Algebra 1 Unit Plan

Office of Curriculum & Instruction

2019-2020 Mathematics Curriculum Guide

Algebra 1

Unit 4: Sequence and Exponential Function

May 1, 2020 – June 22, 2020

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Contents

Curriculum Map

A STORY OF UNITS (Yearlong Pacing Guide)				
Marking Period	**Unit 1** (9/9/19 – 11/13/19)	**Unit 2** (11/14/19- 1/30/20)	**Unit 3** (1/31/20-4/30/20)	**Unit 4** (5/4-20-6/22/20)
Unit Topic	**Linear Functions and Equations**	**System of Linear Equations/Inequalities**	**Quadratic Functions & Polynomials**	**Exponential Functions**
Description	Identify types of functions Create linear functions and equations to model a given situation and solve problems	Create systems of equations/Inequalities to model real-life situations and solve problems Identify types of functions using tables and graphs	Identify quadratic functions; finding key features for the graphs Solve quadratic equations by using tables, graphing, and algebraically. Identify types of polynomials and applying operations of polynomials	Create exponential functions and equations to model real life situations and solve the problems

Unit Overview

Unit 5: Sequences and Exponential Functions

Essential Questions

- How can an arithmetic and geometric sequence be defined recursively and explicitly?
- What are different ways a sequence can be represented?
- What is the difference between arithmetic and geometric sequences?
- What is the difference between linear and exponential functions?
- How do you model a quantity that changes regularly over time by the same percentage?
- How can exponential functions model real life situations?
- How do you write and graph an exponential relationship?

Enduring Understandings

- You can represent arithmetic functions recursively in the form $a_n = a_{n-1} + d$, or explicitly in the form $a_n = a_1 + d(n - 1)$ where d is the common difference.
- You can represent geometric functions recursively in the form $a_n = a_{n-1} \cdot r$, or explicitly in the form $a_n = a_1 \cdot r^{(n-1)}$ where r is the common ratio.
- Sequences can be represented as patterns/pictorially, tables, input/output pairs, descriptions, and graphs.
- Linear relationships, or arithmetic sequences, have a common difference or a constant rate of change. Exponential functions, or geometric sequences, have a common ration, or a constant percent change.
- You can represent repeated multiplication with a function in the form of $y = ab^x$ where b is a positive number other than 1; if b is greater than 1, the function models exponential growth; if b is between 0 and 1, the function models exponential decay.

New Jersey Student Learning Standards (NJSLS)

1) F.LE.1: Distinguish between situations that can be modeled with linear functions and with exponential functions.
   a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
   c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
2) F.LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
3) F.BF.1a: Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
4) F.BF.2: Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. (NOTE: students will not have to translate between two forms in Algebra 1, and this part of the standard will not be assessed)
5) F.IF.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
6) F.IF.3: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.*
7) F.IF.7e: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

8) F.LE.5: Interpret the parameters in a linear or exponential function in terms of a context.
9) A.SSE.1: Interpret expressions that represent a quantity in terms of its context.
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P*.
10) F.IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
11) A.CED.1: Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.
12) A.CED.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Ongoing standards: The following standards should be present in all applicable lessons.

1) N.Q.1: Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2) N.Q.2: Define appropriate quantities for the purpose of descriptive modeling.
3) N.Q.3: Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
4) F.IF.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
5) F.IF.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*[*]

M : Major Content S: Supporting Content A : Additional Content

Student Learning Material

Student Learning Material
Carnegie Learning https://www.carnegielearning.com/login/#/ Course: HSMS Algebra I (@2018) Supplemental resource: https://orange.agilemind.com/LMS/lmswrapper/LMS.html Course: Intensified Algebra I There are five modules in the text. Each module has 2 to 4 topics. Unit 4 consists of Module 3 - Topics 1 and 2 Module 3: Topic 1 begins with geometric sequences that are exponential functions, students write explicit formulas and examine the structure of the resulting function. They apply transformations, sketch graphs, consider whether the Order of Operations applies, and write equations based on described transformations. Topic 2 is tied closely to the work that students just completed in Introduction to Exponential Functions. In Using Exponential Equations, students explore strategies for distinguishing exponential functions that represent growth versus those that represent decay, and methods for solving exponential equations.

Modifications

Special Education/ 504:	English Language Learners:
-Adhere to all modifications and health concerns stated in each IEP. -Give students a MENU options, allowing students to pick assignments from different levels based on difficulty. -Accommodate Instructional Strategies: reading aloud text, graphic organizers, one-on-one instruction, class website (Google Classroom), handouts, definition list with visuals, extended time -Allow students to demonstrate understanding of a problem by drawing the picture of the answer and then explaining the reasoning orally and/or writing , such as Read-Draw-Write -Provide breaks between tasks, use positive reinforcement, use proximity -Assure students have experiences that are on the Concrete- Pictorial- Abstract spectrum by using manipulatives -Common Core Approach to Differentiate Instruction: Students with Disabilities (pg 17-18) -Strategies for Students with 504 Plans	- Use manipulatives to promote conceptual understanding and enhance vocabulary usage - Provide graphic representations, gestures, drawings, equations, realia, and pictures during all segments of instruction - During ALEKS lessons, click on “Español” to hear specific words in Spanish - Utilize graphic organizers which are concrete, pictorial ways of constructing knowledge and organizing information - Use sentence frames and questioning strategies so that students will explain their thinking/ process of how to solve word problems - Utilize program translations (if available) for L1/ L2 students - Reword questions in simpler language - Make use of the ELL Mathematical Language Routines (click here for additional information) -Scaffolding instruction for ELL Learners -Common Core Approach to Differentiate Instruction: Students with Disabilities (pg 16-17)
Gifted and Talented:	**Students at Risk for Failure:**
- Elevated contextual complexity - Inquiry based or open ended assignments and projects - More time to study concepts with greater depth - Promote the synthesis of concepts and making real	- Assure students have experiences that are on the Concrete- Pictorial- Abstract spectrum - Modify Instructional Strategies, reading aloud text, graphic organizers, one-on-one instruction, class website (Google Classroom), inclusion of more visuals and manipulatives, Field Trips, Google Expeditions, Peer

world connections - Provide students with enrichment practice that are imbedded in the curriculum such as: ● Application / Conceptual Development ● Are you ready for more? - Common Core Approach to Differentiate Instruction: Students with Disabilities (pg. 20) - Provide opportunities for math competitions - Alternative instruction pathways available	Support, one on one instruction - Assure constant parental/ guardian contact throughout the year with successes/ challenges - Provide academic contracts to students and guardians - Create an interactive notebook with samples, key vocabulary words, student goals/ objectives. - Always plan to address students at risk in your learning tasks, instructions, and directions. Try to anticipate where the needs will be and then address them prior to lessons. -Common Core Approach to Differentiate Instruction: Students with Disabilities (pg 19)

21st Century Life and Career Skills:

21st Century Life and Career Skills:	
Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study. https://www.state.nj.us/education/cccs/2014/career/9.pdf	
• **CRP1**. Act as a responsible and contributing citizen and employee. • **CRP2**. Apply appropriate academic and technical skills. • **CRP3**. Attend to personal health and financial well-being. • **CRP4**. Communicate clearly and effectively and with reason. • **CRP5**. Consider the environmental, social and economic impacts of decisions. • **CRP6**. Demonstrate creativity and innovation.	• **CRP7**. Employ valid and reliable research strategies. • **CRP8**. Utilize critical thinking to make sense of problems and persevere in solving them. • **CRP9**. Model integrity, ethical leadership and effective management. • **CRP10**. Plan education and career paths aligned to personal goals. • **CRP11**. Use technology to enhance productivity. • **CRP12**. Work productively in teams while using cultural global competence.
Students are given an opportunity to communicate with peers effectively, clearly, and with the use of technical language. They are encouraged to reason through experiences that promote critical thinking and emphasize the importance of perseverance. Students are exposed to various mediums of technology, such as digital learning, calculators, and educational websites.	

Technology Standards

Technology Standards:

All students will be prepared to meet the challenge of a dynamic global society in which they participate, contribute, achieve, and flourish through universal access to people, information, and ideas.

https://www.state.nj.us/education/cccs/2014/tech/

8.1 Educational Technology:

All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.

A. **Technology Operations and Concepts:** Students demonstrate a sound understanding of technology concepts, systems and operations.
B. **Creativity and Innovation:** Students demonstrate creative thinking, construct knowledge and develop innovative products and process using technology.
C. **Communication and Collaboration:** Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
D. **Digital Citizenship:** Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.
E. **Research and Information Fluency:** Students apply digital tools to gather, evaluate, and use of information.
F. **Critical thinking, problem solving, and decision making:** Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.

8.2 Technology Education, Engineering, Design, and Computational Thinking - Programming:

All students will develop an understanding of the nature and impact of technology, engineering, technological design, computational thinking and the designed world as they relate to the individual, global society, and the environment.

A. **The Nature of Technology: Creativity and Innovation-** Technology systems impact every aspect of the world in which we live.
B. **Technology and Society:** Knowledge and understanding of human, cultural, and societal values are fundamental when designing technological systems and products in the global society.
C. **Design:** The design process is a systematic approach to solving problems.
D. **Abilities in a Technological World:** The designed world in a product of a design process that provides the means to convert resources into products and systems.
E. **Computational Thinking: Programming-** Computational thinking builds and enhances problem solving, allowing students to move beyond using knowledge to creating knowledge.

Interdisciplinary Connections

Interdisciplinary Connections:

English Language Arts:	
ELA.Literacy.RI-9-10.4	Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings; analyze the cumulative impact of specific word choices on meaning and tone (e.g., how the language of a court opinion differs from that of a newspaper).
NJSLS ELA-LITERACY.SL.9-10.4	Present information, findings, and supporting evidence clearly, concisely, and logically such that listeners can follow the line of reasoning and the organization, development, substance, and style are appropriate to purpose, audience, and task.
NJSLS .ELA-LITERACY.W.9-10.2.A	Introduce a topic; organize complex ideas, concepts, and information to make important connections and distinctions; include formatting (e.g., headings), graphics (e.g., figures, tables), and multimedia when useful to aiding comprehension.

Pacing Guide

Overview		
Lesson	Topic	Suggesting Pacing
1	Recognizing Patterns and Sequences	2 days
2	Arithmetic and geometric sequences	3 days
3	Determining Recursive and Explicit Expressions from Contexts	3 days

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4	Geometric sequence exponential functions	3 days
5	Rational Exponent and graphs of exponential functions	3 days
n/a	Review	2 days

Summary:

16 days on new content (5 lessons/topics)
1 test day
1 performance task day
1 Benchmark mp4 day
5 Review days for NJSLA
2 ECRS – 1 day
2 flex days
27 days in Unit 4

Algebra 1 Unit 4 May 1st – June 22nd

Calendar

May 2020						
Sun	**Mon**	**Tue**	**Wed**	**Thu**	**Fri**	**Sat**
					1	2
3	4	*5*	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

June 2020						
Sun	**Mon**	**Tue**	**Wed**	**Thu**	**Fri**	**Sat**
	1	**2**	**3**	**4**	**5**	**6**
7	**8**	*9*	**10**	**11**	**12**	**13**
14	**15**	**16**	**17**	**18**	**19**	**20**
21	**22** **Early Dismissal week**	**23**	**24**	**25** **Last day for students**	**26**	**27**
28	**29**					

Assessment Framework

Formative and Summative Assessments					
Assessment	NJSLS	Estimated Time	Date	Format	Graded
Diagnostic/Readiness Assessment	F.LE.2, F.BF.1, F.BF.2, F.IF.3	<½ Block	before Lesson 1	Individual	No Yes (zero weight)
Assessment Checkup #1	F.LE.2, F.BF.1, F.BF.2, F.IF.3	<½ Block	after Lesson 3	Individual	No Yes
Assessment Checkup #2	A.CED.1, A.CED.2, F.IF.2, F.IF.4, F.IF.5, F.LE.1, F.IF.6, F.LE.5, A.SSE.1, F.LE.2, F.IF.7e	<½ Block	after lesson 6	Individual	No Yes
MP4 Benchmark Assessment	All	1 Block		Individual	Yes
ECRS		1 Block		Individual	

Authentic Assessments					
Assessment	NJSLS	Estimated Time	Date	Format	Graded
Performance Assessment #1: Reasoning and modeling with Linear vs. Exponential relationships		½ Block	after Lesson 4	Individual	Yes

NJSLA Window: 5/8/2020 – 5/19/2020

MP 4 Benchmark Assessment: 6/1/2020- 6/12/2020

Lesson Analysis:

Lesson 1: Recognizing Patterns and Sequences

Objectives

- After analyzing sequences given as a description, table, input/outputs, pictures, or terms, students will determine if it represents an arithmetic or geometric sequence, and describe the relationship (as a recursive process or steps for calculation) with at least ___ out of ___ correct on an exit ticket.

Focused Mathematical Practices

- MP 2: Reason abstractly and quantitatively
- MP 7: Look for and make use of structure
- MP 8: Look for and express regularity in repeated reasoning

Vocabulary

- Sequence, arithmetic sequence, geometric sequence, common difference, common ratio

Common Misconceptions/struggles

- Sequences and/or operations involving negative numbers
- Using correct vocabulary

Lesson Clarifications

- It is ***not necessary*** to go through all problems in both sections of Carnegie; teach what is necessary for the objective, concepts, and skills listed in this lesson.

CCSS	**Concepts** What students will know	**Skills** What students will be able to do	Material/ Resource	Suggested Pacing	Assessment Check Point
F.LE.1: Distinguish between situations that can be modeled with linear functions and with exponential functions. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit	***Review*** • ***New*** • Arithmetic sequences have a constant difference between any two consecutive terms; this constant difference is called the common difference. • Geometric sequences have a constant ratio between any two consecutive terms; this constant ratio is called the common ratio.	***Review*** • Identify patterns as repeated addition and repeated multiplication ***New*** • Describe sequences (given in multiple forms) as arithmetic or geometric • Describe sequences as a recursive process or steps for calculation (i.e." in order to get the next term in the sequence, you must add 2 to the last term" or "multiply each term by ½ to find the successive term") • Identify the common difference or common ratio of a sequence	Module 1 Topic 2 Lesson 2.1	2 days	

interval relative to another. F.BF.1a: Write a function that describes a relationship between two quantities. Determine an explicit expression, ***a recursive process, or steps for calculation from a context.***					

Lesson 2: Arithmetic and geometric sequences

Objectives

Determine the next term in a sequence.
Recognize arithmetic sequences and geometric sequences.
Determine the common difference or common ratio for a sequence.
Graph arithmetic and geometric sequences.
Recognize graphical behavior of sequences.
Sort sequences that are represented graphically.

Focused Mathematical Practices

- MP 2: Reason abstractly and quantitatively
- MP 4: Model with mathematics
- MP 7: Look for and make use of structure
- MP 8: Look for and express regularity in repeated reasoning

Vocabulary

Arithmetic Sequence, Common difference, Geometric Sequence, Common Ratio

Common Misconceptions/struggles

- Sequence notation especially in recursive form such as a_{n-1}
- Dividing a term by a number is the same as multiplying the term by fraction of that number.

Lesson Clarifications

- Students are not required to translate between forms of sequences (explicit vs. recursive)
- In this lesson, functions (or formulas) DO NOT need to be written in function notation.
- Writing formulas/functions to represent sequences in the major objective; interpreting parameters is secondary and can be spiraled into upcoming lessons

CCSS	**Concepts** What students will know	**Skills** What students will be able to do	Material/ Resource	Suggested Pacing	Assessment Check Point
F.BF.2: **Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations**, and translate between the two forms. F.LE.2: Construct linear and exponential functions, **including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).**	***Review*** • Arithmetic sequences have a constant difference between any two consecutive terms; this constant difference is called the common difference. • Geometric sequences have a constant ratio between any two consecutive terms; this constant ratio is called the common ratio. ***New*** • A recursive formula expresses each new term of a sequence based on a calculation on the preceding term in the sequence	***New*** • Describe sequences (given in multiple forms) as arithmetic or geometric • Identify the common difference or common ratio of a sequence ***New*** • Write functions (recursively)	Module 1 Topic 2 Lesson 2.2	3 days	

F.BF.1a: Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context. F.LE.5: Interpret the parameters in a linear or exponential function in terms of a context. A.SSE.1: Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*					

Lesson 3: Determining Recursive and Explicit Expressions from Contexts

Objectives

- Given a context students will write an explicit or recursive function to represent it with ___ out of ___ answered correctly on an exit ticket.
- Given a context sequence, students will interpret parameters/parts of the function in terms of the context it represents with ___ out of ___ answered correctly on an exit ticket.

Focused Mathematical Practices

- MP 2: Reason abstractly and quantitatively
- MP 7: Look for and make use of structure
- MP 8: Look for and express regularity in repeated reasoning

Vocabulary

- Exponential function

Common Misconceptions/struggles

- Interpreting the meaning of parameters in a recursive formula
- Using correct notation (a_n is the "n^{th}" term, a_1 is the first term, a_{n-1} the previous term, etc)

Lesson Clarifications

- Students are not required to translate between forms of sequences (explicit vs. recursive)
- Students are not required to graph sequences, however they should be exposed to examples of graphs so that they can build a connection between linear/exponential functions and arithmetic/geometric sequences

CCSS	**Concepts** What students will know	**Skills** What students will be able to do	Material/ Resource	Suggested Pacing	Assessment Check Point
F.IF.3: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.* F.BF.2: **Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations**, and translate between the two forms.	***Review*** • Arithmetic sequences have a constant difference between any two consecutive terms; this constant difference is called the common difference. • Geometric sequences have a constant ratio between any two consecutive terms; this constant ratio is called the common ratio. ***New*** • A recursive formula expresses each new term of a sequence based on a calculation on the preceding term in the sequence	***Review*** • Describe sequences (given in multiple forms) as arithmetic or geometric • Identify the common difference or common ratio of a sequence ***New*** • Write functions (recursively and explicitly) that represent sequences given in multiple forms, including those in a context • Interpret parts of a function for a sequence in terms of the context they represent	Module 1 Topic 2 Lesson 2.3	3 days	

F.LE.2: Construct linear and exponential functions, **including arithmetic and geometric sequences**, **given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).** F.BF.1a: Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.					

Lesson 4: Geometric sequence exponential functions

Objectives

Write a geometric sequence as an exponential function in the form f(x) = ab^x.
Identify the constant ratio and y-intercept in different representations of exponential functions.
Recognize when a relationship is exponential.
Use algebra to show that, for an exponential function $f(x) = a \cdot b^x$ the ratio $\frac{f(x+1)}{f(x)}$ is constant and equal to common ration b, and the y-intercept is represented by the ordered pair (0, a).

Focused Mathematical Practices

- MP 2: Reason abstractly and quantitatively
- MP 3: Construct viable arguments and critique the reasoning of others
- MP 4: Model with mathematics
- MP 6: Attend to precision

Vocabulary

- Simple interest, compound interest

Common Misconceptions/struggles

-

Lesson Clarifications

- Suggested outline: spend 2 days on CL ST 5.1, then 1 day on Engage NY L-21
- Continue to incorporate the following standards: N.Q, F.IF.2, F.IF.5

CCSS	**Concepts** What students will know	**Skills** What students will be able to do	Material/ Resource	Suggested Pacing	Assessment Check Point
F.LE.1: Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a	***Review*** • A linear function can be derived from an arithmetic sequence; graphs of arithmetic sequences follow the behavior of a linear graph that is discrete because the domain is in the integers • An exponential function can be derived from a geometric sequence; graphs of geometric sequences whose common ratio is greater than zero and not equal to 1 follow the behavior of an exponential graph that	***Review*** • Write linear functions (in function notation) to model problem situations • Evaluate functions given a value in function notation; interpret what these values mean in a context • Identify linear relationships and their common difference ***New*** • Identify the common ratio from different representations such as graphs, equation and table • Write exponential functions (in function	CL ST Module 3 Lesson 3.1 Engage NY L-21	3 days	

constant percent rate per unit interval relative to another. A.CED.1: Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*. A.CED.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. F.IF.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. F.LE.5: Interpret the parameters in a linear or exponential function in terms of a context. A.SSE.1: Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*	is discrete because the domain is in the integers ***New*** • Common ratio needs to be greater than 1 in order to have exponential growth • Common ratio needs to be between 0 and 1 for exponential decay • All geometric sequences are functions; however, only some geometric sequences are exponential functions.	notation) to model problem situations • Calculate and interpret the rate of change of an exponential function • Describe the domain of an exponential function			

Lesson 5: Rational Exponent and graphs of exponential functions

Objectives

- Rewrite powers with rational exponents as radical expressions.
- Rewrite radical expressions as powers with rational exponents.
- Use the properties of exponents to interpret output values for non-integer input values in exponential functions.
- Construct exponential functions and identify a common ratio between output values in a graph, a table, and the equation.
- Solve simple exponential equations using common bases

Focused Mathematical Practices

- MP 1: Make sense of problems and persevere in solving them
- MP 2: Reason abstractly and quantitatively
- MP 4: Model with mathematics
- MP 5: Use appropriate tools strategically

Vocabulary

- Horizontal asymptote, extracting square roots

Common Misconceptions/struggles

Students may enter $\sqrt{2}$ in the calculator and think the result is a terminating decimal that can be written as a rational number. Discuss the limitations of technology such that all nonterminating decimals appear to terminate.

Lesson Clarifications

- Continue to incorporate the following standards: N.Q, F.IF.2, F.IF.5

CCSS	**Concepts** What students will know	**Skills** What students will be able to do	Material/ Resource	Suggested Pacing	Assessment Check Point
F.IF.7e: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. F.IF.4: For a function that models a relationship between	***New*** • Exponential relationships can be modeled using a graph where the y-int and horizontal asymptote represent a characteristic in the context of the problem it represents	***Review*** • Graphing functions using a calculator or table of values • Evaluate functions given a value in function notation; interpret what these values mean in a context • Write exponential functions (in function notation) to model problem situations • Describe the domain of an exponential function ***New*** • Graph exponential functions (with and without a calculator)	CL ST Module 3 Lesson 3.2	3 days	

two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* A.CED.1: Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*. A.CED.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.		• Interpret key features of an exponential graph in a context (y-intercepts, asymptotes, intervals of increasing/decreasing) • Re-write expressions as rational exponents • Solve exponential equation with same BASE			

5 Practices for Orchestrating Productive Mathematics Discussions	
Practice	Description/ Questions
1. Anticipating	What strategies are students likely to use to approach or solve a challenging high-level mathematical task? How do you respond to the work that students are likely to produce? Which strategies from student work will be most useful in addressing the mathematical goals?
2. Monitoring	Paying attention to what and how students are thinking during the lesson. Students working in pairs or groups Listening to and making note of what students are discussing and the strategies they are using Asking students questions that will help them stay on track or help them think more deeply about the task. (Promote productive struggle)
3. Selecting	This is the process of deciding the ***what*** and the ***who*** to focus on during the discussion.
4. Sequencing	What order will the solutions be shared with the class?
5. Connecting	Asking the questions that will make the mathematics explicit and understandable. Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations.

Ideal Math Block

The following outline is the department approved ideal math block for grades 9-12.

1) Do Now (7-10 min)
    a. Serves as review from last class' or of prerequisite material
    b. Provides multiple entry points so that it is accessible by all students and quickly scaffolds up

2) Starter/Launch (5 min)
    a. Designed to introduce the lesson
    b. Uses concrete or pictorial examples
    c. Attempts to bridge the gap between grade level deficits and rigorous, on grade level content
    d. Provides multiple entry points so that it is accessible by all students and quickly scaffolds up

3) Mini-Lesson (15-20 min)
    a. Design varies based on content
    b. May include an investigative approach, direct instruction approach, whole class discussion led approach, etc.
    c. Includes CFU's
    d. Anticipates misconceptions and addresses common mistakes

4) Class Activity (25-30 min)
    a. Design varies based on content
    b. May include partner work, group work/project, experiments, investigations, game based activities, etc.

5) Independent Practice (7-10 min)
    a. Provides students an opportunity to work/think independently

6) Closure (5-10 min)
    a. Connects lesson/activities to big ideas
    b. Allows students to reflect and summarize what they have learned
    c. May occur after the activity or independent practice depending on the content and objective

7) DOL (5 min)
    a. Exit ticket

Ideal Math Block with Intervention Stations

Whole Group Instruction	50 min	INSTRUCTION (Grades 9 – 12) Daily Routine: Mathematical Content or Language Routine Anchor Task: Anticipate, Monitor, Select, Sequence, Connect Collaborative Work* Guided Practice Independent Work (Demonstration of Student Thinking)		TOOLS Manipulatives RESOURCES Agile Mind
Rotation Stations (Student Notebooks & Chromebooks Needed)	1-2X 35 min	STATION 1: Focus on current Grade Level Content STUDENT EXPLORATION* Independent or groups of 2-3 Emphasis on MP's 3, 6 (Reasoning and Precision) And MP's 1 & 4 (Problem Solving and Application) TOOLS/RESOURCES Agile Mind Math Journals	STATION 2: Focus on Student Needs TECH STATION Independent TOOLS/ RESOURCES Khan Academy Approved Digital Provider Fluency Practice	TEACHER STATION: Focus on Grade Level Content; heavily scaffolded to connect deficiencies TARGETED INSTRUCTION 4 – 5 Students TOOLS/ RESOURCES Agile Homework Manipulatives
	5 min	INSTRUCTION Exit Ticket (Demonstration of Student Thinking) TOOLS/RESOURCES Notebooks or Exit Ticket Slips		

Sample of Performance Tasks:

Reasoning Task (Extending the Definitions of Exponents)

NJSLS: F.LE.A, A.CED.A

A biology student is studying bacterial growth. She was surprised to find that the population of the bacteria doubled every hour. The following table was the data that she recorded for the first 4 hours.

Hours into study	0	1	2	3	4
Population (thousands)	4	8	16	32	64

Part A: Write an equation for P, the population of the bacteria, as a function of time, t. Then justify your equation.

Part B: The student conducting the study wants to create a table with more entries; specifically, she wants to fill in the population at each half hour. However, she forgot to make these measurements so she wants to estimate the values.

Instead she notes that the population increases by the same factor each hour, and reasons that this property should hold over each half-hour interval as well.

Complete the table below, and ***decide what constant factor, x, she should multiply the population by each half hour*** in order to produce consistent results. (Show your work or explain how you arrive your answer)

Hours into Study	0	1/2	1	3/2	2	5/2	3
Population (thousands)	4		8		16		32

Part C: Use the equation from Part A and the process that you find the growth factor for each half hour on Part B to explain why it makes sense that we define $2^{1/2}$ to be $\sqrt{2}$.

Part D: Julie working on the same problem makes the following claim:

The bacterial growth is an exponential function. The population doubles in 1 hour, then half that growth occurs in the first half-hour and the other half occurs in the second half-hour.

Is Julie's conjecture correct in this bacteria population problem? Explain your reasoning.

Extended Constructed Response (ECR)

Math Department ECR Protocol

ECR Protocol

(Extended Constructed Response)

Issuing

- Moving forward ECR'S will be disseminated by the first of each month and collected by the end of each month
- Method of Issuing: email and post on the website

Dissemination

- Teachers can elect to print copies for each student or use the Smartboard to project the ECR. (Note: Student work will be included in Student Portfolios)
- Students should be given up to 30 minutes depending on the complexity of the ECR
- Assure appropriate testing environment
- ECR should be completed independently

Scoring

- Conversion tables are available in the *Assessment & Data in Mathematics Bulletin* for genesis inputting purposes
- ECR's will count as Authentic Assessments
- Naming Protocol "Course Month ECR" (ex: Grade 6 October ECR)

Collection

- ECR's will be collected & kept in student portfolios
- Student work will be reviewed during CPT's

ECR Conversion Chart

Points	Genesis Conversion	Points	Genesis Conversion	Points	Genesis Conversion
0	55	0	55	0	55
1	59	1	69	1	69
2	69	2	79	2	89
3	79	3	89	3	100
4	89	4	100		
5	100				

Link of Unit 4 ECRs

https://www.dropbox.com/sh/hmvby1p19x0xqhi/AAAjA-dI8610Yt65VyD3E5Dfa?dl=0

Algebra 1 Unit 4 May 1st – June 22nd

Intervention Plan

Scope of Sequence				
Marking Period	MP 1 (9/9/19 – 11/13/19)	MP 2 (11/14/19- 1/30/20)	MP 3 (1/31/20-4/9/20)	**MP 4 (4/10-20-6/22/20)**
Unit Topic	Foundation of Algebra	Introduction to functions and equations	Rate of Change	Linear Functions
Description	Investigate the use of variables to represent unknowns and to generalize relationships. In addition, the unit also reviews important graphing skills of Algebra.	Introduce different mathematical representations to represent patterns and relationships; begin to use multiple representations of proportional and non-proportional situations.	Explore rate of change in motion problems and other situations; and model data sets that have a constant rate of change with a linear function.	Make connections between rate of change and slope. Introduce the linear function rule y=mx+b. Explore the value of m in the linear function rule and the relationships between slopes of parallel and perpendicular lines and connect the equations of lines to their graphs and descriptions.

Marking Period 4 Intervention Overview

Student Learning Material: Agile Mind Intensified Algebra I: https://orange.agilemind.com

Topics	Number of lessons	Topic Descriptions	NJSLS
The material incorporates review and repair strategies for pre-algebra concepts and skills, standards prior to Algebra I have been included.			
10. Understanding slope and intercepts	5-6 lessons	This topic relates the constant rate of change of a linear function, the slope of the line that is the linear function's graph, and the value of m in the linear function rule y = mx + b. Students explore this connection using tables, graphs, and function rules. It also develops students' understanding of the x- and y-intercepts of the graph of a linear model and the relationship between the intercepts and the situation being modeled. Students learn to find the values of the intercepts directly from linear function rules expressed in slope intercept form (y = mx + b) or standard form (Ax + By = C).	**A-CED A. 2**. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **F-IF B**. Interpret functions that arise in applications in terms of the context 4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. **F.-IF.6.** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **F-IF.C. 7**.Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **F-IF.C. 9**. Compare properties of two functions each represented in a different way. Linear, Quadratic, and Exponential Models★ — **F-LE A. 1**. Distinguish between situations that can be modeled with linear functions and with exponential functions
11. Parallel and Perpendicular Lines	3 lessons	Focus Skill work: Writing and analyzing various forms of equations of lines including standard form, slope-intercept form, and point slope form while practicing the distributive property and collecting like terms.	**A.CED.A.2**: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales **F.IF.C.7**: .Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases

			F.LE.2: . Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
12.Creating Linear Models for Data	7 lessons	This topic revisits analyzing rate of change to determine whether using a linear model to represent data is appropriate. It also develops the pointslope form for the equation of a line, explicitly connects the point-slope and slope intercept forms, and introduces students to the idea of transformations of functions by transforming the basic function y=x to create linear models for data. Linear regression and correlation is explored when finding the line of best fit for a data set.	**F.BF.A.3**: Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. S★**-ID B. 6.** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions b. fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. c. Fit a linear function for a scatter plot that suggests a linear association. **S.ID.C. 7**. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. **S.ID.C 8**. Compute (using technology) and interpret the correlation coefficient of a linear fit. **S.ID.C.9.** Distinguish between correlation and causation.

Multiple Representations

Exponential Function

Table

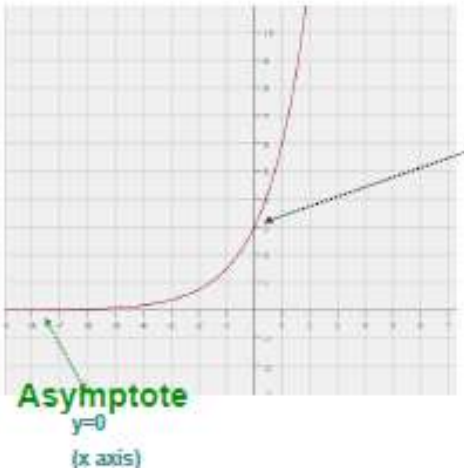
x	0	1	2	3	4	5	6	n
y	3	6	12	24	48	96	192	?

3(2) $3(2)(2) = 3(2)^2$ $3(2)(2)(2) = 3(2)^3$ $3(2)(2)(2)(2) = 3(2)^4$ $3(2)(2)(2)(2)(2) = 3(2)^5$

Initial number

Growth Factor

Graph



Algebraic (Function Notation)

$f(x) = 3(2)^x$

variable

Growth Factor

Initial number

Appendix A – Acronyms

#	Acronym	Meaning
1	AA	Authentic Assessment
2	AM	Agile Minds
3	AR	Additional Resources
4	CCSS	Common Core State Standards
5	CFU	Check for understanding
6	CL	Carnegie Learning
7	CL SA	Carnegie Learning Student Assignments
8	CL SP	Carnegie Learning Skills Practice
9	CL ST	Carnegie Learning Student Text
10	EOY	End of Year (assessment)
11	MP	Math Practice
12	MYA	Mid-Year Assessment (same as PBA)
13	PBA	Problem Based Assessment (same as MYA)
14	PLD	Performance Level Descriptors
15	SAP	Student Assessment Portfolio
16	SMP	Standards for Mathematical Practice