Algebra 1 Unit Plan

Unit 4: Polynomial Operations and Solving Quadratic Equations
March – April

ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

Contents

Curriculum Map

A STORY OF UNITS					
	SEP OCT	NOV	DEC JAN	FEB MAR	APR MAY JUN
Alg1 Tier 1/2	QUANTITAIVE RELATIONSHIPS, GRAPHS,	LINEAR EQUATIONS, INEQUALITIES, FUNCTION	QUADRATIC RELATIONSHIPS AND FUNCTION	SOLVING QUADRATIC EQUATION	SEQUENCE AND EXPONENTIAL FUNCTION
	Identify types of function with graphs and tables	Creating linear functions/equations and inequalities to model situation given and solve the problems	Identify quadratic functions, find key features for their graphs	Sove quadratic equations by graph, table, & algebraically	Identify exponential function, use the function to model situation given and solve problems

Unit Overview

Unit 4: Polynomial Operations and Solving Quadratic Equations
Essential Questions
➢ How are polynomial expressions similar to integers? ➢ What do the zeros of a polynomial function tell you about its graph? ➢ How can you write quadratic expressions and functions in different but equivalent forms, and what do the different forms tell you about the function's graph? ➢ How do you solve a quadratic equation? ➢ When will a quadratic equation give you real solutions? Non-real solutions? ➢ How do you solve systems of equations involving quadratics? ➢ What are the properties of real numbers?
Enduring Understandings
• Polynomials, like integers, can be added, subtracted, and multiplied by combining like terms and applying the distributive property. • The zeros of a function are the values in which the output is equal to zero, and where its graph crosses the x-axis. The zeros are easily identified when the function is written in factored form. • Quadratic functions (and expressions) can be written in factored form to reveal the zeros of the quadratic graph by a process called factoring. • Quadratic functions (and expressions) can be written in vertex form to reveal the vertex of the quadratic graph by a process called completing the square. • Quadratic equations can be solved using many techniques including factoring, completing the square, using the quadratic formula, and taking the square root. • Quadratic equations will either have 2 real and rational solutions, 2 real and irrational solutions, 1 real and rational solution, or 0 real solutions. This can be determined by the discriminant. • One method for solving systems of equations involving quadratics includes determining the points of intersection between their graphs.
Common Core State Standards
1) A.APR.1: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. 2) A.APR.3: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. 3) A.SSE.2: Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$. 4) A.SSE.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. 5) F.IF.8: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context 6) F.IF.7a: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* Graph linear and quadratic functions and show intercepts, maxima, and minima. 7) A.REI.4: Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in *x* into an

equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

8) A.REI.11: Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

9) N.RN.3: Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

M : Major Content S: Supporting Content A : Additional Content

21 Century Career Ready Practice

CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP3. Attend to personal health and financial well-being.
CRP4. Communicate clearly and effectively and with reason.
CRP5. Consider the environmental, social and economic impacts of decisions.
CRP6. Demonstrate creativity and innovation.
CRP7. Employ valid and reliable research strategies.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP10. Plan education and career paths aligned to personal goals.
CRP11. Use technology to enhance productivity.
CRP12. Work productively in teams while using cultural global competence.

Calendar

March 2019						
Sun	**Mon**	**Tue**	**Wed**	**Thu**	**Fri**	**Sat**
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

April 2019

Sun	Mon	Tue	Wed	Thu	Fri	Sat
31	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

Assessment Framework

Formative and Summative Assessments						
Assessment	CCSS	Estimated Time	Date	Format	SAP	Graded
Diagnostic/Readiness Assessment *Unit 3 Diagnostic (In Supplemental Resources on Dropbox)*	A.APR.1, A.APR.3, A.SSE.2, A.REI.4, A.REI.11, N.RN.3	<½ Block	2/9/15 or before Lesson 1	Individual	No	Yes (zero weight)
Assessment Checkup #1	A.APR.1, A.APR.3, A.SSE.2, A.SSE.3	<½ Block	2/24/15 or after Lesson 3	Individual	No	Yes
Assessment Checkup #2	A.SSE.2, A.SSE.3, A.REI.4, F.IF.8, F.IF.7a	<½ Block	3/9/15 or after lesson 7	Individual	No	Yes
Unit 4 Assessment	All	1 Block	3/19/15	Individual		Yes

Authentic Assessments						
Assessment	CCSS	Estimated Time	Date	Format	SAP	Graded
Performance Assessment: Reasoning with Quadratic Equations	A.REI.4	½ Block	3/13/15 or after Lesson 9	Individual	Yes	Yes
F.IF.8 Task #1	F.IF.8	½ Block	During or after Lesson 7	Optional	Optional	Optional
F.IF.8 Task #2	F.IF.8	½ Block	During or after Lesson 7	Optional	Optional	Optional

Scope and Sequence

Overview		
Lesson	Topic	Suggesting Pacing
1	CL ST 12.1: Adding and Subtracting Polynomials	2 days
2	CL ST 12.2: Multiplying Polynomials	2 days
3	CL ST 12.3: Factoring Polynomials	3 days
4	CL ST 12.4: Solving Quadratics by Factoring	2 days
5	CL ST 12.5: Special Products	2 days
6	CL ST 12.6: Approximating & Rewriting Radicals/Solving by Taking Square root	2 days
7	CL ST 12.7: Completing the Square	3 days
8	CL ST 13.1: Quadratic Formula	4 days
9	CL ST 13.4: Systems of Quadratic Equations	1 day
10	CL ST 14.1: Real Numbers	1 day
11	Review	1 day
Summary: 22 days on new content (11 lessons/topics) 1 review day 1 test day 1 flex day **25 days in Unit 4**		

Lesson 1: Adding and Subtracting Polynomials

Objectives

- Using algebra tiles to represent polynomials, students add and/or subtract polynomials with ___ out of ___ answered correctly on an exit ticket.
- By performing an error analysis and critiquing the work of others, students will justify or refute algebraic statements about adding or subtracting polynomials by scoring a ___ out of 5 on the exit ticket (Rubric)

Focused Mathematical Practices

- MP 2: Reason abstractly and quantitatively
- MP 3: Construct viable arguments and critique the reasoning of others
- MP 5: Use appropriate tools strategically (use the models provided)
- MP 7: Look for and make use of structure

Vocabulary

- Polynomial, term, coefficient, monomial, binomial, trinomial, degree of a term, degree of a polynomial

Common Misconceptions

- Distributing the "-" in subtracting polynomials
- Determining the degree of a polynomial if it does not begin with the leading term
- Correctly combining the like terms
- How to add/subtract when there are no matching terms [i.e. $(2x^2 + 5) – (6x – 2)$]
- How to represent quantities that add up to zero (i.e. $-2x + 2x$)

Lesson Clarifications

- Suggested outline:
  - Day 1: Spend ~15 minutes on the diagnostic, and observe student's work while they are taking it to use as formative assessment. Spend the rest of the block introducing terms, how to classify polynomials, and how to use algebra tiles to model simple addition and subtraction problems.
  - Day 2: Continue to use algebra tiles if necessary, however incorporate more difficult problems. Also incorporate opportunities for students to justify or refute algebraic statements about adding/subtracting polynomials.
- Instruction, materials, and DOL should include polynomials of various degrees, variables, and question types (non-routine problems).
- Pages 710 – 714 are not necessary and supplement book as necessary to meet CCSS, objectives, concepts, and skills.

CCSS	**Concepts** What students will know	**Skills** What students will be able to do	Material/ Resource	Suggested Pacing	Assessment Check Point
A.APR.1: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	***Review*** • $-(a + b) = -a – b$ • Only terms with the same variables and variable powers can be added or subtracted ***New*** • Polynomials are mathematical expressions involving the sum of powers in one or more variables multiplied by	***Review*** • Combine like terms • Apply the distributive property (i.e. $2x – (x – 5)$) ***New*** • Add and subtract polynomials • Provide reasoning to justify or refute algebraic statements about adding or subtracting polynomials	CL ST 12.1	2 days	Adding and Subtracting Polynomials Exit Ticket

	coefficients • Polynomials are closed under the operations of addition and subtraction				

Lesson 2: Multiplying Polynomials

Objectives

- After exploring and applying the area model, students will multiply polynomials with ___ out of ____ answered correctly on an exit ticket.
- By performing an error analysis and critiquing the work of others, students will justify or refute algebraic statements about adding or subtracting polynomials by scoring a ___ out of 5 on the exit ticket (Rubric)
- Using polynomial operations, students will identify (without calculator) and verify (with calculator) equivalent quadratic functions with ___ out of ____ answered correctly on an exit ticket.

Focused Mathematical Practices

- MP 2: Reason abstractly and quantitatively
- MP 3: Construct viable arguments and critique the reasoning of others
- MP 5: Use appropriate tools strategically (use the models provided)
- MP 7: Look for and make use of structure

Vocabulary

Common Misconceptions

- Students struggle with large polynomial products (i.e. binomial times a trinomial) because they don't understand the multiplication as the distributive property (for example, they have memorized FOIL but don't know what to do beyond that)
- Integer operations, including signs of integers
- Multiplication of powers (properties of exponents)

Lesson Clarifications

- Student text may need to be supplemented with additional instructional strategies, activities, and problems.
- Instruction, materials, and DOL should include polynomials of various degrees, variables, and question types (non-routine problems).
- Include opportunities for students to make connection between $(ax + b)(ax + b) = (ax + b)^2$; this will be revisited in 12.5
- Include problems that involve a simple context (i.e. area, perimeter, etc)
- Utilize the video on resources.carneglielearning.com on multiplying binomials
- Avoid using calculator (unless using it to graph equivalent functions and compare graphs).

CCSS	**Concepts** What students will know	**Skills** What students will be able to do	Material/ Resource	Suggested Pacing	Assessment Check Point
A.APR.1: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add,	***Review*** • $x^a(x^b) = x^{a+b}$ ***New*** • Multiplying polynomials is applying the distributive property several times (depending on the	***Review*** • Combine like terms • Apply the distributive property [i.e. $3x(x – 5)$] ***New*** • Multiply polynomials • Provide reasoning to justify or refute algebraic	CL ST 12.2	2 days	Multiplying polynomials exit ticket

subtract, and multiply polynomials. A.SSE.2: Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.	number of terms in one of the factors) • Polynomials are closed under the operations of Multiplication • Quadratic functions can be written in many different but equivalent forms; equivalent forms can be verified by multiplying or expanding the function or using a graphing calculator to show they have the same graph.	statements about multiplying polynomials • Identify and verify equivalent quadratic functions in standard form, factored form, and vertex form.			

Lesson 3: Factoring Polynomials

Objectives

- Using greatest common factors and/or algebra tiles to create rectangles that represent polynomials, students will factor a quadratic expression with ___ out of ___ answered correctly on an exit ticket.
- By applying the factoring technique, students will write quadratic functions in factored form, and construct a sketch of its graph with ___ out of ___ answered correctly on an exit ticket.
- By analyzing the factored form of a polynomial function, students will understand the connection between the function and the zeros of its graph with ___ out of ___ answered correctly on an exit ticket.

Focused Mathematical Practices

- MP 2: Reason abstractly and quantitatively
- MP 5: Using appropriately tools strategically
- MP 7: Look for and make use of structure
- MP 8: Express regularity in repeated reasoning

Vocabulary

- factor, factored form

Common Misconceptions

- Identifying that the zeros are the opposite of the r1 and r2 seen in the factored form of a quadratic function

Lesson Clarifications

- Multiple strategies for factoring should be introduced.
- Provide opportunities for students to develop understanding of the definition of a zero in a function and what the factored form of a polynomial function reveals about its graph.
- Avoid using calculator (unless using it to graph equivalent functions and compare graphs).
- Suggestion outline:
  - Day 1: Introduce GCF strategy (including co-constructing rules for factoring a quadratic trinomial into two binomials) and algebra tiles strategy. Students practice simpler examples and justify the equivalence of two expressions using multiplication
  - Day 2: Continue with Day 1 strategies and introduce more difficult examples. Solidify the definition of factored form of a polynomial function and zeros. Allow students to write equivalent quadratic functions in factored form and construct sketches of their graphs. Include examples of polynomial functions (degree 3) that are already given in factored form or as a product of a linear factor and quadratic factor. This connection will be similar to solving quadratics by factoring, without explicitly describing it like that (i.e. the roots/zeros are the values that make the output equal to zero…solve by inspection)
  - Day 3: Review and practice skills and concepts from Day 1 & 2; spend about ½ the block given checkpoint #1.

CCSS	**Concepts** What students will know	**Skills** What students will be able to do	Material/ Resource	Suggested Pacing	Assessment Check Point
A.SSE.2: Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as*	***Review*** • The leading coefficient of a quadratic function determines its concavity; if a is positive, the graph is concave up and if a is negative, the graph is	***Review*** • Graph a quadratic function given in factored form • Identify and verify equivalent quadratic functions in standard form, factored form, and	CL ST 12.3	3 days	Factoring Polynomials Exit Ticket

$(x^2 - y^2)(x^2 + y^2)$. A.APR.3: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. A.SSE.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* a. Factor a quadratic expression to reveal the zeros of the function it defines. F.IF.7a: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* Graph linear and quadratic functions and show intercepts, maxima, and minima.	concave down. • In factored form, f(x) = a(x – r1)(x – r2), a quadratic graph has zeros, or x-intercepts/roots, at r1 and r2. • Quadratic functions can be written in many different but equivalent forms; equivalent forms can be verified by multiplying or expanding the function or using a graphing calculator to show they have the same graph. ***New*** • A quadratic expression in standard form can be factored (using various strategies) so that it can be written as a product of linear factors. • The zeros of a polynomial function are the values in which the output is equal to zero, and where its graph crosses the x-axis. The zeros are easily identified when the function is written in factored form.	vertex form. ***New*** • Factor a quadratic expression/function • Determine the zeros of a polynomial function and construct a sketch of its graph (without a calculator)			

Lesson 4: Solving Quadratics by Factoring

Objectives
- By applying the factoring technique, students will solve quadratic equations with ___ out of ___ questions answered correctly on an exit ticket.
- After exploring how to solve quadratics by factoring, students will describe its connection to finding the zeros of a quadratic function in factored form with ___ out of ___ questions answered correctly on an exit ticket.

Focused Mathematical Practices
- MP 1: Make sense of problems and persevere in solving them
- MP 5: Use appropriate tools strategically
- MP 6: Attend to precision
- MP 7: Look for and make use of structure

Vocabulary
- zeros, roots, Zero Product Property, Converse of Multiplication Property of Zero

Common Misconceptions
- Identifying that the zeros are the opposite of the r1 and r2 seen in the factored form of a quadratic function

Lesson Clarifications
- Continue to make connections between solving quadratic equations and the factored form/zeros of a quadratic function. Students should still be determining zeros/roots/x-intercepts and constructing sketches of quadratic functions.
- Include examples where the given equation is not in standard form and has non-integer coefficients.
- Avoid using calculator (unless using it to graph equivalent functions and compare graphs).

CCSS	**Concepts** What students will know	**Skills** What students will be able to do	Material/ Resource	Suggested Pacing	Assessment Check Point
A.REI.4: Solve quadratic equations in one variable. a. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and **factoring**, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	***Review*** • The leading coefficient of a quadratic function determines its concavity; if a is positive, the graph is concave up and if a is negative, the graph is concave down. • In factored form, f(x) = a(x – r1)(x – r2), a quadratic graph has zeros, or x-intercepts/roots, at r1 and r2. • Quadratic functions can be written in many different but equivalent forms; equivalent forms can be verified by multiplying or expanding the function	***Review*** • Graph a quadratic function (without calculator) • Factor a quadratic expression/function • Determine the zeros of a polynomial function (given in factored form) and construct a sketch of its graph ***New*** • Solve a quadratic equation using factoring (when function is given in various forms) • Justify solutions by substituting values for x back into the original equation. • Understand similarities	CL ST 12.4	2 days	Solving Quadratics by Solving Exit Ticket

A.SSE.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* a. Factor a quadratic expression to reveal the zeros of the function it defines. A.APR.3: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. F.IF.7a: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* Graph linear and quadratic functions and show intercepts, maxima, and minima.	or using a graphing calculator to show they have the same graph. • A quadratic expression in standard form can be factored (using various strategies) so that it can be written as a product of linear factors. • ***New*** • The zeros of a polynomial/quadratic function are the values in which the output is equal to zero, and where its graph crosses the x-axis. The zeros are easily identified when the function is written in factored form, by setting each linear factor equal to zero. • The Zero Product Property states that if the product of two or more factors is equal to zero, then at least one factor must be equal to zero. If a(b) = 0, then a = 0 or b = 0.	between solving a quadratic equation by factoring and finding the zeros of a quadratic function in factored form			

Lesson 5: Special Products

Objectives

- By applying the area model, students will understand the connection between the factored form and expanded form of perfect square trinomials with ___ out of ___ correct on an Exit Ticket.
- After looking for patterns in forms of expressions, students will identify ways to rewrite polynomial expressions and equations with ___ out of ___ answered correctly on an Exit Ticket.
- After learning a new factoring technique, students will solve quadratic equations, and find zeros of and graph quadratic functions with ___ out of ___ answered correctly on an Exit Ticket.

Focused Mathematical Practices

- MP 7: Look for and make use of structure
- MP 8: Express regularity in repeated reasoning

Vocabulary

- Difference of two squares, perfect square trinomial

Common Misconceptions

- Not realizing that the sum of "-ax" and "ax" equals zero (avoid the use of "cancelling out")
- Confusing 2x and x^2 – for example, when dealing with perfect square trinomials students make errors on when to add 7+7 and when to multiply 7*7.
- Trying to factor the sum of two squares. For example $x^2 + 25 = (x + 5)^2$ or $(x + 5)(x – 5)$. See page 753, #6.

Lesson Clarifications

- Do not include the sum/difference of two cubes, however see Exit Ticket items to see the limitations and level of difficulty of how this standard could be assessed
- A.SSE.2 is the primary standard in this lesson. The lesson analysis below contains an extensive list of the possible Concepts and Skills used in this lesson. Keep in mind that most are a review from prior lessons. Content and instruction should be spiraling from lesson to lesson. Avoid teaching new concepts and skills in isolated chunks (i.e. continue to have students practice solving quadratic equations and justifying their solutions, finding zeros and graphing quadratic functions, and verify how two expressions/functions are equivalent using technology)
- Avoid the use of calculators except when graphing to verify equivalent functions.
- Limit polynomial expressions/functions to the power of 2.

CCSS	Concepts What students will know	Skills What students will be able to do	Material/ Resource	Suggested Pacing	Assessment Check Point
A.SSE.2: Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.* A.REI.4: Solve quadratic equations in one variable. a. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and **factoring**, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	***Review*** • The leading coefficient of a quadratic function determines its concavity; if a is positive, the graph is concave up and if a is negative, the graph is concave down. • In factored form, f(x) = a(x – r1)(x – r2), a quadratic graph has zeros, or x-intercepts/roots, at r1 and r2. • Quadratic functions can be written in many different but equivalent forms; equivalent forms can be verified by multiplying or expanding the function or using a graphing calculator to show they have the same graph. • A quadratic expression in standard form can be factored (using various strategies) so that it can be written as a product of linear factors.	***Review*** • Factor and a quadratic expression/function • Multiply two binomials • Solve a quadratic equation using factoring • Graph a quadratic function (without calculator) ***New*** • Recognize situations when a polynomial can be written in a more simplified or desired form; rewrite quadratic equations in different but equivalent forms in order to solve them. • Find dimensions of squares given its area in the form of a quadratic expression	CL ST 12.5	2 days	Special Products Exit Ticket

A.SSE.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* a. Factor a quadratic expression to reveal the zeros of the function it defines. A.APR.3: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	• The zeros of a polynomial/quadratic function are the values in which the output is equal to zero, and where its graph crosses the x-axis. The zeros are easily identified when the function is written in factored form, by setting each linear factor equal to zero. • The Zero Product Property states that if the product of two or more factors is equal to zero, then at least one factor must be equal to zero. If a(b) = 0, then a = 0 or b = 0. ***New*** • An expression in the form $a^2 - b^2$ can be written as $(a - b)(a + b)$; this is called the difference of two squares. • An expression in the form of $a^2 + 2ab + b^2$ can be written in the form $(a + b)(a + b)$ or $(a + b)^2$; this is called a perfect square trinomial. Similarly, $a^2 - 2ab + b^2 = (a - b)(a - b) = (a - b)^2$				

Lesson 6: Approximating and rewriting radicals/Solve by taking square root

Objectives

- Using inverse operations and properties of square roots, students will solve quadratic equations, determine their roots and create a sketch of its graph with ___ out of ___ correct on an Exit Ticket.
- After exploring several scenarios with quadratic equations, students will recognize situations when a quadratic equation results in no real solutions or 1 real solution with ___ out of ___ correct on an Exit Ticket.

Focused Mathematical Practices

- MP 2: Reason abstractly and Quantitatively
- MP 7: Look for and make use of structure

Vocabulary

- Square root, positive/principal square root, negative square root, radical expression, radicand

Common Misconceptions

- Not understanding why we take the positive and negative square root of a number (memorizing vs. understanding that 9*9 = 81 but -9*-9 also = 81)

Lesson Clarifications

- Solving quadratics by inspection and taking the square root is the primary focus for this lesson; approximating an rewriting radicals is secondary and can always be spiraled in to future lessons (i.e. revisit in a Do Now prior to the quadratic formula)
- Instructional materials will need to be supplemented with additional and more rigorous examples.
- Provide opportunities for students to recognize and reason when an equation will result in no real solutions; have students explain what this means graphically.

CCSS	**Concepts** What students will know	**Skills** What students will be able to do	Material/ Resource	Suggested Pacing	Assessment Check Point
A.REI.4: Solve quadratic equations in one variable. a. Solve quadratic equations by **inspection (e.g., for $x^2 = 49$), taking square roots**, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. A.APR.3: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	***Review*** • The leading coefficient of a quadratic function determines its concavity; if a is positive, the graph is concave up and if a is negative, the graph is concave down. • The zeros of a polynomial/quadratic function are the values in which the output is equal to zero, and where its graph crosses the x-axis. ***New*** • Quadratic equations in some forms can be solved by using inverse operations; the inverse operation of something "squared" is taking the positive and negative square root. • Negative numbers do not have real roots. Zero has only 1 square root, which is also zero. • A quadratic equation/function that results in no real solutions/roots will have a graph that never	***Review*** • Solving equations using inverse operations • Justify solutions by substituting values back into original equation. • Graph a quadratic function (without calculator) ***New*** • Solve quadratic equations using inverse operations, including taking the positive and negative square roots. • Find zeros of quadratic functions by using inverse operations including taking the positive and negative square roots; create a sketch of its graph. • Describe situations and provide reasoning for when a quadratic equation/function would result in no real solutions/roots or 1 real solution/root; describe the graph of these situations (this may involve solving literal equations)	CL ST 12.6	2 days	Solving by Taking Square Roots Exit Ticket

	crosses the x-axis.				

Lesson 7: Completing the Square

Objectives

- After an algebra tiles activity, students will understand how the completing the square technique as a way to rewrite quadratic expressions with ___ out of ___ correct on an Exit Ticket.
- By applying the process of completing the square, students will find roots of quadratic equations and graph its function with ___ out of ___ correct on an Exit Ticket.
- By applying the process of completing the square, students will write a quadratic function in vertex form to reveal extreme value with ___ out of ___ correct on an Exit Ticket.
- After exploring several scenarios with quadratic equations, students will recognize situations when a quadratic equation results in no real solutions or 1 real solution with ___ out of ___ correct on an Exit Ticket.

Focused Mathematical Practices

- MP 2: Reason abstractly and Quantitatively
- MP 5: Use appropriate tools strategically
- MP 7: Look for and make use of structure
- MP 8: Express regularity in repeated reasoning

Vocabulary

- Completing the square, properties of equality, additive identity property, additive inverse property

Common Misconceptions

- Making mistakes with negative signs. For example: writing $x^2 - 6x + 9 = (x + 3)^2$ or completing $x^2 - 6x$ with -9.

Lesson Clarifications

- Suggested outline:
  - Day 1: explore the concept of completing the square with algebra tiles; write a quadratic function in vertex form.
  - Day 2: continue to use completing the square to write quadratic functions in vertex form and also find their roots/zeros and create a graph.
  - Day 3: incorporate multi-step or application problems
- Expose students to problems with more difficult numbers (i.e. odd or non-integer “b” values, or when a ≠ 1), but don’t let it be a major emphasis.
- The lesson analysis below contains an extensive list of the possible Concepts and Skills used in this lesson. Keep in mind that most are a review from prior lessons. Content and instruction should be spiraling from lesson to lesson. Avoid teaching new concepts and skills in isolated chunks (i.e. continue to have students practice solving quadratic equations and justifying their solutions, finding zeros and graphing quadratic functions, and verify how two expressions/functions are equivalent using technology, describing situations that would result in no real solution/roots and describe their graphs)
- Opportunities for optional Authentic Assessment Tasks: F.IF.8 Task #1, F.IF.8 Task #2

CCSS	**Concepts** What students will know	**Skills** What students will be able to do	Material/ Resource	Suggested Pacing	Assessment Check Point
A.REI.4: Solve quadratic equations in one variable. b. Solve quadratic equations by	***Review*** • The leading coefficient of a quadratic function determines its concavity; if a is	***Review*** • Finding roots/zeros of quadratic equations using inverse operations, including taking the	CL ST 12.7 F.IF.8 Task #1 F.IF.8	3 days	Completing the Square Exit Ticket

inspection (e.g., for $x^2 = 49$), taking square roots, **completing the square**, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. A.SSE.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* b. **Complete the square** in a quadratic expression to reveal the maximum or minimum value of the function it defines. F.IF.8: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and **completing the square in a quadratic function** to show zeros, **extreme values, and symmetry** of the graph, and **interpret these in terms of a context**	positive, the graph is concave up and if a is negative, the graph is concave down. • The zeros of a polynomial/quadratic function are the values in which the output is equal to zero, and where its graph crosses the x-axis. • Quadratic equations in some forms can be solved by using inverse operations; the inverse operation of something "squared" is taking the positive and negative square root. • Negative numbers do not have real roots. Zero has only 1 square root, which is also zero. • A quadratic equation/function that results in no real solutions/roots will have a graph that never crosses the x-axis. ***New*** • The additive inverse property states that the quantity c + (-c) = 0 and the additive identity property states that if a = b, then a + c + (-c) = b** • If a = b, then the properties of equality state that a + c = b + c or a – c = b – c ** • Completing the square can be used to change a quadratic function from standard form into vertex form in order to find different key features of the function • One way to factor quadratic functions that	square roots • Justify solutions by substituting values back into original equation. • Graph a quadratic function (without calculator) • Describe situations and provide reasoning for when a quadratic equation/function would result in no real solutions/roots or 1 real solution/root; describe the graph of these situations (this may involve solving literal equations) ***New*** • Complete the square of a quadratic expression to create a perfect square trinomial • Solve/find roots of a quadratic equation using completing the square process. • Write a quadratic expression/function in vertex form using completing the square process • Solve multi-step or application problems that involve writing a quadratic function in vertex form and interpreting it in a context	Task #2		

A.APR.3: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	do not have whole number factors is by completing the square • Completing the square is a process that allows you to factor a completed trinomial square by factoring it as a square of a binomial and then finding the square root.				

Lesson 8: Quadratic Formula

Objectives

- Using the quadratic formula, students will find roots of quadratic equations and graph its function with ___ out of ___ correct on an Exit Ticket.
- After exploring several scenarios with quadratic equations, students will provide reasoning for situations that result in no real solutions or 1 real solution with ___ out of ___ correct on an Exit Ticket.
- Given a quadratic equation in any form, students will choose the best method for solving it, and verify the solution graphically with ___ out of ___ correct on an Exit Ticket.

Focused Mathematical Practices

- MP 2: Reason abstractly and Quantitatively
- MP 5: Use appropriate tools strategically
- MP 7: Look for and make use of structure

Vocabulary

- Quadratic formula, discriminant

Common Misconceptions

- Identifying the values of a, b, and c and substituting them correctly into the formula (particularly with negative signs)
- Simplifying radicals that are not perfect squares
- Knowing how to manipulate equations not given in standard form or with integer coefficients AND knowing the most efficient method for solving

Lesson Clarifications

- Suggested outline:
  - Day 1: Derive the quadratic formula using the process of completing the square; use quadratic formula to solve equations and sketch graph (keep equations/examples simple).
  - Day 2: Continue to solve equations using the quadratic formula and sketch graphs; introduce more complex scenarios (equation not given in standard form; b or c is zero; a, b, or c are non-integers; solutions are irrational); Show how solutions can be verified graphically by finding points of intersection (i.e. $x - 2 = x^2 + 3x - 5$ by graphing either side of the equal sign as a function and finding points of intersection); introduce scenarios that result in 1 real solution or no real solutions and examine their accompanying graphs.
  - Day 3: formalize a way to determine the number of real solutions (discriminant); understand the connection between the number of real solutions and its graph; provide reasoning for when equations would result in 0, 1, or 2 real solutions.
  - Day 4: Review all methods for solving quadratics (factoring, taking square root, completing square, quadratic formula, graphing/finding points of intersection) and understand cases when one method is more preferred; performance task/authentic assessment.
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CCSS	**Concepts** What students will know	**Skills** What students will be able to do	Material/ Resource	Suggested Pacing	Assessment Check Point
A.REI.4: Solve quadratic equations in one variable. a. Use the method of completing the square	***Review*** • The leading coefficient of a quadratic function determines its	***Review*** • Justify solutions by substituting values back into original equation.	CL ST 13.1 Performance Assessment (reasoning with quadratic	4 days	Quadratic formula Exit Ticket

to transform any quadratic equation in *x* into an equation of the form $(x - p)^2 = q$ that has the same solutions. **Derive the quadratic formula from this form**. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the **quadratic formula** and factoring, as appropriate to the initial form of the equation. **Recognize when the quadratic formula gives complex solutions** and write them as $a \pm bi$ for real numbers *a* and *b*. A.APR.3: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. A.REI.11: Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive	concavity; if a is positive, the graph is concave up and if a is negative, the graph is concave down. • The zeros of a polynomial/quadratic function are the values in which the output is equal to zero, and where its graph crosses the x-axis. • Negative numbers do not have real square roots. Zero has only 1 square root, which is also zero. • A quadratic equation/function that results in no real solutions/roots will have a graph that never crosses the x-axis; A quadratic equation/function that results in 1 real solution/root will have a graph that crosses the x-axis once (at the vertex). ***New*** • The quadratic formula is $x = \frac{-b \pm\sqrt{b^2 - 4ac}}{2a}$ • The quadratic formula was derived from the process of completing the square; this formula can be used to find the solutions of a quadratic equation. • The discriminant is $b^2 - 4ac$ and can be used to determine how many real solutions a quadratic equation has; if the discriminant is positive, it will have 2 real solutions; zero – 1 real solution; negative –	• Graph a quadratic function (without calculator) using zeros and concavity • Describe situations and provide reasoning for when a quadratic equation/function would result in no real solutions/roots or 1 real solution/root; describe the graph of these situations (this may involve solving literal equations) ***New*** • Solve/find roots of a quadratic equation using the quadratic formula. • Verify solutions algebraically and graphically (by graphing both sides of an equation as a function and finding points of intersection) • ***Use the discriminant*** to Describe situations and provide reasoning for when a quadratic equation/function would result in no real solutions/roots or 1 real solution/root; describe the graph of these situations (this may involve solving literal equations) • Determine which strategy for solving quadratic equations is most efficient based on the given equation; justify solutions using multiple strategies.	equations)			

approximations. Include cases where *f*(*x*) and/or *g*(*x*) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions	no real solutions. • Depending on the form of the original equation, one method for solving a quadratic equation may be more efficient and preferred over another. • Given a quadratic equation f(x) = g(x), the x-values of the points of intersection between f(x) and g(x) are the roots or solutions of the equation.				

Lesson 9: Systems of quadratic equations

Objectives

- By examining graphs of functions, students will solve a system of quadratic and/or linear functions with ___ out of ___ correct on an Exit Ticket.
- Given a pair of quadratic and/or linear functions, students will provide reasoning about the number of solutions in the system with ___ out of ___ correct on an Exit Ticket.

Focused Mathematical Practices

- MP 2: Reason abstractly and Quantitatively
- MP 5: Use appropriate tools strategically
- MP 7: Look for and make use of structure

Vocabulary

-

Common Misconceptions

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Lesson Clarifications

- Given the quadratic equation f(x) = g(x), make a connection between the solutions to the quadratic equation (0, 1, or 2 real solutions) to the number of solutions of the system of equations containing f(x) and g(x), or the number of points of intersection

CCSS	**Concepts** What students will know	**Skills** What students will be able to do	Material/ Resource	Suggested Pacing	Assessment Check Point
A.REI.4: Solve quadratic equations in one variable. a. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots,	***Review*** • A quadratic equation/function that results in no real solutions/roots will have a graph that never crosses the x-axis; A quadratic	***Review*** • Justify solutions by substituting values back into original equation. • Verify solutions algebraically and graphically (by graphing both sides of an equation	CL ST 13.4	1 day	Systems of quadratics Exit Ticket

completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as *a* ± *bi* for real numbers *a* and *b*. A.REI.11: Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f*(*x*) and *y* = *g*(*x*) intersect are the solutions of the equation *f*(*x*) = *g*(*x*); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f*(*x*) and/or *g*(*x*) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions	equation/function that results in 1 real solution/root will have a graph that crosses the x-axis once (at the vertex). ***New*** • Given a quadratic equation f(x) = g(x), the number of points of intersection between the graphs of f(x) and g(x) is the same as the number of solutions in the equation f(x) = g(x); the x-values of the points of intersection are the solutions or roots to the equation.	as a function and finding points of intersection) • Describe situations and provide reasoning for when a quadratic equation/function would result in no real solutions/roots or 1 real solution/root; describe the graph of these situations (this may involve solving literal equations) • Determine which strategy for solving quadratic equations is most efficient based on the given equation; justify solutions using multiple strategies. ***New*** • Find solutions to systems of linear and/or quadratic equations, algebraically or graphically • Describe situations and provide reasoning for when a **system of quadratic/linear equations** would result in 2 real solutions, no real solutions/roots or 1 real solution/root; describe the graphs of these situations (points of intersection).			

Lesson 10: Real Numbers

Objectives

- After learning about the real number system, students will base explanations and reasoning on properties of rational and irrational numbers with ___ out of ___ correct on an Exit Ticket.

Focused Mathematical Practices

- MP 2: Reason abstractly and Quantitatively
- MP 3: Construct viable arguments and critique the reasoning of others
- MP 6: Attend to precision

Vocabulary

- Natural numbers, whole numbers, closed, counterexample, integers, rational numbers, irrational numbers, real numbers

Common Misconceptions

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Lesson Clarifications

- Lesson should focus more on reasoning on the properties of real numbers (critiquing others' work, providing counterexamples to disprove a statement, justifying whether rational or irrational numbers are closed under a particular operation, etc) and less on the classification and definition of numbers. However, definitions should be applied to support an argument, conjecture, or justification.
- Lesson should focus mainly on the properties of rational numbers vs. irrational numbers, and whether or not they are closed under addition, subtraction, or multiplication.

CCSS	**Concepts** What students will know	**Skills** What students will be able to do	Material/ Resource	Suggested Pacing	Assessment Check Point
N.RN.3: Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	***Review*** • Definitions of natural numbers, whole numbers, and integers ***New*** • Definitions of real, rational, and irrational numbers; a rational number consists of all numbers that can be written as a/b where a and b are integers and b≠0. • Rational numbers are closed under addition, subtraction, multiplication, and division. • Division by 0 is not defined; if should not be considered when determining closure properties.	***Review*** • ***New*** • Determine whether irrational or rational numbers are closed under a particular operation; provide ***mathematically based*** reasoning as to why or why not.	CL ST 14.1 Real Numbers Task	1 day	Real Numbers Exit Ticket

Ideal Math Block

The following outline is the department approved ideal math block for grades 9-12.

1) Do Now (7-10 min)
    a. Serves as review from last class' or of prerequisite material
    b. Provides multiple entry points so that it is accessible by all students and quickly scaffolds up

2) Starter/Launch (5 min)
    a. Designed to introduce the lesson
    b. Uses concrete or pictorial examples
    c. Attempts to bridge the gap between grade level deficits and rigorous, on grade level content
    d. Provides multiple entry points so that it is accessible by all students and quickly scaffolds up

3) Mini-Lesson (15-20 min)
    a. Design varies based on content
    b. May include an investigative approach, direct instruction approach, whole class discussion led approach, etc.
    c. Includes CFU's
    d. Anticipates misconceptions and addresses common mistakes

4) Class Activity (25-30 min)
    a. Design varies based on content
    b. May include partner work, group work/project, experiments, investigations, game based activities, etc.

5) Independent Practice (7-10 min)
    a. Provides students an opportunity to work/think independently

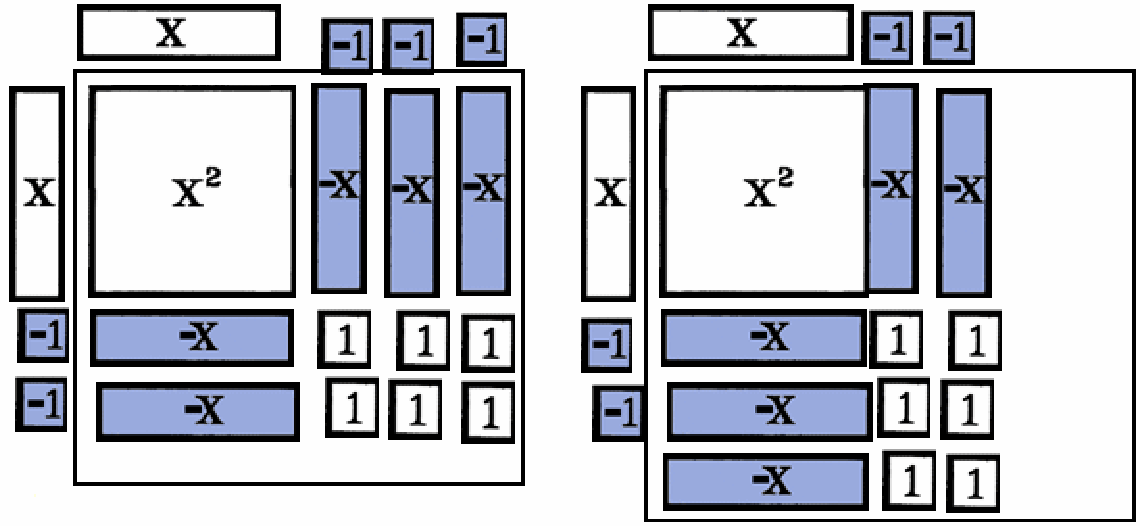
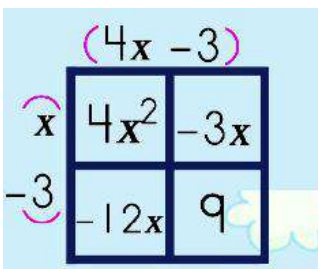
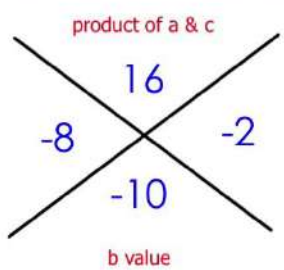
6) Closure (5-10 min)
    a. Connects lesson/activities to big ideas
    b. Allows students to reflect and summarize what they have learned
    c. May occur after the activity or independent practice depending on the content and objective

7) DOL (5 min)
    a. Exit ticket

Sample Lesson Plan

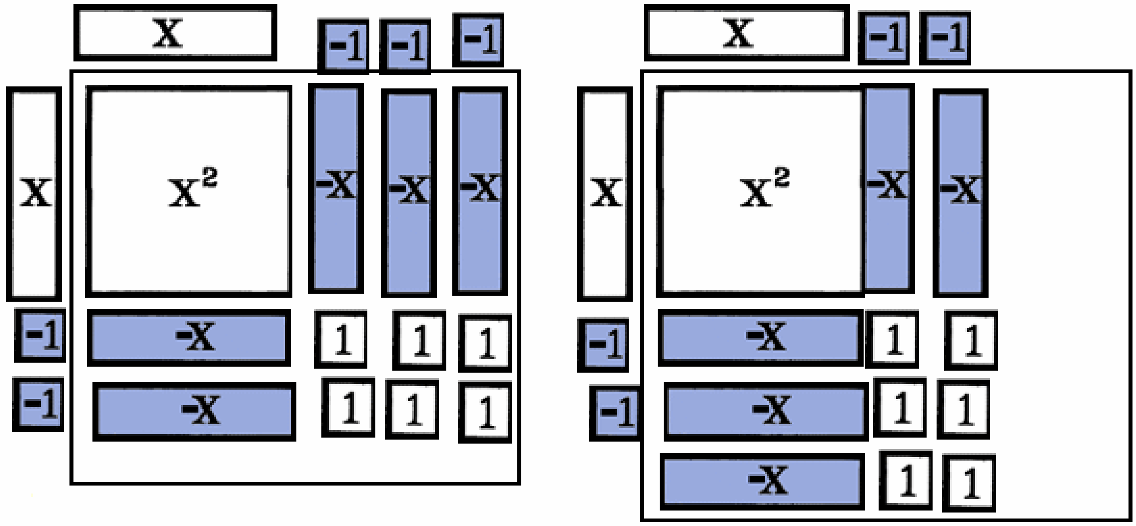
Lesson		**Days**	1
Objective		**CCSS**	
Learning activities/strategies			
Differentiation			
Assessment			
Common Misconceptions			

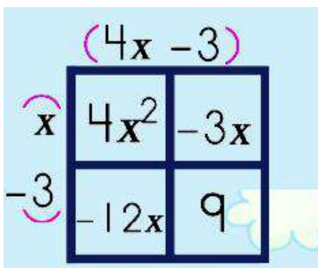
Multiple Representations

Polynomial Operations	
Concrete and/or pictorial	i. $x^2 - 5x + 6$ is $(x-2)(x-3)$ $x^2 - 5x + 6$ is $(x-3)(x-2)$  
Abstract	**Factor. $n^2 - 10n + 16$** **Diamond Method**  $=(n - 8)(n - 2)$ $x^2 - x - 30 = \underbrace{x^2 - 6x}_{group} + \underbrace{5x - 30}_{group}$ $= x(x-6) + 5(x-6)$ $= (x-6)(x+5)$

<table>
<tr><th colspan="2">Polynomial Operations</th></tr>
<tr><td>Concrete and/or pictorial</td><td>

i. $x^2 - 5x + 6$ is $(x-2)(x-3)$ $x^2 - 5x + 6$ is $(x-3)(x-2)$

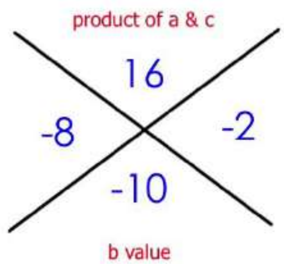




</td></tr>
<tr><td>Abstract</td><td>

Factor. $n^2 - 10n + 16$

Diamond Method



$=(n - 8)(n - 2)$

$$x^2 - x - 30 = \underbrace{x^2 - 6x}_{group} + \underbrace{5x - 30}_{group}$$
$$= x(x-6) + 5(x-6)$$
$$= (x-6)(x+5)$$

</td></tr>
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Completing the Square

What is needed to create a perfect square trinomials for :

$x^2 + 4x + ?$ → $x^2 + 4x + 4$

Use the algebra tiles to create a **square**.
What tiles will be needed to complete the square?

$X^2 - 6x + ?$ → X^2 **-6x** + 9

Use the algebra tiles to create a **square**.
Red tiles represent negative values.
What tiles will be needed to complete the square

* After viewing several examples, help students to see the pattern for finding the needed constant term. ***"Take half of the coefficient of the x-term and square it.***

Transfer concrete model to symbolic expression

Example:

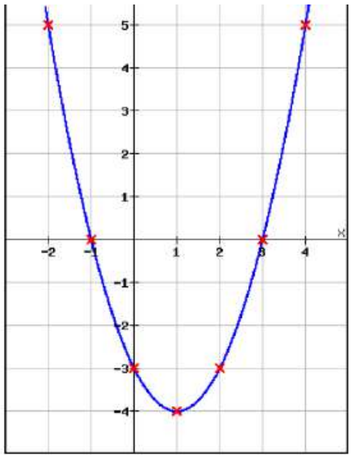
"Take half of the coefficient of the x-term and square it.

$X^2 + 10x + ?$ → $x^2 + 10x + 25$

Half of 10 = 5 then 5 square = 25

Quadratic function

Make connection between equations and graphs



Vertex: (1, -4) ---- minimum/maximum
Axis of symmetry --- $x = 1$
Vertex form: $y = a(x-h)2 + k$
$y = a(x - 1)^2 + (-4)$

Zeros: $x = 3$, $x = -1$ ---- solutions/roots (*x-intercept*)
Factor form: $y = a(x - x_1)(x - x_2)$
$y = a(x - 3)(x - -1)$

y-intercept: (0, -3)
Standard form: $y = ax^2 + bx + c$
$y = ax2 + bx + (-3)$
a is a positive number (because the graph is upward)

Appendix A – Acronyms

#	Acronym	Meaning
1	AA	Authentic Assessment
2	AM	Agile Minds
3	AR	Additional Resources
4	CCSS	Common Core State Standards
5	CFU	Check for understanding
6	CL	Carnegie Learning
7	CL SA	Carnegie Learning Student Assignments
8	CL SP	Carnegie Learning Skills Practice
9	CL ST	Carnegie Learning Student Text
10	EOY	End of Year (assessment)
11	MP	Math Practice
12	MYA	Mid-Year Assessment (same as PBA)
13	PBA	Problem Based Assessment (same as MYA)
14	PLD	Performance Level Descriptors
15	SAP	Student Assessment Portfolio
16	SMP	Standards for Mathematical Practice