to the y values. Yes, rational functions and square root functions are the most notable functions that have restricted domains. • The properties of functions and function operations are used to model and analyze real-world applications and relationships. • Using the parent graph and rules of vertex form, cubic functions can be manipulated to be easily graphed. • By definition of inverse, the input and output are switched. The line is where x and y variables are equal. This is called the line of symmetry. • The inverse of a function may or may not be a function.
Areas of Focus: Proficiencies **(New Jersey Student Learning Standards)**	**Lessons**
Students will: F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* F-IF.C.7.B Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	**Lesson 1:** • Graph cubic functions in vertex form using transformations. **Lesson 2:** • Perform operations with functions and state the domain of the resulting function. • Perform a composition of two functions. **Lesson 3:**

A-APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiple polynomials. F-BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. F-BF.B.4 Find inverse functions. F-BF.B.4.A Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x3 or f(x) = (x+1)/(x-1) for x ≠ 1. F-BF.B.4.B (+) Verify by composition that one function is the inverse of another. F-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*	• Find the inverse of a function graphically and algebraically. • Perform the horizontal line test to determine if a relation has an inverse function. **Lesson 4:** • Recognize whether a function is even, odd, or neither based on its graph. • Algebraically show that a function is even, odd, or neither. **Lesson 5:** • Evaluate and graph piecewise functions.
Differentiation	**Assessments**
Interdisciplinary Connections: • Students can use The Ring Nebula to write an inverse model that gives the age of the nebula as a function of its volume.	**Formative Assessments:** • Students can explain how to perform the function operations of $f(x)+g(x), f(x)-g(x), f(x)\cdot g(x)$

Technology Integration: ● http://www.mathsisfun.com/data/function-grapher.php **Media Literacy Integration:** ● Students can use a composition of functions to calculate total percent discounts of advertised sale items. **Global Perspectives:** ● Students can compute income taxes in a a tax bracketed system.	$f(x)/g(x)$ and $f(g(x))$. ● Students can explain how a function and its inverse are related. **Summative Assessments, Projects, and Celebrations:** ● One sectional quiz and one test. ● Students can write a piecewise function given specific criteria and use the model to answer questions. For example, the minimum payment on a credit card is based on the total amount owed. A credit card company uses the following rules: For a bill less than \$10 the entire amount is due. For a bill of at least \$10 but less than \$500, the minimum due is \$10. There is a minimum of \$30 due on a bill of at least \$500 but less than \$1000, a minimum of \$50 due on a bill of at least \$1000, but less than \$1500, and a minimum of \$70 is due on bills \$1500 or more. Find the function f that describes the minimum payment due on a bill of x dollars. Graph the function.

Unit 5: Radicals/Rational Exponents/Powers/Roots	
Big Ideas: *Course Objectives/Content Statement(s)* ● Perform Radical Operations ● Using Properties of Exponents to Simplify Fractional Roots ● Solving Radical Equations ● Graphing Square Root and Cube Root Functions using Transformations	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
● Why is $\sqrt{125} = 5\sqrt{5}$? ● Why is it necessary for roots to "match" in order to add or subtract radicals? ● Why can we rationalize that ? ● How do you find the solutions to a radical equation? What are extraneous solutions?	**Students will understand that:** ● Both $\sqrt{125}$ and $5\sqrt{5}$ have the same approximate decimal value. They will then notice that 125 is equivalent to the perfect square, 25, times the non-perfect square, 5, thus making the two radical expressions equivalent. ● Just like variables must be alike to combine like terms, so must radicals as they too are two

• How can vertex form be applied to square root and cube root functions?	numerical values being multiplied together. They will see the relationship between $3x + 4y - 2x + y$ and $3\sqrt{2} + 4\sqrt{3} - 2\sqrt{2} + \sqrt{y}$ • Power rules can be applied to radicals. Since $\sqrt{x^2} = x$ and $(x^2)^{1/2} = x$, a ½ power must be the same operation as a square root. • You can write a radical expression in an equivalent form using fractional or rational exponents. • Radical equations can be solved by applying properties of equations. Squaring both sides of an equation can introduce extraneous solutions. • Using the parent graph and rules of vertex form, square root and cube root functions can be manipulated to be easily graphed. • The square root function has a restricted domain.
Areas of Focus: Proficiencies (New Jersey Student Learning Standards)	**Lessons**
Students will: A-REI.A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. N-RN.A.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5. N-RN.A.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents. N-RN.B.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. F-IF.C.7.B Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	**Lesson 1:** • Simplify expressions using rules of exponents. **Lesson 2:** • Evaluate n^{th} roots of real numbers using radical notation and rational exponent notation both by hand and with a calculator. **Lesson 3:** • Define radicand, index, like radicals. • Perform operations with radical expressions (addition, subtraction, multiplication, division). • Rationalize denominators using conjugate (also complex conjugates). • Simplify radicals using absolute value where appropriate. **Lesson 4:** • Solve radical equations. • Find extraneous solutions and reason about what it means to be an extraneous solution. **Lesson 5:** • Graph cube root and square root functions in vertex form with transformations. • Find the domain and range of cube root and

F-BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

square root functions and write in interval notation.
- Connect their graphs to inverses.

Differentiation

Interdisciplinary Connections:
- Students can connect concepts learned in physics to find the time that it takes a pendulum to complete a swing if its length is 10inches, using the formula $T = 2\pi\sqrt{\frac{L}{384}}$, where T represents time in seconds and L represents the length of the pendulum in inches.
- Students can approximate the surface area of a human that has a mass of a given quantity.

Technology Integration:
- http://www.youtube.com/watch?v=gEw8xpb1aRA
- Students can use mathisfun to explore fractional exponents.

Media Literacy Integration:
- Students can research the Mandelbrot set and explain how imaginary numbers are used to create these fractals.

Global Perspectives:
- Students can use Philip Darlinton's discovery to calculate the number of reptile and amphibian species areas around the globe can support.
- Students can research the tsunami warning system used today to warn people when a tsunami might arrive.

Assessments

Formative Assessments:
- Students can complete the sample bell ringer problems below:

  1. Radicals.
     Simplify.$(\sqrt{10} + \sqrt{6})(\sqrt{30} - \sqrt{18})$
     Simplify. $\sqrt{\frac{1}{18}}$
  2. Rational exponents.
     Simplify. $(-64)^{2/3}$
  3. Radical equations.
     Solve. $5 + \sqrt{x+4} = 12$
     Solve. $-(x+6)^{1/3} = 2$

- Students can complete the table below to see the difference between $\sqrt{x^2}$ and $(\sqrt{x})^2$.

x	$(\sqrt{x^2})$	$(\sqrt{x})^2$	$(\sqrt[3]{x^3})$	$(\sqrt[3]{x})^3$
1				
-1				
64				
-64				

Summative Assessments, Projects, and Celebrations:
- Two sectional quizzes and one test.
- Students can demonstrate understanding of rational exponents by completing the Find Someone Who Can Activity

<table>
<tr><th colspan="2">Unit 6: Logarithmic and Exponential Functions</th></tr>
<tr><td colspan="2">Big Ideas: Course Objectives/Content Statement(s)
Graphs of Exponential and Logarithmic Functions
Logarithms and their Properties
Natural Logarithms and their Properties
Word Problems with Logarithmic and Exponential Functions</td></tr>
<tr><th>Essential Questions
What provocative questions will foster inquiry, understanding, and transfer of learning?</th><th>Enduring Understandings
What will students understand about the big ideas?</th></tr>
<tr><td>How are exponents and logarithms related?
What is an asymptote?
What is a logarithm and what is it used for?
How are properties of exponents applied to logarithms?
What is the importance of logarithms in real life data?</td><td>Students will understand that:
The exponential and logarithmic functions are inverses.
An asymptote is a line that the graph approaches as you move away from the origin.
A logarithm is a function that allows one to find the solution to a variable exponential value.
Logarithms and exponents have common properties.
The summation of logarithms is derived from multiplying values with like-bases, subtraction from division, and multiplication from power rules.
Since logarithms can solve for exponential variables, it is often used to solve for rate and time in interest problems.</td></tr>
<tr><th>Areas of Focus: Proficiencies
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<tr><td>Students will:

A-SSE.A.1.B Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.

F-IF.C.7.E Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.</td><td>Lesson 1:
Graph exponential functions in vertex form and state the domain, range and horizontal asymptote.
Lesson 2:
Solve exponential equations both with like and unlike bases that do not require logs.
Lesson 3:
Switch between logarithmic and exponential form.</td></tr>
</table>

F-IF.C.8.B Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)12^t$, $y = (1.2)^t/10$, and classify them as representing exponential growth or decay.

F-BF.B.5 (+) Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.

F-LE.A4 Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or e; evaluate the logarithm using technology.

F-LE.B.5. Interpret the parameters in a linear or exponential function in terms of a context.

- Evaluate logarithms without the calculator.
- Evaluate logarithms using the change of base formula and the calculator.

Lesson 4:
- Graph logarithmic functions in vertex form and state the domain, range and vertical asymptote.

Lesson 5:
- Use properties to expand and condense logarithms.

Lesson 6:
- Solve logarithmic equations and exponential equations requiring logarithms with all bases.

Lesson 7:
- Evaluate, expand, and condense logarithms of base e.

Lesson 8:
- Solve word problems related to logarithms (compound interest, growth and decay word problems).

Differentiation	Assessments

Interdisciplinary Connections:
- Students can solve a variety of science related problems. For example, how many days will it take a culture of bacteria to increase from 2000 to 50,000 if the growth rate per day is 93.2%?

  Carl plans to invest $500 at an interest rate of 8.25%, compounded continuously. How long will it take him to have $2000 in his account?

Technology Integration:
- Students can use a graphing calculator to relate the functions $f(x) = \log x$ and $f(x) = 10^x$.
- Students can use Desmos to explore transformational changes when logarithmic and exponential functions are written in vertex form.

Formative Assessments:
- Students can complete the sample bell-ringer problems below:
  1. Evaluate. $\log_7 \frac{1}{49}$
  2. Solve. $\log_4 x = \frac{3}{2}$
  3. Solve. $2\log x + \log 3 = \log 27$
  4. Solve. $\log_2 6 - \log_2 (x + 4) = 3$
  5. Solve. $-3(11^x) - 6 = -22$
  6. Expand. $\log 10x^2y$
  7. Solve. $\ln 5x + \ln x = 7$
  8. Graph the function $f(x) = 2^x + 3$ and its inverse on the same coordinate plane. Then find the equation of the inverse function.

Summative Assessments, Projects, and Celebrations:
- Two sectional quizzes, one mid-unit test, one end of unit test.

Media Literacy Integration: ● Students can use the internet to explore the change in interest rates over the past 10 years, both in interest charged and interest earned. How can this affect an investment of $10,000? Now have students look at the current interest rates around the world and compare them with our… Are they better or worse? Convert all units to dollars during the comparisons. **Global Perspectives:** ● Students can examine wind energy from different countries and create exponential models over a course of 10 to 15 years.	● Students can research and study the effects of Hurricane Fran when it hit North Carolina in 1996. They can use the information from the National Weather Service to create exponential models and answer questions as it relates to the hurricane's devastation.

Unit 7: Polynomial Functions	
Big Ideas: *Course Objectives/Content Statement(s)* ● Polynomial Operations ● The Remainder and Factor Theorem ● Sketching Graphs Polynomial Functions ● Quadratic Factoring Techniques	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
● How are the properties of real numbers related to polynomial expressions? ● What is synthetic division and why is it useful? Why must we include zeros for missing powers? ● What is the relationship between a polynomial and it's factors? ● What are roots to polynomial equations? ● What types of roots exist for polynomial equations? ● How are factors, zeros and x-intercepts related? How are factors and roots related? ● What are the differences between even and odd polynomials graphically?	**Students will understand that:** ● That the rules for powers are derived from basic concepts: adding powers: $(x^2)(x^3) = (x \cdot x)(x \cdot x \cdot x) = x^5$ subtracting powers: $\frac{x^3}{x^4} = \frac{x \cdot x \cdot x}{x \cdot x \cdot x \cdot x} = \frac{1}{x}$ negative exponents: $\frac{1}{x} = \frac{x^3}{x^4} = x^{-1}$ zero power: $1 = \frac{x^3}{x^3} = x^0$ multiplying powers: $(x^3)^2 = (x^3)(x^3) = (x \cdot x \cdot x)(x \cdot x \cdot x) = x^6$

- How do you compose functions? - What are the characteristics of a polynomial function? - How does left/right and even/odd behavior help in graphing polynomial functions?	- A polynomial times another polynomial is simply the act of distributing the terms in the first polynomial to the terms in the other polynomial. We sometimes use "FOIL" for a binomial times a binomial. - Synthetic division is a quick way to divide two polynomials without using long division. We must include zeros for missing powers in synthetic division because each position represents a power of the variable. If you do not put in zeros, then an expression such as $x^4 + x^2 + 2$ will be written as 1 1 2, but this represents $x^2 + x + 2$. We would, instead, enter 1 0 0 1 2. - Factoring is the process of breaking down a polynomial into the multiplication of two or more polynomial or monomial expressions. Similar to a number that can be factored into its prime factors, a polynomial can be factored into it's composite terms. - Roots are where the polynomial equation equals zero. Real zeros cross or touch the *x*-axis. - There are rational, irrational, and imaginary roots in polynomial equations. Number of real and imaginary roots always equals the degree of the polynomial. - If $(x - a)$is a factor of a polynomial, then the polynomial has value zero when $x = a$. If *a* is a real number, then the graph of the polynomial has $(a, 0)$ as an x-intercept. - A polynomial function has distinguishing "behaviors'. You can look at its algebraic form and know something about its graph or look at its graph and know something about its algebraic form. - In an even polynomial, the end behaviors of the polynomial either both increase or both decrease, whereas in an odd polynomial, one will increase while the other will decrease.
Areas of Focus: Proficiencies (New Jersey Student Learning Standards)	**Lessons**
Students will: A-APR.A.1 Understand that polynomials form a system	**Lesson 1:**

analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. A-SSE.A.1.A Interpret parts of an expression, such as terms, factors, and coefficients. A-SSE.A.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$. A-APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiple polynomials. A-APR.B.2 Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x). A-APR.B.3 Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial. A-APR.C4 Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples. F-IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. F-IF.C.8.A Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. F-IF.C.7.C Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. N-CN.C.8 (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as (x + 2i)(x – 2i).	● Recognize characteristics of a polynomial (degree, leading coefficient, standard form, monomial, binomial, trinomial, etc…). ● Perform polynomial operations (addition, subtraction, multiplication). **Lesson 2:** ● Factor polynomials over the reals (certain polynomials may need to be factored more than once). ● Solve polynomial equations using the Zero-Product property. ● Revisit factoring using the sum/difference of cubes. **Lesson 3:** ● Divide polynomials using long and synthetic division ● Recognize when to use synthetic division over long division **Lesson 4:** ● Find the number of maximum turns (degree-1). ● Find relative extrema on the calculator. ● Describe the end behavior of a polynomial function. **Lesson 5:** ● Use the remainder theorem to evaluate polynomials ● Use the factor theorem to write a polynomial in factored form. **Lesson 6:** ● Apply the rational zero test and synthetic division to find all rational zeros of a polynomial. ● Define multiplicity and use it sketch a graph of a polynomial function written in factored form. ● Describe how zeros, factors, and solutions are related. ● Use the Fundamental Theorem of Algebra to determine the number of solutions of a polynomial/draw a connection to the linear factorization theorem. ● Find all (real and complex) zeros of a polynomial function.

	Lesson 7: ● Graph polynomials using degree, the leading coefficient, multiplicity, end behavior and maximum number of turns.
Differentiation	**Assessments**
Interdisciplinary Connections: ● Students can connect the concept of Punnett Squares from biology to polynomial multiplication. **Technology Integration:** ● Students can use the graphing calculator to graph polynomial functions, requiring them to change the viewing window so that it displays important characteristics of the graph. **Media Literacy Integration:** ● Students can watch the factoring video and create their own video about a concept in this unit. http://www.youtube.com/user/WSHSmath#p/u/2/OFSrINhfNsQ **Global Perspectives:** ● Students can research statistical population data of American Indian, Eskimo and Aleut people to create polynomial models that approximate future populations.	**Formative Assessments:** ● Students can complete the sample bell-ringer problems below: 1. Simplify using rules of exponents. $\left(\frac{2^3m^2n^{-2}}{p^{-4}}\right)^2$ 2. Multiply polynomials. $(2a-1)(8a-5)(2a+1)$ $(4p^2-1)^2$ $(6n^2-6n-5)(7n^2+6n-5)$ $(x^{-2})^3 \cdot (x^2y^3)^4$ 3. Factor completely. $x+x^2y+x^3y^2$ $12p^3-21p^2+28p-49$ $6x^2+5x-6$ $16a^4-1$ 4. Divide using long division. $(2x^3+5x^2-2x-15) \div (2x-3)$ 5. Divide using synthetic division. $(x^3-13x^2+9) \div (x+2)$ 6. Find the remaining factors of $(x^4+14x^3+51x^2+54x)$ if $(x+9)$ is a factor. 7. Find the zeros of the polynomial. Then sketch the graph. $f(x)=x^4+2x^3-8x^2-18x-9$ 8. Sketch a possible graph of $f(x)=-2(x-3)^2(3x+4)^3(x^2+1)$ and determine the number of *x*-intercepts of the function. 9. Use quadratic techniques to solve $x^4+x^2-20=0$ **Summative Assessments, Projects, and Celebrations:** ● One or two sectional quizzes, one mid-unit test and one end of unit test. ● Students can play the "numbers game" and discuss findings: Choose any number. Multiply that number by 3.

	Add the sum of your number and 8 to the number you got when you multiplied. Now divide by the sum of your number and 2. Is your answer 4? Why does this work? - Choose any #: x - Multiply by 3: $3x$ - Add the sum of your number and 8 to the previous result: $3x + (x + 8)$ or $4x + 8$ - Divide this result by your number plus 2: $\frac{4x+8}{x+2}$ Students can use long division or factoring and reducing to simplify and discover that regardless of what number is chosen for x, the expression simplifies to a value of 4.

Unit 8: Trigonometric Functions	
Big Ideas: *Course Objectives/Content Statement(s)* ● Right Triangle Trigonometry ● Exploring Angles and Their Measure ● Extending Trigonometry to Circular Functions ● Graphing Trigonometric Functions	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
● How are the six basic trig functions related? ● How are radian and degree measures used in real-world settings? ● What is a periodic function? ● How do amplitudes, periods, phase shifts, vertical shifts and co-functions relate to the graphs of translated sine and cosine functions?	**Students will understand that:** ● Angles can be measured in radians and degrees. ● Trigonometric functions can be extended to all real numbers, using the unit circle. ● The cosine function corresponds with the x-coordinate of the point where the terminal side of the angle intersects the unit circle. The sine function corresponds with the y-coordinate of the point where the terminal side of the angle intersects the unit circle.

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- Trigonometric functions are periodic. Their graphs are unique because they are continuous curves that repeat themselves.
- Changes to the algebraic equation of a function cause predictable changes to the function's graph.

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<tr><th>Areas of Focus: Proficiencies
(New Jersey Student Learning Standards)</th><th>Lessons</th></tr>
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Students will:

F-TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

F-TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

F-TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*

F-TF.C.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

F-IF.C.7.E Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F-TF.A.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

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Lesson 1:

- Evaluate all trigonometric functions (sin, cos, tan, csc, sec, cot) of acute angles with and without the calculator.

Lesson 2:

- Extend the unit circle beyond acute angles
- Decide whether angles are coterminal.
- Find coterminal and reference angles.
- Rewrite radian measures as degree and degree as radian.
- Draw angles of rotation.

Lesson 3:

- Find reference angles and use them to evaluate all six trig functions of any angle.

Lesson 4:

- Define amplitude and period and find each for sine and cosine functions.
- Graph sine and cosine functions using vertical shifts, phase shifts and period changes.

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<tr><th>Differentiation</th><th>Assessments</th></tr>
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Interdisciplinary Connections:

- Students can refer to a physics text and look through the section on waves, especially light and sound, with a focus on the meanings of period and amplitude.

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Formative Assessments:

- Students can convert from radians to degrees and degrees to radians.
  Ex. Convert $\frac{2\pi}{9}$ to degrees.
- Students can use the special right triangle relationships, coterminal and reference angles to evaluate trigonometric functions on the unit

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Technology Integration: ● Students can use a graphing calculator or online graphing applets to visualize the effect of changing the period and amplitude of a trigonometric graph vs. its parent function. **Media Literacy Integration:** ● Students can research some of the largest ferris wheels in the world and compare the maximum heights of riders. **Global Perspectives:** ● Students can research sundials and their use of shadows falling on a calibrated scale to tell time. In some primitive regions of Egypt, sundials are still used to tell time. *(See Chapters 13 and 14 for extensions of these ideas)	circle. Ex. Evaluate $cos\ \frac{17\pi}{3}$ Ex. Evaluate $csc\ 495°$ ● Students can find the six trigonometric functions of any angle. Ex. The point $(1, -\sqrt{2})$ lies on the terminal side of angle θ. Calculate the six trig functions of θ. ● Students can define the terms of cycle and period. ● Students can determine the domain and range of a graph of sine or cosine, without sketching the graph. **Summative Assessments, Projects, and Celebrations:** ● One sectional quiz and one unit test. ● Students can create and use diagrams to model examples of the terms: arc, standard position, radian, and coterminal angles and display these in the classroom as a visual. ● Students can create a tessellation using their knowledge of angles gained from this unit (see p. 873 in text).

Career-Ready Practices
CRP1: Act as a responsible and contributing citizen and employee. **CRP2**: Apply appropriate academic and technical skills. **CRP3**: Attend to personal health and financial well-being. **CRP4**: Communicate clearly and effectively and with reason. **CRP5**: Consider the environmental, social and economic impacts of decisions. **CRP6**: Demonstrate creativity and innovation. **CRP7**: Employ valid and reliable research strategies. **CRP8**: Utilize critical thinking to make sense of problems and persevere in solving them. **CRP9**: Model integrity, ethical leadership and effective management. **CRP10**: Plan education and career paths aligned to personal goals. **CRP11**: Use technology to enhance productivity. **CRP12**: Work productively in teams while using cultural global competence.

Supports for English Language Learners		
Sensory Supports	**Graphic Supports**	**Interactive Supports**
Real-life objects	Charts	In pairs or partners
Manipulatives	Graphic Organizers	In triands or small groups
Pictures	Tables	In a whole group
Illustrations, diagrams & drawings	Graphs	Using cooperative group
Magazines & Newspapers	Timelines	Structures
Physical activities	Number lines	Internet / Software support
Videos & Film		In the home language
Broadcasts		With mentors
Models & Figures		

Intervention Strategies		
Accommodations	**Interventions**	**Modifications**
Allow for verbal responses	Multi-sensory techniques	Modified tasks/expectations
Repeat/confirm directions	Increase task structure (e.g. directions, checks for understanding, feedback	Differentiated materials
Permit response provided via computer or electronic device	Increase opportunities to engage in active academic responding	Individualized assessment tools based on student need
Audio Books	Utilize pre-reading strategies and activities previews, anticipatory guides, and semantic mapping	Modified assessment grading