Time Extended Construction Response (ECR) Task - Algebra II November

Name:_____Date:_____

For this task, the letter i denotes the imaginary unit, that is, $= \sqrt{-1}$.

a) For each integer k from 0 to 8, write i^k in the form a + bi. Write your answers by completing the table below.

k	i^k	$a + bi$
0		
1		
2		
3		
4		
5		
6		
7		
8		

b) Describe the pattern you observe, and algebraically prove your observation. In particular, simplify i^{195}.

c) Write each of the following expressions in the form a + bi.

- $i^2 + i + 1$
- $i^3 + i^2 + i + 1$
- $i^4 + i^3 + i^2 + i + 1$
- $i^5 + i^4 + i^3 + i^2 + i + 1$
- $i^6 + i^5 + i^4 + i^3 + i^2 + i + 1$
- $i^7 + i^6 + i^5 + i^4 + i^3 + i^2 + i + 1$
- $i^8 + i^7 + i^6 + i^5 + i^4 + i^3 + i^2 + i + 1$

Task is worth a total of **10 points.**

<table>
<tr><th colspan="2">Rubric Part A</th></tr>
<tr><td>Score</td><td>Description</td></tr>
<tr><td>2</td><td>Student response includes the following elements
❖ Computation component = 2 points
Students have written i^k for each integer from 0 to 8 in the form $a + bi$

Sample Student Response:

<table>
<tr><th>k</th><th>i^k</th><th>$a + bi$</th></tr>
<tr><td>0</td><td>$i^0 = 1$</td><td>$1 + 0i$</td></tr>
<tr><td>1</td><td>$i^1 = i$</td><td>$0 + i$</td></tr>
<tr><td>2</td><td>$i^2 = -1$</td><td>$-1 + 0i$</td></tr>
<tr><td>3</td><td>$i^3 = -i$</td><td>$0 - i$</td></tr>
<tr><td>4</td><td>$i^4 = 1$</td><td>$1 + 0i$</td></tr>
<tr><td>5</td><td>$i^5 = i$</td><td>$0 + i$</td></tr>
<tr><td>6</td><td>$i^6 = -1$</td><td>$-1 + 0i$</td></tr>
<tr><td>7</td><td>$i^7 = -i$</td><td>$0 - i$</td></tr>
<tr><td>8</td><td>$i^8 = 1$</td><td>$1 + 0i$</td></tr>
</table>
</td></tr>
<tr><td>1</td><td>Student response includes at least 5 (out of 9) correct answers</td></tr>
<tr><td>0</td><td>Student response is incorrect or irrelevant</td></tr>
</table>

Rubric Part B	
Score	**Description**
2	Student response includes the following elements ❖ Reasoning component = 1 point Students correctly describe the pattern they see ❖ Computation component = 1 point Students algebraically prove their answer and compute i^{195} Sample Student Response: b. We observe that the pattern of powers of i is cyclical, repeating every 4 exponents. When the exponent is an integer multiple of 4, the result is a 1. Exponents which are one more than a multiple of 4 give a result of i, and so on. To make this precise, we simply observe that any integer can be written as a multiple of 4, plus either 0, 1, 2, or 3. We can justify this pattern as follows: To compute i^n, we write $n = 4k + a$ (where a is 0, 1, 2, or 3), and then observe $$i^n = i^{4k+a} = (i^4)^k \times i^a = a^k \times i^a = i^a.$$ That is, we can compute i^n by computing i^a, where a is the remainder upon dividing n by 4. Since 195 is three more than the multiple $192 = 4 \cdot 48$, we have $i^{195} = i^3 = -i$.
1	Student response includes 1 of the 2 elements
0	Student response is incorrect or irrelevant

Rubric Part C	
Score	**Description**
6	Student response includes the following elements ❖ Reasoning component = 1 point Algebraic or written explanation for solving the equation ❖ Computation component = 1 point Calculating the correct answer Sample Student Response: c. Here are the algebraic solutions: • $i^2+i+1=-1+i+1=i$ • $i^3+i^2+i+1=-i+-1+i+1=0$ • $i^4+i^3+i^2+i+1=1+-i+-1+i+1=1$ • $i^5+i^4+i^3+i^2+i+1=i+1+-i+-1+i+1=1+i$ • $i^6+i^5+i^4+i^3+i^2+i+1=-1+i+1+-i+-1+i+1=i$ • $i^7+i^6+i^5+i^4+i^3+i^2+i+1=-i+-1+i+1+-i+-1+i+1=0$ • $i^8+i^7+i^6+i^5+i^4+i^3+i^2+i+1=1+-i+-1+i+1+-i+-1+i+1=1$
5	Student response includes 6/7 correct answers
4	Student response includes 5/7 correct answers
3	Student response includes 4/7 correct answers
2	Student response includes 3/7 correct answers
1	Student response includes 1 or 2 correct answers
0	Student response is incorrect or irrelevant

Glow	Grow
