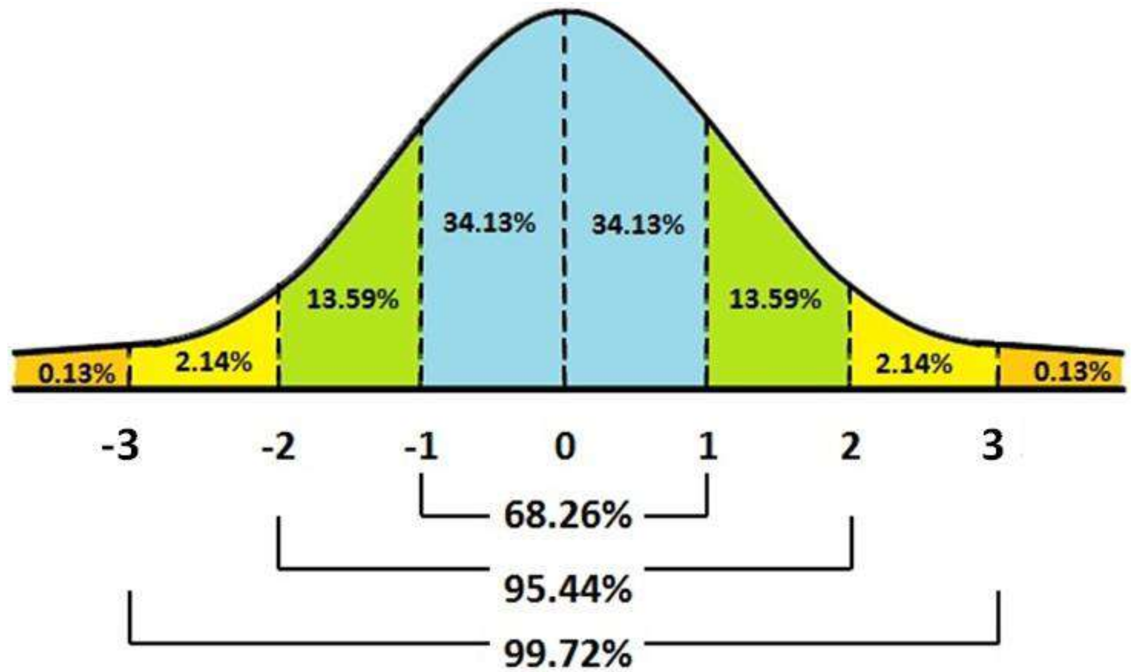
Amherst County Public Schools

Algebra 2 – Curriculum Pacing



6/5/2018

Nine Week	SOL	Curriculum Pages	Time Allotment
1	**Overview:**		
	Topic 1 - Factoring - AII.1c		**10 days**
	Topic 2 - Solving Quadratic Equations, Simplifying Radicals and Complex Numbers - AII.1b,c, AII.2, AII3.b, AII.9		**17 days**
	Topic 3 - Rational Expressions and Equations - AII.1a, AII.3c		**13 days**
	Review and Assessment		**5 days**
	TOTAL		**45 days**
	Topic 1 - Factoring **AII.1** The student will c) factor polynomials completely in one or two variables.		**10 days**
	SMART Goal Factoring Test/1st Day Procedures		1
	Factoring review: GCF, difference of perfect squares, trinomials	U4 6-7, old curriculum	5
	Factor polynomials (cubes and grouping)	U5 27-29	2
	Review and test		2
	Topic 2 - Solving Quadratic Equations, Simplifying Radicals and Complex Numbers **AII.1** The student will b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents; and c) factor polynomials completely in one or two variables. **AII.2** The student will perform operations on complex numbers and express the results in simplest form using patterns of the powers of *i*. **AII.3** The student will solve b) quadratic equations over the set of complex numbers; **AII.9** The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve practical problems, using mathematical models of quadratic and exponential functions.		**17 days**
	Radicals review	U4 16	1
	nth root - simplifying radicals (from unit 6)	U6 4-6	1

	add/sub/multiply radicals (from unit 6)	U6 7-9	1
	divide radicals - rationalize the denominator (from unit 6)	U6 10-13	1
	Rational exponents, operations with rational exponents (from unit 6)	U6 14-16	1
	Pure imaginary numbers	U4 22-24	1
	complex numbers add/sub/mult/div	U4 26-31	2
	Solving Quadratic Equations by factoring	U4 10-15	2
	Solving Quadratic Equations by square root property	U4 17-19, 22-23, 25	1
	Pre-AP Solving Quadratic Equations by CTS	U4 32-35	
	Solving Quadratic Equations by Quadratic Formula	U4 38-41	2
	Discriminant	U4 42-48	1
	Quadratic and Linear Regression	U2 18-19, U4 57-58	1
	review and test		2
	Topic 3 - Rational Expressions and Equations **AII.1** The student will a) add, subtract, multiply, divide, and simplify rational algebraic expressions; **AII.3** The student will solve c) equations containing rational algebraic expressions;		**13 days**
	Simplifying Rational expression, mult/div/add/sub, complex fractions	U8 4-11, 14-24	7
	Solving Rational equations	U8 43-46	3
	review and test		3
	9-week assessment Review and Test		**5 days**
	Overview		
	Topic 4 - Absolute Value Equations and Inequalities - AII.3a		**15 days**

2	**Topic 5 - Radical Expressions and Equations and Polynomial Equations - AII.1b,c, AII.3d**		**10 days**
	Topic 6 - Graphing - Linear, Quadratic, Absolute Value, Radical Equations - AII.6a,b, AII.7a,b,d,e,f,g,h		**12 days**
	Review and Assessment		**4 days**
	After 9-week assessment - Non-linear Systems of equations - AII.4		**4 days**
	TOTAL		**45 days**
	Topic 4 - Absolute Value Equations and Inequalities **AII.3** The student will solve a) absolute value linear equations and inequalities;		**15 days**
	Real Number Systems and Properties	U1 4-6	1
	Order of Operations, evaluating expressions, simplifying expressions	U1 7-10	1
	Multi-step equations	U1 11-16	1
	Special cases (inf and no solution) eq	old curriculum	1
	Solve abs value eq	U1 17-20	2
	Review Solving Ineq, multi-step, compound, interval notation	U1 25-28, 35-36, old curriculum	3
2	Solve abs value ineq	U1 29-32	3
	Review and Test	U137-40	3
	Topic 5 - Radical Expressions and Equations and Polynomial Equations **AII.1** The students will b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents c) factor polynomials completely in one or two variables. **AII.3** The student will solve d) equations containing radical expressions.		**10 days**
	Radicals and Rational exponents review	U6 17-20	1
	Solving radical equations	U6 23-26	2

	Monomial and Polynomial review	U5 4-9	1
	Factor polynomials (cubes and grouping)	U5 27-29	1
	More practice with factoring polynomials	U5 30-34	1
	Solving polynomial equations by factoring	U5 35-39	1
	Review and test		3
	Topic 6 - Graphing - Linear, Quadratic, Absolute Value, Radical Equations **AII.6** For polynomial functions, the student will a) recognize the general shape of function families; and b) use knowledge of transformations to convert between equations and the corresponding graphs of functions. **AII.7** The student will investigate and analyze square root, cube root families algebraically and graphically. Key concepts include a) domain, range; b) intervals in which a function is increasing or decreasing; d) zeros; e) intercepts; f) values of a function for elements in its domain; g) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs; h) end behavior;		**12 days**
	Graphing Absolute Value equations and inequalities by table Domain/range	U3 10-14	1
	Parent functions, transformations, vertex form Domain/range	U3 17-21	1
	Graphing Standard form of quadratic equations and ineq Domain/range	U3 24-28	1
	Graphing Vertex form of quadratic equations Domain/range	U3 29-32	1
	Pre-AP - Convert to Vertex Form using CTS	U3 33-36	
	Review Linear, Quadratic and Absolute Value functions Domain, Range, Increasing/Decreasing, End behavior	U3 37, 40-43	2
	Solving Quadratic Equations by graphing (real zeros, roots, x-intercepts)	U4 4-5	1
	Graphing radical functions - square root and cube roots	U6 27-32	2

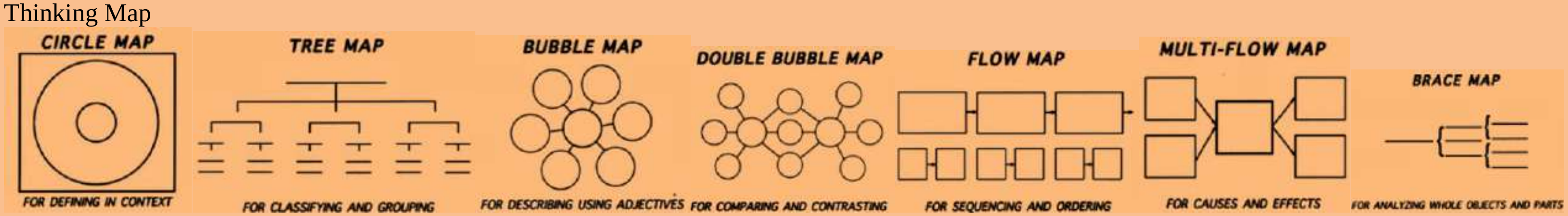
	review and test		3
	Review and 9-week assessment		**4 days**
	(after the 9-week assessment in December before the end of Q2) **AII.4** The student will solve systems of linear-quadratic and quadratic-quadratic equations, algebraically and graphically. (this is NOT tested on the 2nd NWA)		**4 days**
	Systems of equations by graphing, substitution and addition/elimination	U2 26-34	2
	Non-linear systems of equations	old curriculum	2
3	**Overview**		
	Topic 7 - Graphing Polynomial, Rational, Exponential and Logarithmic Functions - AII.1c, AII.6a,b, AII.7a,b,c,d,e,f,g,h,i		**16 days**
	Topic 8 - Operations with Functions - AII.7f,g,j,k, AII.9		**9 days**
	Topic 9 - Variation, Sequence/Series - AII.5, AII.10		**13 days**
	Review and Assessment		**5 days**
	TOTAL		**43 days**
	Topic 7 - Graphing Polynomial, Rational, Exponential and Logarithmic Functions **AII.1** The student will c) factor polynomials completely in one or two variables. **AII.6** For polynomial functions, the student will a) recognize the general shape of function families; and b) use knowledge of transformations to convert between equations and the corresponding graphs of functions. **AII.7** The student will investigate and analyze polynomial function families algebraically and graphically. Key concepts include a) domain, range, and continuity; b) intervals in which a function is increasing or decreasing; c) extrema; d) zeros; e) intercepts; f) values of a function for elements in its domain; g) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs; h) end behavior; i) vertical and horizontal asymptotes;		**16 days**
	Graphing Polynomial functions Parent functions, end behavior, turning points, inc/dec	U5 10-15	2

	roots/zeros of a polynomial function, multiplicity	U5 15-22	1
	Graphing Rational functions	U8 32-42	3
	Graphing Exponential equations	U7 4-9	2
	Introduction to logarithms, convert exp/log forms	U7 16-18	2
	Graphing log equations	U7 24-27	2
	Natural Logs	U7 44-47	1
	test and review		3
	Topic 8 - Operations with Functions **AII.7** The student will investigate and analyze polynomial function families algebraically and graphically. Key concepts include f) values of a function for elements in its domain; g) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs; j) inverse of a function; k) composition of functions algebraically and graphically. **AII.9** The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve practical problems, using mathematical models of quadratic and exponential functions.		**9 days**
	Operations with functions and composition (using equations and graphs)	U2 4-9, U6 46-49 old curriculum	3
	Inverse relations and functions, graphing inverses one-to-one, horizontal line test	U6 35-39	2
	regression Linear, quadratic, quadratic, exponential, cubic, quartic	U5 51-52, U7 52-55	1
	review and test		3
	Topic 9 - Variation, Sequence/Series **AII.5** The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve practical problems, including writing the first *n* terms, determining the n^{th} term, and evaluating summation formulas. Notation will include å and a_n. **AII.10** The student will represent and solve problems, including practical problems, involving inverse variation, joint variation, and a combination of direct and inverse variations.		**13 days**
	Variation - direct, inverse and combined	U8 49-53	3
	Introductions to sequences, series and summation notation	U10 4-9	1

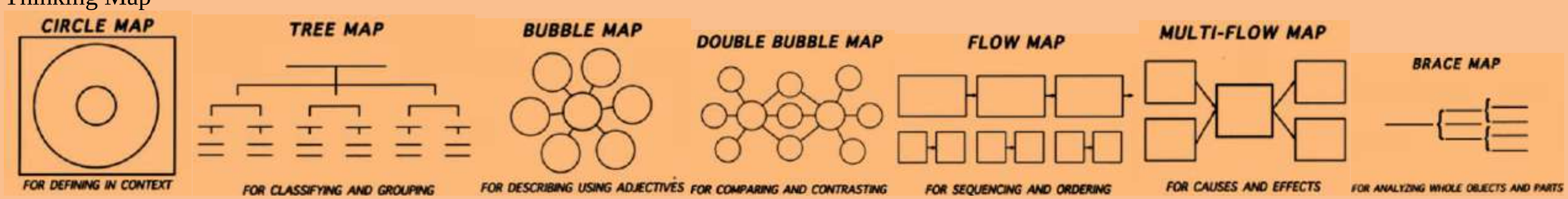
	Arithmetic Sequences and series	U10 10-13	2
	Geometric sequences	U10 16-19	1
	Geometric series, infinite geometric series	U10 20-23	2
	arithmetic vs geometric	U10 24-28	1
	review and test		3
	9-week review and assessment		**5 days**
4	**Overview**		
	Topic 10 - Statistics - Combinations/Permutations/Normal Curve - AII.11a,b,c, AII.12		**15 days**
	SOL Review and test		**20 days**
	After SOL - additional topics, expedited retake review and 9-week assessment		**10 days**
	TOTAL		**45 days**
	Topics 10 - Statistics **AII.11** The student will a) identify and describe properties of a normal distribution; b) interpret and compare *z*-scores for normally distributed data; and c) apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve. **AII.12** The student will compute and distinguish between permutations and combinations.		**15 days**
	Permutation and combinations	U11 4-8	4
	Measures of center and variation, normal distribution	U11 25-30	2
	z-scores, standard normal curve, probability under the curve	U11 31-34	6
	test and review		3
	SOL Review - all SOLs		20 days
	After the SOL test:		10 days
	Completing the Square, Conics, etc... as time permits		

SOL	[AII.1 - Resources]()
Standard	The student will a) add, subtract, multiply, divide, and simplify rational algebraic expressions; b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents; and c) factor polynomials completely in one or two variables.
Days	16
Key Vocabulary	Rational, Binomial, Factors, Quadratic, Radical, Rationalizing the denominator, Solutions, GCF, Trinomial, Difference of squares, Like terms, radicand, index, conjugate, complete factorization, sum of cubes, difference of cubes, rational exponents,
Essential Questions	How are radical expressions simplified? How is conversion between radical and rational exponents completed? How do you know when radicals can be added or subtracted? When is a polynomial completely factored? What are the patterns to investigate when factoring a polynomial? How is a rational expression simplified? How are multiplying and dividing rational expressions related? What is the relationship between the factor of a polynomial and the graph of the polynomial? What is the relationship between the vocabulary terms root, zeros, solutions, and x-intercepts? How can you verify factors of a polynomial with a graph?
Foundational Objectives	A.2 The student will perform operations on polynomials, including a) applying the laws of exponents to perform operations on expressions; b) adding, subtracting, multiplying, and dividing polynomials; and c) factoring completely first- and second-degree binomials and trinomials in one variable. A.3 The student will simplify a) square roots of whole numbers and monomial algebraic expressions; b) cube roots of integers; and c) numerical expressions containing square or cube roots.

Thinking Map

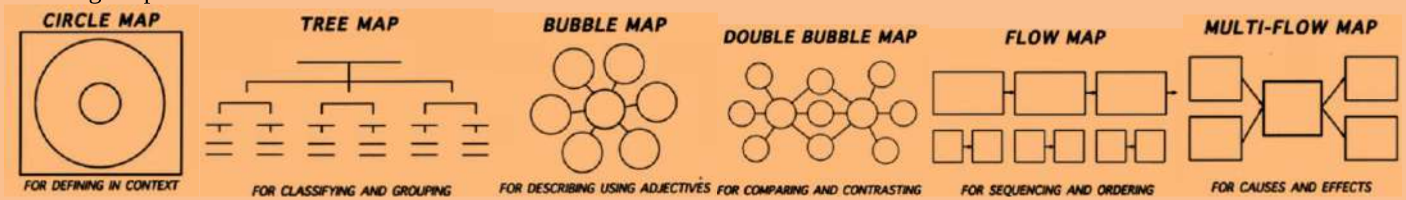


SOL	[AII.2 - Resources]
Standard	The student will perform operations on complex numbers and express the results in simplest form using patterns of the powers of i.
Days	3
Key Vocabulary	Complex, Conjugate, Real, Properties, ***i***, distributive, associative, commutative, identity, inverse, reflexive, symmetric, imaginary
Essential Questions	How are real, imaginary, and complex numbers related? Why do we use conjugates to rationalize the denominator? Other than counting, how could we simplify the powers of ***i*** Why are complex numbers necessary? What is the connection between imaginary solutions and the graph of a function?
Foundational Objectives	A.3 The student will simplify a) square roots of whole numbers and monomial algebraic expressions;
Thinking Map	



SOL	AII.3 - Resources
Standard	The student will solve a) absolute value linear equations and inequalities; b) quadratic equations over the set of complex numbers; c) equations containing rational algebraic expressions; and d) equations containing radical expressions
Days	24
Key Vocabulary	Roots, Quadratic Formula, Complete the Square, Discriminant, Solutions, Absolute Value, Verify, Extraneous, absolute value equations, absolute value inequalities, compound inequality, set notation, interval notation,
Essential Questions	How is an absolute value equation solved? How can the solution for an absolute value inequality be described? What are the characteristics of an absolute value function? Why can an absolute value equation take on more than one solution? How can there be no solution or infinitely many solutions for an absolute inequality? What is the difference between absolute value equations with 0, 1, or 2 solutions? When working with square roots, when do numbers not exist? when working with denominators, what do we need to avoid? Why is it advantageous to know a variety of way to solve quadratic equations? How are the solutions to these equations related to the roots, zeros and x-intercepts of the graph?
Foundational Objectives	A.1 The student will b) evaluate algebraic expressions for given replacement values of the variables. A.4 The student will solve a) multistep linear equations in one variable algebraically; b) quadratic equations in one variable algebraically; A.5 The student will a) solve multistep linear inequalities in one variable algebraically and represent the solution graphically; b) represent the solution of linear inequalities in two variables graphically; c) solve practical problems involving inequalities; and d) represent the solution to a system of inequalities graphically. AFDA.2 The student will use knowledge of transformations to write an equation, given the graph of a linear, quadratic, exponential, and logarithmic function.
Succeeding Objectives	MA.10 The student will use parametric equations to model and solve practical problems.

Thinking Map

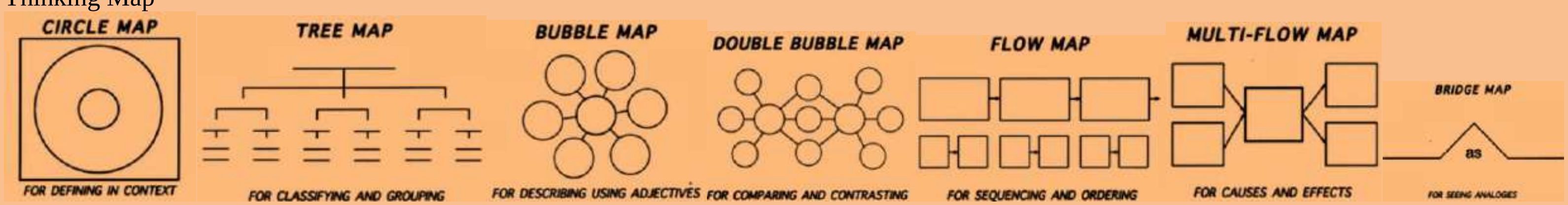


SOL	AII.4 - Resources
Standard	The student will solve systems of linear-quadratic and quadratic-quadratic equations, algebraically and graphically.
Days	4
Key Vocabulary	Parabolas, System of equations, Solutions, Infinite solutions, No Solutions, Coordinates of Intersection, Verify, Substitution, Elimination
Essential Questions	How can we determine which method would be most efficient? How can you determine how many answers a system might have?
Foundational Objectives	A.4 The student will solve d) systems of two linear equations in two variables algebraically and graphically; and e) practical problems involving equations and systems of equations.
Succeeding Objectives	MA.11 The student will use matrices to organize data and will add and subtract matrices, multiply matrices, multiply matrices by a scalar, and use matrices to solve systems of equations.

Thinking Map

SOL	AII.5 - Resources
Standard	The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve practical problems, including writing the first n terms, determining the n th term, and evaluating summation formulas. Notation will include and an.
Days	7
Key Vocabulary	Sequence, Series, Arithmetic, Geometric, Patterns, Explicit, Recursive, Summation, Sigma, ***n***th term, Geometric Means, Arithmetic Means
Essential Questions	How do you distinguish between a sequence and a series? How do you distinguish between an arithmetic and geometric sequence/series? What are the advantages of using a recursive formula or an explicit formula?
Succeeding Objectives	MA.13 The student will determine the sum of finite and infinite convergent series.

Thinking Map



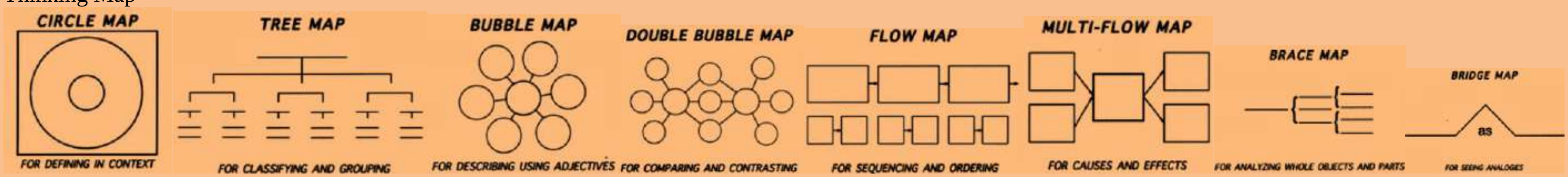
SOL	AII.6 - Resources
Standard	For absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions, the student will a) recognize the general shape of function families; and b) use knowledge of transformations to convert between equations and the corresponding graphs of functions.
Days	7
Key Vocabulary	Parent Function, Parabola, Radical, Vertex, Pre-image, Image, Anchor Graph, Translations, Reflections, Dilations, Logarithmic, Exponential, Cube Root
Essential Questions	How can transformations be applied to real world situations? What are the similarities and differences between the images and preimages generated with transformations? What are the differences between the various parent functions? (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions)
Foundational Objectives	A.7 The student will investigate and analyze linear and quadratic function families and their characteristics both algebraically and graphically, including a) determining whether a relation is a function; b) domain and range; c) zeros; d) intercepts; e) values of a function for elements in its domain; and f) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs. AFDA.1 The student will investigate and analyze linear, quadratic, exponential, and logarithmic function families and their characteristics. Key concepts include a) domain and range; b) intervals on which a function is increasing or decreasing; c) absolute maxima and minima; d) zeros; e) intercepts; f) values of a function for elements in its domain; g) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs; h) end behavior; and i) vertical and horizontal asymptotes. AFDA.2 The student will use knowledge of transformations to write an equation, given the graph of a linear, quadratic, exponential, and logarithmic function.
Succeeding Objectives	MA.1 The student will investigate and identify the properties of polynomial, rational, piecewise, and step functions and sketch the graphs of the functions. MA.2 The student will investigate and identify the characteristics of exponential and logarithmic functions to graph the function, solve equations, and solve practical problems.

Thinking Map

SOL	AII.7 - Resources
Standard	The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic function families algebraically and graphically. Key concepts include a) domain, range, and continuity; b) intervals in which a function is increasing or decreasing; c) extrema; d) zeros; e) intercepts; f) values of a function for elements in its domain; g) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs; h) end behavior; i) vertical and horizontal asymptotes; j) inverse of a function; and k) composition of functions algebraically and graphically.
Days	17
Key Vocabulary	Parent Function, Parabola, Radical, Vertex, Logarithmic, Exponential, Cube Root, zero, extrema, intercepts, domain, range, continuity, tables, end behavior, asymptotes, inverse of a function, composition of function, function families, discontinuities, removable, nonremovable, hole, relative extrema, absolute extrema.
Essential Questions	Why do discontinuities occur? What does it mean for a function to be continuous or discontinuous? How are extrema related to the increasing/decreasing intervals of a function? How do you sketch the graph of a polynomial using relative mins, maxs and end behavior? How are zeros of a function related to the x-intercepts of a function? Why do vertical asymptotes occur where they do? How does end behavior relate to horizontal asymptotes? How does the graph of an inverse function relate to the graph of the original function?
Foundational Objectives	A.7 The student will investigate and analyze linear and quadratic function families and their characteristics both algebraically and graphically, including a) determining whether a relation is a function; b) domain and range; c) zeros; d) intercepts; e) values of a function for elements in its domain; and f) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs. AFDA.1 The student will investigate and analyze linear, quadratic, exponential, and logarithmic function families and their characteristics. Key concepts include a) domain and range; b) intervals on which a function is increasing or decreasing; c) absolute maxima and minima; d) zeros; e) intercepts;

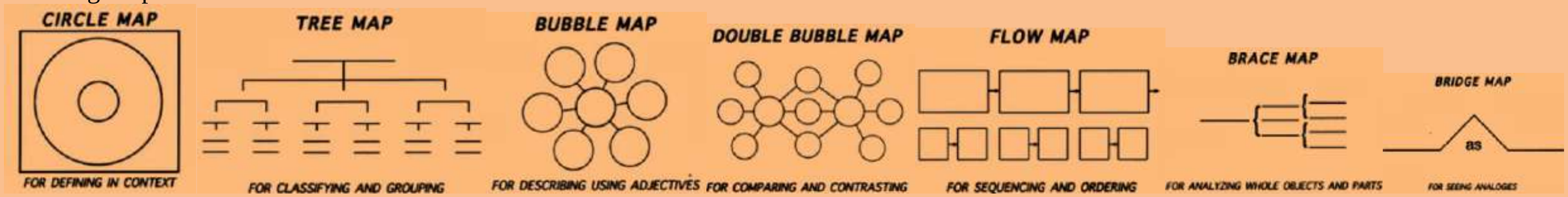
	f) values of a function for elements in its domain; g) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs; h) end behavior; and i) vertical and horizontal asymptotes. AFDA.2 The student will use knowledge of transformations to write an equation, given the graph of a linear, quadratic, exponential, and logarithmic function. AFDA.4 The student will use multiple representations of functions for analysis, interpretation, and prediction.
Succeeding Objectives	MA.3 The student will apply compositions of functions and inverses of functions to practical situations and investigate and verify the domain and range of resulting functions. MA.4 The student will determine the limit of an algebraic function, if it exists, as the variable approaches either a finite number or infinity. MA.5 The student will investigate and describe the continuity of functions.

Thinking Map



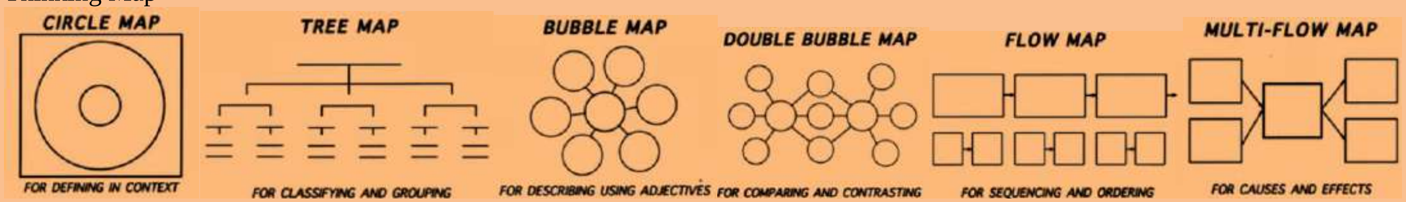
SOL	AII.8 - Resources
Standard	The student will investigate and describe the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph, and factors of a polynomial expression.
Days	1
Key Vocabulary	solutions, zeros, x-intercepts, factors, Fundamental Theorem of Algebra, degree, real, imaginary, complex
Essential Questions	How are the solutions, ,zeros, x-intercepts, and factors of a polynomial expression related?
Foundational Objectives	A.7 The student will investigate and analyze linear and quadratic function families and their characteristics both algebraically and graphically, including a) determining whether a relation is a function; b) domain and range; c) zeros; d) intercepts; e) values of a function for elements in its domain; and f) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs. AFDA.1 The student will investigate and analyze linear, quadratic, exponential, and logarithmic function families and their characteristics. Key concepts include a) domain and range; b) intervals on which a function is increasing or decreasing; c) absolute maxima and minima; d) zeros; e) intercepts; f) values of a function for elements in its domain; g) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs; h) end behavior; and i) vertical and horizontal asymptotes. AFDA.4 The student will use multiple representations of functions for analysis, interpretation, and prediction.
Succeeding Objectives	MA.2 The student will investigate and identify the characteristics of exponential and logarithmic functions to graph the function, solve equations, and solve practical problems.

Thinking Map



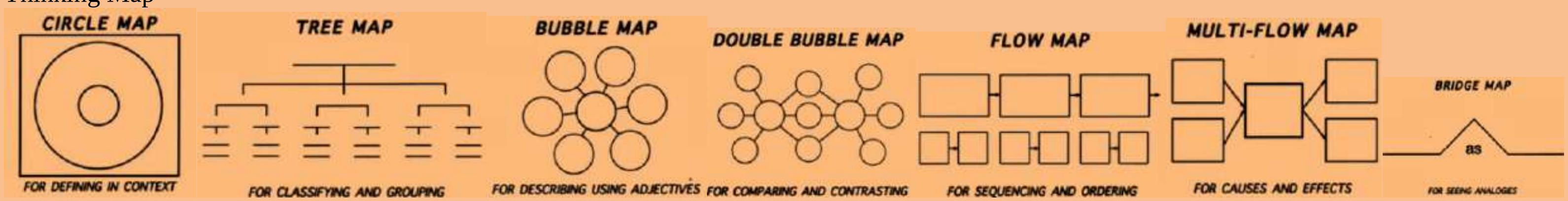
SOL	AII.9 - Resources
Standard	The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve practical problems, using mathematical models of quadratic and exponential functions.
Days	3
Key Vocabulary	collect, analyze, regression, curve of best fit, predictions, models, scatterplots, coefficient
Essential Questions	How do you determine the curve of best fit and decide if it is a good model of the data? How can we use the curve of best fit to make predictions about the data set? What kind of real life scenarios would best be modeled with quadratic regression?
Foundational Objectives	A.9 The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve practical problems, using mathematical models of linear and quadratic functions. AFDA.3 The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve practical problems using models of linear, quadratic, and exponential functions. AFDA.4 The student will use multiple representations of functions for analysis, interpretation, and prediction.
Succeeding Objectives	MA.6 The student will investigate, graph, and identify the properties of conic sections from equations in vertex and standard form.

Thinking Map



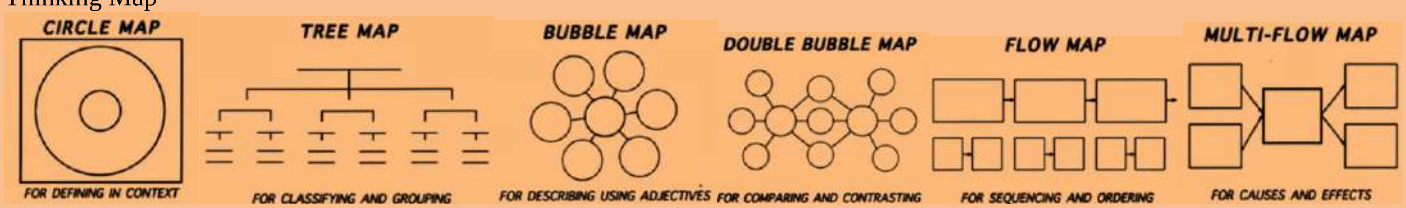
SOL	AII.10 - Resources
Standard	The student will represent and solve problems, including practical problems, involving inverse variation, joint variation, and a combination of direct and inverse variations.
Days	3
Key Vocabulary	Variation, inverse, joint, direct, constant of proportionality, constant of variation,
Essential Questions	What real world models involve direct, inverse and joint variation? How can you determine whether a direct variation exists in a data set or practical situation? How can you determine whether an inverse variation exists in a data set or practical situation?
Foundational Objectives	A.8 The student, given a data set or practical situation, will analyze a relation to determine whether a direct or inverse variation exists, and represent a direct variation algebraically and graphically and an inverse variation algebraically.

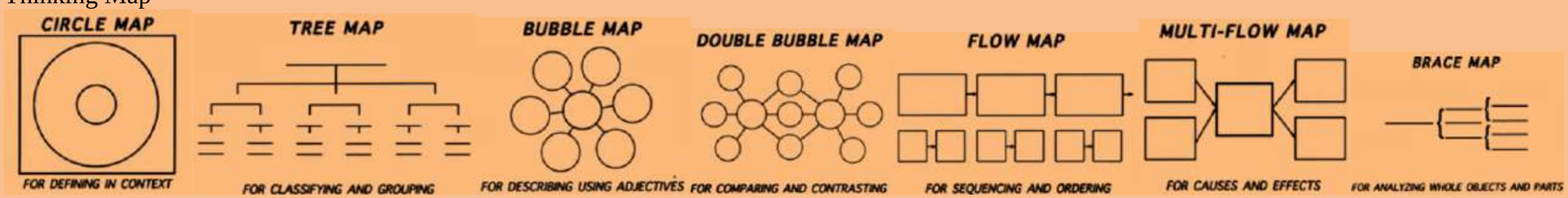
Thinking Map



SOL	[AII.11 - Resources]
Standard	The student will a) identify and describe properties of a normal distribution; b) interpret and compare z-scores for normally distributed data; and c) apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve.
Days	8
Key Vocabulary	normal distribution, z-scores, area under the curve, variation, standard deviation, mean, Empirical Rule (68 95 99.7), percentile
Essential Questions	How is the standard normal distribution related to probability? How does the standard deviation and mean affect the graph of the normal distribution? How are z-scores helpful in real world situations?
Foundational Objectives	AFDA.7 The student will a) identify and describe properties of normal distribution; b) interpret and compare z-scores for normally distributed data; and c) apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve.

Thinking Map



SOL	[AII.12- Resources]
Standard	The student will compute and distinguish between permutations and combinations.
Days	4
Key Vocabulary	Fundamental Counting Principle, Permutation, Combination, Factorial, Circular permutation,
Essential Questions	What real world applications could incorporate calculating possible permutations and combinations?
Thinking Map 	

Virginia 2016 *Mathematics Standards of Learning Curriculum Framework*
Introduction

The 2016 *Mathematics Standards of Learning Curriculum Framework*, a companion document to the 2016 *Mathematics Standards of Learning*, amplifies the *Mathematics Standards of Learning* and further defines the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and Curriculum Framework are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and *Curriculum Framework* into a broader, locally designed curriculum. The *Curriculum Framework* delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students.

The *Curriculum Framework* also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the *Curriculum Framework*.

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the *Curriculum Framework* facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The *Curriculum Framework* is divided into two columns: Understanding the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction. The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s).

Essential Knowledge and Skills
This section provides a detailed expansion of the mathematics knowledge and skills that each student should know and be able to demonstrate. This is not meant to be an exhaustive list of student expectations.

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "… the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade three state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades four through seven, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning.

Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of grade four. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. "Algebra readiness" describes the mastery of, and the ability to apply, the *Mathematics Standards of Learning*, including the Mathematical Process Goals for Students, for kindergarten through grade eight. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 *Mathematics Standards of Learning* form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics.

Equity

> "Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
>
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

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Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop problem-solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.

AII.1 The student will

a) add, subtract, multiply, divide, and simplify rational algebraic expressions;
b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents; and
c) factor polynomials completely in one or two variables.

Understanding The Standard	Essential Knowledge And Skills
• Computational skills applicable to numerical fractions also apply to rational expressions involving variables. • Radical expressions can be written and simplified using rational exponents. • Only radicals with a common radicand and index can be added or subtracted, which may require rewriting a radical with a lower base and different index. • A relationship exists among arithmetic complex fractions, algebraic complex fractions, and rational numbers. • The complete factorization of polynomials has occurred when each factor is a prime polynomial. • Pattern recognition can be used to determine complete factorization of a polynomial. • Polynomials may be factored in various ways, including, but not limited to grouping or applying general patterns such as difference of squares, sum and difference of cubes, and perfect square trinomials.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Add, subtract, multiply, and divide rational algebraic expressions. (a) • Simplify a rational algebraic expression with monomial or binomial factors. Algebraic expressions should be limited to linear and quadratic expressions. (a) • Recognize a complex algebraic fraction, and simplify it as a quotient or product of simple algebraic fractions. (a) • Simplify radical expressions containing positive rational numbers and variables. (b) • Convert between radical expressions and expressions containing rational exponents. (b) • Add and subtract radical expressions. (b) • Multiply and divide radical expressions. Simplification may include rationalizing denominators. (b) • Factor polynomials in one or two variables with no more than four terms completely over the set of integers. Factors of the polynomial should be constant, linear, or quadratic. (c) • Verify polynomial identities including the difference of squares, sum and difference of cubes, and perfect square trinomials. (c)

AII.2 The student will perform operations on complex numbers and express the results in simplest form using patterns of the powers of *i*.

Understanding the Standard	Essential Knowledge and Skills
• A complex number multiplied by its conjugate is a real number. • Equations having no real number solutions may have solutions in the set of complex numbers. • Algebraic properties apply to complex numbers as well as real numbers. • All complex numbers can be written in the form $a + bi$ where a and b are real numbers and i is the imaginary unit that satisfies the equation $i^2 = -1$ (e.g., $3 + 2i$; $\pm\sqrt{-9} = 0 \pm 3i$; $5 = 5 + 0i$).	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize that the square root of –1 is represented as i. • Simplify radical expressions containing negative rational numbers and express in $a + bi$ form. • Simplify powers of i. • Add, subtract, and multiply complex numbers.

AII.3 The student will solve
a) absolute value linear equations and inequalities;
b) quadratic equations over the set of complex numbers;
c) equations containing rational algebraic expressions; and
d) equations containing radical expressions.

Understanding the Standard	Essential Knowledge and Skills
• A quadratic function whose graph does not intersect the x-axis has roots with imaginary components. • The quadratic formula can be used to solve any quadratic equation. • The quadratic formula can be derived by applying the completion of squares to any quadratic equation in standard form. • The value of the discriminant of a quadratic equation can be used to describe the number and type of solutions. • Solutions of quadratic equations are real or a sum or difference of a real and imaginary component. • Complex solutions occur in conjugate pairs. • Quadratic equations with exactly one real root can be referred to as having one distinct root with a multiplicity of two. For instance, the quadratic equation, $x^2 - 4x + 4$, has two identical factors, giving one real root with a multiplicity of two. • The definition of absolute value (for any real numbers a and b, where $b \geq 0$, if $\|a\| = b$, then $a = b$ or $a = -b$) is used in solving absolute value equations and inequalities. • Absolute value inequalities in one variable can be solved algebraically using a compound statement. • Compound statements representing solutions of an inequality in one variable can be represented graphically on a number line.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve absolute value linear equations or inequalities in one variable algebraically. (a) • Represent solutions to absolute value linear inequalities in one variable graphically. (a) • Solve a quadratic equation over the set of complex numbers algebraically. (b) • Calculate the discriminant of a quadratic equation to determine the number and type of solutions. (b) • Solve rational equations with real solutions containing factorable algebraic expressions algebraically and graphically. Algebraic expressions should be limited to linear and quadratic expressions. (c) • Solve an equation containing no more than one radical expression algebraically and graphically. (d) • Solve equations and verify algebraic solutions using a graphing utility. (a, b, c, d)

AII.3 The student will solve
a) absolute value linear equations and inequalities;
b) quadratic equations over the set of complex numbers;
c) equations containing rational algebraic expressions; and
d) equations containing radical expressions.

Understanding the Standard	Essential Knowledge and Skills
• Practical problems can be interpreted, represented, and solved using equations and inequalities. • The process of solving equations can lead to extraneous solutions. • An extraneous solution is a solution of the simplified form of an equation that does not satisfy the original equation. • Equations can be solved in a variety of ways. • The zeros, roots, or solutions of a function are the values of x that make $f(x) = 0$ • The real zeros of a function are the x-intercepts of that function. • Radical expressions may be converted to expressions using rational exponents. • The equation of an inverse variation is a rational function. • Solutions and intervals may be expressed in different formats, including set notation, using equations and inequalities, or interval notation. – Examples may include:	

Equation/Inequality	Set Notation	Interval Notation
$x = 3$	$\{3\}$	
$x = 3$ or $x = 5$	$\{3, 5\}$	
$0 \leq x < 3$	$\{x \mid 0 \leq x < 3\}$	$[0, 3)$
$y \geq 3$	$\{y: y \geq 3\}$	$[3, \infty)$
Empty (null) set $\emptyset$	$\{\ \}$	

AII.4 The student will solve systems of linear-quadratic and quadratic-quadratic equations, algebraically and graphically.

Understanding the Standard	Essential Knowledge and Skills
• Quadratic equations included in this standard will only include those that can be represented as parabolas of the form $y = ax^2 + bx + c$ where $a \neq 0$. • Solutions of a system of equations are numerical values that satisfy every equation in the system. • A linear-quadratic system of equations may have zero, one, or two solutions. • A quadratic-quadratic system of equations may have zero, one, two, or an infinite number of solutions. • The coordinates of points of intersection in any system of equations are solutions to the system. • Practical problems can be interpreted, represented, and solved using systems of equations.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the number of solutions to a linear-quadratic and quadratic-quadratic system of equations in two variables. • Solve a linear-quadratic system of two equations in two variables algebraically and graphically. • Solve a quadratic-quadratic system of two equations in two variables algebraically and graphically. • Solve systems of equations and verify solutions of systems of equations with a graphing utility.

AII.5 The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve practical problems, including writing the first *n* terms, determining the n^{th} term, and evaluating summation formulas. Notation will include $\sum$ and a_n.

Understanding the Standard	Essential Knowledge and Skills
• Sequences and series arise from practical situations. • The study of sequences and series is an application of the investigation of patterns. • A sequence is a function whose domain is the set of natural numbers. • Sequences can be defined explicitly and recursively.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Distinguish between a sequence and a series. • Generalize patterns in a sequence using explicit and recursive formulas. • Use and interpret the notations $\sum$, n, n^{th} term, and a_n. • Given the formula, determine a_n (the n^{th} term) for an arithmetic or a geometric sequence. • Given formulas, write the first n terms and determine the sum, S_n, of the first n terms of an arithmetic or geometric series. • Given the formula, determine the sum of a convergent infinite series. • Model practical situations using sequences and series.

AII.6 will **For absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic functions, the student will**

a) recognize the general shape of function families; and
b) use knowledge of transformations to convert between equations and the corresponding graphs of functions.

Understanding the Standard	Essential Knowledge and Skills
• The transformation of a function, called a pre-image, changes the size, shape, and/or position of the function to a new function, called the image. • The graphs/equations for a family of functions can be determined using a transformational approach. • The graph of a parent function is an anchor graph from which other graphs are derived using transformations. • Transformations of functions may require the domain to be restricted. • Transformations of graphs include – Translations (horizontal and/or vertical shifting of a graph); – Reflections (over the x-axis and/or y-axis); and – Dilations (horizontal or vertical stretching and compressing of graphs). • The reflection of a function over the line $y = x$ represents the inverse of a function.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize the general shape of function families. (a) • Recognize graphs of parent functions. (a) • Identify the graph of a function from the equation. (b) • Write the equation of a function given the graph. (b) • Graph a transformation of a parent function, given the equation. (b) • Identify the transformation(s) of a function. Transformations of exponential and logarithmic functions, given a graph, should be limited to a single transformation. (b) • Investigate and verify transformations of functions using a graphing utility. (a, b)

AII.7 The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic function families algebraically and graphically. Key concepts include
a) domain, range, and continuity;
b) intervals in which a function is increasing or decreasing;
c) extrema;
d) zeros;
e) intercepts;
f) values of a function for elements in its domain;
g) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs;
h) end behavior;
i) vertical and horizontal asymptotes;
j) inverse of a function; and
k) composition of functions, algebraically and graphically.

Understanding the Standard	Essential Knowledge and Skills
• Functions may be used to model practical situations. • Functions describe the relationship between two variables where each input is paired to a unique output. • Function families consist of a parent function and all transformations of the parent function. • The domain of a function is the set of all possible values of the independent variable. • The range of a function is the set of all possible values of the dependent variable. • For each *x* in the domain of *f*, *x* is a member of the input of the function *f*, *f(x)* is a member of the output of *f*, and the ordered pair (*x*, *f(x)*) is a member of *f*. • A function is said to be continuous on an interval if its graph has no jumps or holes in that interval.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the domain, range, zeros, and intercepts of a function presented algebraically or graphically, including graphs with discontinuities. (a, d, e) • Describe a function as continuous or discontinuous. (a) • Given the graph of a function, identify intervals on which the function (linear, quadratic, absolute value, square root, cube root, polynomial, exponential, and logarithmic) is increasing or decreasing. (b) • Identify the location and value of absolute maxima and absolute minima of a function over the domain of the function graphically or by using a graphing utility. (c) • Identify the location and value of relative maxima or relative minima of a function over some interval of the domain graphically or by using a graphing utility. (c)

AII.7 The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic function families algebraically and graphically. Key concepts include

a) domain, range, and continuity;
b) intervals in which a function is increasing or decreasing;
c) extrema;
d) zeros;
e) intercepts;
f) values of a function for elements in its domain;
g) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs;
h) end behavior;
i) vertical and horizontal asymptotes;
j) inverse of a function; and
k) composition of functions, algebraically and graphically.

Understanding the Standard	Essential Knowledge and Skills
• The domain of a function may be restricted algebraically, graphically, or by the practical situation modeled by a function. • Discontinuous domains and ranges include those with removable (holes) and nonremovable (asymptotes) discontinuities. • A function can be described on an interval as increasing, decreasing, or constant over a specified interval or over the entire domain of the function. • A function, *f(x)*, is increasing over an interval if the values of *f(x)* consistently increase over the interval as the *x* values increase. • A function, *f(x)*, is decreasing over an interval if the values of *f(x)* consistently decrease over the interval as the *x* values increase. • A function, *f(x)*, is constant over an interval if the values of *f(x)* remain constant over the interval as the *x* values increase.	• For any *x* value in the domain of *f*, determine *f(x)*. (f) • Represent relations and functions using verbal descriptions, tables, equations, and graphs. Given one representation, represent the relation in another form. (g) • Describe the end behavior of a function. (h) • Determine the equations of vertical and horizontal asymptotes of functions (rational, exponential, and logarithmic). (i) • Determine the inverse of a function (linear, quadratic, cubic, square root, and cube root). (j) • Graph the inverse of a function as a reflection over the line $y = x$. (j) • Determine the composition of two functions algebraically and graphically. (k) • Investigate and analyze characteristics and multiple representations of functions with a graphing utility. (a, b, c, d, e, f, g, h, i, j, k)

AII.7 **The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic function families algebraically and graphically. Key concepts include**

a) domain, range, and continuity;
b) intervals in which a function is increasing or decreasing;
c) extrema;
d) zeros;
e) intercepts;
f) values of a function for elements in its domain;
g) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs;
h) end behavior;
i) vertical and horizontal asymptotes;
j) inverse of a function; and
k) composition of functions, algebraically and graphically.

Understanding the Standard	Essential Knowledge and Skills
• Solutions and intervals may be expressed in different formats, including set notation, using equations and inequalities, or interval notation. Examples may include:	

Equation/Inequality	Set Notation	Interval Notation
$x = 3$	$\{3\}$	
$x = 3$ or $x = 5$	$\{3, 5\}$	
$0 \leq x < 3$	$\{x \mid 0 \leq x < 3\}$	$[0, 3)$
$y \geq 3$	$\{y: y \geq 3\}$	$[3, \infty)$
Empty (null) set $\emptyset$	$\{ \}$	

- A function, f, has an absolute maximum located at $x = a$ if $f(a)$ is the largest value of f over its domain.
- A function, f, has an absolute minimum located at $x = a$ if $f(a)$ is the smallest value of f over its domain.
- Relative maximum points occur where the function changes from increasing to decreasing.
- A function, f, has a relative maximum located at $x = a$ over some

AII.7 The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic function families algebraically and graphically. Key concepts include

a) domain, range, and continuity;
b) intervals in which a function is increasing or decreasing;
c) extrema;
d) zeros;
e) intercepts;
f) values of a function for elements in its domain;
g) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs;
h) end behavior;
i) vertical and horizontal asymptotes;
j) inverse of a function; and
k) composition of functions, algebraically and graphically.

Understanding the Standard	Essential Knowledge and Skills
open interval of the domain if $f(a)$ is the largest value of f on the interval. • Relative minimum points occur where the function changes from decreasing to increasing. • A function, f, has a relative minimum located at $x = a$ over some open interval of the domain if $f(a)$ is the smallest value of f on the interval. • A value x in the domain of f is an x-intercept or a zero of a function f if and only if $f(x) = 0$. • Given a polynomial function $f(x)$, the following statements are equivalent for any real number, k, such that $f(k) = 0$: – k is a zero of the polynomial function $f(x)$ located at $(k, 0)$; – k is a solution or root of the polynomial equation $f(x) = 0$; – the point $(k, 0)$ is an x-intercept for the graph of $f(x) = 0$; and – $(x - k)$ is a factor of $f(x)$.	

AII.7 The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic function families algebraically and graphically. Key concepts include

a) domain, range, and continuity;
b) intervals in which a function is increasing or decreasing;
c) extrema;
d) zeros;
e) intercepts;
f) values of a function for elements in its domain;
g) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs;
h) end behavior;
i) vertical and horizontal asymptotes;
j) inverse of a function; and
k) composition of functions, algebraically and graphically.

Understanding the Standard	Essential Knowledge and Skills
• Connections between multiple representations (graphs, tables, and equations) of a function can be made. • End behavior describes the values of a function as x approaches positive or negative infinity. • If (a, b) is an element of a function, then (b, a) is an element of the inverse of the function. • The reflection of a function over the line $y = x$ represents the inverse of the reflected function. • A function is invertible if its inverse is also a function. For an inverse of a function to be a function, the domain of the function may need to be restricted. • Exponential and logarithmic functions are inverses of each other. • Functions can be combined using composition of functions. • Two functions, $f(x)$ and $g(x)$, are inverses of each other if $f(g(x)) = g(f(x)) = x$.	

AII.8 The student will investigate and describe the relationships among solutions of an equation, zeros of a function, *x*-intercepts of a graph, and factors of a polynomial expression.

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<tr><th>Understanding the Standard</th><th>Essential Knowledge and Skills</th></tr>
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- The *Fundamental Theorem of Algebra* states that, including complex and repeated solutions, an n^{th} degree polynomial equation has exactly n roots (solutions).
- Solutions of polynomial equations may be real, imaginary, or a combination of real and imaginary.
- Imaginary solutions occur in conjugate pairs.
- Given a polynomial function $f(x)$, the following statements are equivalent for any real number k, such that $f(k) = 0$:
  - k is a zero of the polynomial function $f(x)$ located at $(k, 0)$;
  - k is a solution or root of the polynomial equation $f(x) = 0$;
  - the point $(k, 0)$ is an x-intercept for the graph of polynomial $f(x) = 0$; and
  - $(x - k)$ is a factor of polynomial $f(x)$.
- Polynomial equations may have fewer distinct roots than the order of the polynomial. In these situations, a root may have "multiplicity." For instance, the polynomial equation $y = x^3 - 6x^2 + 9x$ has two identical factors, $(x - 3)$, and one other factor, x. This polynomial equation has two distinct, real roots, one with a multiplicity of 2.

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The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Define a polynomial function in factored form, given its zeros.
- Determine a factored form of a polynomial expression from the x-intercepts of the graph of its corresponding function.
- For a function, identify zeros of multiplicity greater than 1 and describe the effect of those zeros on the graph of the function.
- Given a polynomial equation, determine the number and type of solutions.

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AII.9 The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve practical problems, using mathematical models of quadratic and exponential functions.

Understanding the Standard	Essential Knowledge and Skills
• Data and scatterplots may indicate patterns that can be modeled with an algebraic equation. • The curve of best fit for the relationship among a set of data points can be used to make predictions where appropriate. • Knowledge of transformational graphing using parent functions can be used to verify a mathematical model from a scatterplot that approximates the data. • Graphing utilities can be used to collect, organize, represent, and generate an equation of a curve of best fit for a set of data. • Data that fit quadratic ($y = ax^2 + bx + c$), and exponential ($y = ab^x$) models arise from practical situations. • Rounding that occurs during intermediate steps of problem solving may reduce the accuracy of the final answer. • Evaluation of the reasonableness of a mathematical model of a practical situation involves asking questions including: – "Is there another curve (quadratic or exponential) that better fits the data?" – "Does the curve of best fit make sense?" – "Could the curve of best fit be used to make reasonable predictions?"	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine an equation of the curve of best fit, using a graphing utility, given a set of no more than 20 data points in a table, graph, or practical situation. • Make predictions, using data, scatterplots, or the equation of the curve of best fit. • Solve practical problems involving an equation of the curve of best fit. • Evaluate the reasonableness of a mathematical model of a practical situation.

AII.10 The student will represent and solve problems, including practical problems, involving inverse variation, joint variation, and a combination of direct and inverse variations.

Understanding the Standard	Essential Knowledge and Skills
• Practical problems can be represented and solved by using direct variation, inverse variation, joint variation, and a combination of direct and inverse variations. • A direct variation represents a proportional relationship between two quantities. The statement "*y* is directly proportional to *x*" is translated as $y = kx$. • The constant of proportionality (k) in a direct variation is represented by the ratio of the dependent variable to the independent variable and can be referred to as the constant of variation. • A direct variation can be represented by a line passing through the origin. • An inverse variation represents an inversely proportional relationship between two quantities. The statement "*y* is inversely proportional to *x*" is translated as $y = \frac{k}{x}$. • The constant of proportionality (k) in an inverse variation is represented by the product of the dependent variable and the independent variable and can be referred to as the constant of variation. • The graph of an inverse variation is a rational function. • Joint variation is a combination of direct variations. The statement "*y* varies jointly as *x* and *z*" is translated as $y = kxz$. • The value of the constant of proportionality is typically positive when applied in practical situations.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Given a data set or practical situation, write the equation for an inverse variation. • Given a data set or practical situation, write the equation for a joint variation. • Solve problems, including practical problems, involving inverse variation, joint variation, and a combination of direct and inverse variations.

AII.11 The student will

a) identify and describe properties of a normal distribution;

b) interpret and compare *z*-scores for normally distributed data; and

c) apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve.

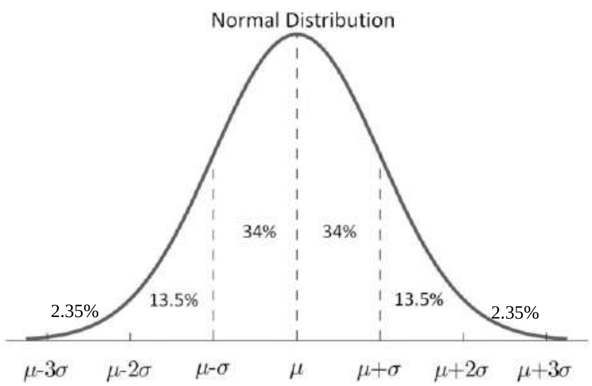
Understanding the Standard	Essential Knowledge and Skills
• The focus of this standard is on the interpretation of descriptive statistics, *z*-scores, probabilities, and their relationship to the normal curve in the context of a data set. • Descriptive statistics include measures of center (mean, median, mode) and dispersion or spread (variance and standard deviation). • Variance (σ^2) and standard deviation (σ) measure the spread of data about the mean in a data set. • Standard deviation is expressed in the original units of measurement of the data. • The greater the value of the standard deviation, the further the data tends to be dispersed from the mean. • In order to develop an understanding of standard deviation as a measure of dispersion (spread), students should have experiences analyzing the formulas for and the relationship between variance and standard deviation. • A normal distribution curve is the family of symmetrical, bell-shaped curves defined by the mean and the standard deviation of a data set. The arithmetic mean (μ) is located on the line of symmetry of the curve and is approximately equivalent to the median and mode of the data set. • The normal curve is a probability distribution and the total area under the curve is 1.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the properties of a normal distribution. (a) • Describe how the standard deviation and the mean affect the graph of the normal distribution. (a) • Solve problems involving the relationship of the mean, standard deviation, and z-score of a normally distributed data set. (b) • Compare two sets of normally distributed data using a standard normal distribution and *z*-scores, given the mean and standard deviation. (b) • Represent probability as area under the curve of a standard normal distribution. (c) • Use the graphing utility or a table of Standard Normal Probabilities to determine probabilities associated with areas under the standard normal curve. (c) • Use a graphing utility to investigate, represent, and determine relationships between a normally distributed data set and its descriptive statistics. (a, b, c)

AII.11 The student will

a) identify and describe properties of a normal distribution;

b) interpret and compare *z*-scores for normally distributed data; and

c) apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve.

Understanding the Standard	Essential Knowledge and Skills
• For a normal distribution, approximately 68 percent of the data fall within one standard deviation of the mean, approximately 95 percent of the data fall within two standard deviations of the mean, and approximately 99.7 percent of the data fall within three standard deviations of the mean. This is often referred to as the Empirical Rule or the 68-95-99.7 rule.  NOTE: This chart illustrates percentages that correspond to subdivisions in one standard deviation increments. Percentages for other subdivisions require the table of Standard Normal Probabilities or a graphing utility. • The mean and standard deviation of a normal distribution affect the location and shape of the curve. The vertical line of symmetry of the normal distribution falls at the mean. The greater the standard deviation, the wider (“flatter” or “less peaked”) the distribution of the data.	

AII.11 The student will

a) identify and describe properties of a normal distribution;
b) interpret and compare *z*-scores for normally distributed data; and
c) apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve.

Understanding the Standard	Essential Knowledge and Skills
• A *z*-score derived from a particular data value tells how many standard deviations that data value falls above or below the mean of the data set. It is positive if the data value lies above the mean and negative if the data value lies below the mean. • A standard normal distribution is the set of all *z*-scores. The mean of the data in a standard normal distribution is 0 and the standard deviation is 1. This allows for the comparison of unlike normal data. • The table of Standard Normal Probabilities and graphing utilities may be used to determine normal distribution probabilities. • Given a *z*-score (*z*), the table of Standard Normal Probabilities (*z*-table) shows the area under the curve to the left of *z*. This area represents the proportion of observations with a *z*-score less than the one specified. Table rows show the *z*-score's whole number and tenths place. Table columns show the hundredths place. • Graphing utilities can be used to represent a normally distributed data set and explore relationships between the data set and its descriptive statistics.	

AII.12 The student will compute and distinguish between permutations and combinations.

Understanding the Standard	Essential Knowledge and Skills
• The *Fundamental Counting Principle* states that if one decision can be made *n* ways and another can be made *m* ways, then the two decisions can be made *nm* ways. • A permutation is the number of possible ways to arrange a group of objects without repetition and when order matters (e.g., the outcome 1, 2, 3 is different from the outcome 3, 2, 1 when order matters; therefore, both arrangements would be included in the possible outcomes). • A combination is the number of possible ways to select or arrange objects when there is no repetition and order does not matter (e.g., the outcome 1, 2, 3 is the same as the outcome 3, 2, 1 when order does not matter; therefore, both arrangements would not be included in the possible outcomes).	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and contrast permutations and combinations. • Calculate the number of permutations of *n* objects taken *r* at a time. • Calculate the number of combinations of *n* objects taken *r* at a time. • Use permutations and combinations as counting techniques to solve practical problems. • Calculate and verify permutations and combinations using a graphing utility.