7.6 Lesson

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Writing a polynomial as a product of factors is called *factoring*. When the terms of a polynomial have a common factor, you can factor the polynomial as shown below.

Key Idea

Factoring Polynomials Using the GCF

Step 1: Find the greatest common factor (GCF) of the terms.

Step 2: Use the Distributive Property to write the polynomial as a product of the GCF and its remaining factors.

EXAMPLE 1 Factoring Polynomials

Factor each polynomial.

a. $2x^2 + 18$

Step 1: Find the GCF of the terms.

$$2x^2 = 2 \cdot x \cdot x$$
$$18 = 2 \cdot 3 \cdot 3$$

The GCF is 2.

Step 2: Write the polynomial as a product of the GCF and its remaining factors.

$$2x^2 + 18 = 2(x^2) + 2(9) \qquad \text{Factor out GCF.}$$
$$= 2(x^2 + 9) \qquad \text{Distributive Property}$$

b. $15y^3 + 10y^2$

Step 1: Find the GCF of the terms.

$$15y^3 = 3 \cdot 5 \cdot y \cdot y \cdot y$$
$$10y^2 = 2 \cdot 5 \cdot y \cdot y$$

The GCF is $5 \cdot y \cdot y = 5y^2$.

Step 2: Write the polynomial as a product of the GCF and its remaining factors.

$$15y^3 + 10y^2 = 5y^2(3y) + 5y^2(2) \qquad \text{Factor out GCF.}$$
$$= 5y^2(3y + 2) \qquad \text{Distributive Property}$$

Study Tip

When you factor a polynomial, you *undo* the multiplication of its factors.

On Your Own

Now You're Ready
Exercises 6–11

Factor the polynomial.

1. $5z^2 + 30$ **2.** $3x^2 + 14x$ **3.** $8y^2 - 24y$

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7.6 Practice A

Factor the polynomial.

1. $3x + 12$

2. $7p^2 - 21p$

3. $2s^3 + 10s^2$

4. $25z^3 - 15z^2$

5. $3d^2 - 9d + 6$

6. $2u^2 - 10u - 12$

7. The profit of a company can be represented by $8c^2 - 4c$. Factor the polynomial.

Solve the equation.

8. $4x - 12 = 0$

9. $6u - 8 = 0$

10. $9p^2 + 6p = 0$

11. $15z^2 - 9z = 0$

12. $2w^2 = -14w$

13. $5t^2 = 20t$

6.2

Factoring - Grouping

Objective: Factor polynomials with four terms using grouping.

The first thing we will always do when factoring is try to factor out a GCF. This GCF is often a monomial like in the problem $5xy + 10xz$ the GCF is the monomial $5x$, so we would have $5x(y + 2z)$. However, a GCF does not have to be a monomial, it could be a binomial. To see this, consider the following two example.

Example 1.

$$\begin{array}{rl} 3ax - 7bx & \text{Both have } x \text{ in common, factor it out} \\ x(3a - 7b) & \text{Our Solution} \end{array}$$

Now the same problem, but instead of x we have $(2a + 5b)$.

Example 2.

$$\begin{array}{rl} 3a(2a+5b) - 7b(2a+5b) & \text{Both have } (2a+5b) \text{ in common, factor it out} \\ (2a+5b)(3a-7b) & \text{Our Solution} \end{array}$$

In the same way we factored out a GCF of x we can factor out a GCF which is a binomial, $(2a + 5b)$. This process can be extended to factor problems where there is no GCF to factor out, or after the GCF is factored out, there is more factoring that can be done. Here we will have to use another strategy to factor. We will use a process known as grouping. Grouping is how we will factor if there are four terms in the problem. Remember, factoring is like multiplying in reverse, so first we will look at a multiplication problem and then try to reverse the process.

Example 3.

$$\begin{array}{rl} (2a+3)(5b+2) & \text{Distribute } (2a+3) \text{ into second parenthesis} \\ 5b(2a+3) + 2(2a+3) & \text{Distribute each monomial} \\ 10\text{ab} + 15b + 4a + 6 & \text{Our Solution} \end{array}$$

The solution has four terms in it. We arrived at the solution by looking at the two parts, $5b(2a + 3)$ and $2(2a + 3)$. When we are factoring by grouping we will always divide the problem into two parts, the first two terms and the last two terms. Then we can factor the GCF out of both the left and right sides. When we do this our hope is what is left in the parenthesis will match on both the left and right. If they match we can pull this matching GCF out front, putting the rest in parenthesis and we will be factored. The next example is the same problem worked backwards, factoring instead of multiplying.

Example 4.

$10\text{ab}+15b+4a+6$	Split problem into two groups
$\boxed{10\text{ab}+15b}\boxed{+4a+6}$	GCF on left is $5b$, on the right is 2
$\boxed{5b(2a+3)}\boxed{+2(2a+3)}$	$(2a+3)$ is the same! Factor out this GCF
$(2a+3)(5b+2)$	Our Solution

The key for grouping to work is after the GCF is factored out of the left and right, the two binomials must match exactly. If there is any difference between the two we either have to do some adjusting or it can't be factored using the grouping method. Consider the following example.

Example 5.

$6x^2+9xy-14x-21y$	Split problem into two groups
$\boxed{6x^2+9xy}\boxed{-14x-21y}$	GCF on left is $3x$, on right is 7
$\boxed{3x(2x+3y)}\boxed{+7(-2x-3y)}$	The signs in the parenthesis don$'t$ match!

when the signs don't match on both terms we can easily make them match by factoring the opposite of the GCF on the right side. Instead of 7 we will use -7. This will change the signs inside the second parenthesis.

$\boxed{3x(2x+3y)}\boxed{-7(2x+3y)}$	$(2x+3y)$ is the same! Factor out this GCF
$(2x+3y)(3x-7)$	Our Solution

Often we can recognize early that we need to use the opposite of the GCF when factoring. If the first term of the first binomial is positive in the problem, we will also want the first term of the second binomial to be positive. If it is negative then we will use the opposite of the GCF to be sure they match.

Example 6.

$5xy-8x-10y+16$	Split the problem into two groups
$\boxed{5xy-8x}\boxed{-10y+16}$	GCF on left is x, on right we need a negative,
	so we use -2
$\boxed{x(5y-8)}\boxed{-2(5y-8)}$	$(5y-8)$ is the same! Factor out this GCF
$(5y-8)(x-2)$	Our Solution

Sometimes when factoring the GCF out of the left or right side there is no GCF to factor out. In this case we will use either the GCF of 1 or -1. Often this is all we need to be sure the two binomials match.

Example 7.

$$\begin{array}{rl}
12ab-14a-6b+7 & \text{Split the problem into two groups}\\
\boxed{12\text{ab}-14a}\boxed{-6b+7} & \text{GCF on left is } 2a\text{, on right, no GCF, use} -1\\
\boxed{2a(6b-7)}\boxed{-1(6b-7)} & (6b-7) \text{ is the same! Factor out this GCF}\\
(6b-7)(2a-1) & \text{Our Solution}
\end{array}$$

Example 8.

$$\begin{array}{rl}
6x^3-15x^2+2x-5 & \text{Split problem into two groups}\\
\boxed{6x^3-15x^2}\boxed{+2x-5} & \text{GCF on left is } 3x^2\text{, on right, no GCF, use 1}\\
\boxed{3x^2(2x-5)}\boxed{+1(2x-5)} & (2x-5) \text{ is the same! Factor out this GCF}\\
(2x-5)(3x^2+1) & \text{Our Solution}
\end{array}$$

Another problem that may come up with grouping is after factoring out the GCF on the left and right, the binomials don't match, more than just the signs are different. In this case we may have to adjust the problem slightly. One way to do this is to change the order of the terms and try again. To do this we will move the second term to the end of the problem and see if that helps us use grouping.

Example 9.

$$\begin{array}{rl}
4a^2-21b^3+6ab-14ab^2 & \text{Split the problem into two groups}\\
\boxed{4a^2-21b^3}\boxed{+6ab-14ab^2} & \text{GCF on left is 1, on right is } 2ab\\
\boxed{1(4a^2-21b^3)}\boxed{+2ab(3-7b)} & \text{Binomials don}'t \text{ match! Move second term to end}\\
4a^2+6ab-14ab^2-21b^3 & \text{Start over, split the problem into two groups}\\
\boxed{4a^2+6\text{ab}}\boxed{-14ab^2-21b^3} & \text{GCF on left is } 2a\text{, on right is} -7b^2\\
\boxed{2a(2a+3b)}\boxed{-7b^2(2a+3b)} & (2a+3b) \text{ is the same! Factor out this GCF}\\
(2a+3b)(2a-7b^2) & \text{Our Solution}
\end{array}$$

When rearranging terms the problem can still be out of order. Sometimes after factoring out the GCF the terms are backwards. There are two ways that this can happen, one with addition, one with subtraction. If it happens with addition, for example the binomials are $(a+b)$ and $(b+a)$, we don't have to do any extra work. This is because addition is the same in either order $(5+3=3+5=8)$.

Example 10.

$$\begin{array}{rl} 7+y-3xy-21x & \text{Split the problem into two groups} \\ \boxed{7+y}\boxed{-3xy-21x} & \text{GCF on left is } 1\text{, on the right is } -3x \\ \boxed{1(7+y)}\boxed{-3x(y+7)} & y+7 \text{ and } 7+y \text{ are the same, use either one} \\ (y+7)(1-3x) & \text{Our Solution} \end{array}$$

However, if the binomial has subtraction, then we need to be a bit more careful. For example, if the binomials are $(a-b)$ and $(b-a)$, we will factor out the opposite of the GCF on one part, usually the second. Notice what happens when we factor out -1.

Example 11.

$$\begin{array}{rl} (b-a) & \text{Factor out } -1 \\ -1(-b+a) & \text{Addition can be in either order, switch order} \\ -1(a-b) & \text{The order of the subtraction has been switched!} \end{array}$$

Generally we won't show all the above steps, we will simply factor out the opposite of the GCF and switch the order of the subtraction to make it match the other binomial.

Example 12.

$$\begin{array}{rl} 8xy-12y+15-10x & \text{Split the problem into two groups} \\ \boxed{8xy-12y}\boxed{15-10x} & \text{GCF on left is } 4y\text{, on right, } 5 \\ \boxed{4y(2x-3)}\boxed{+5(3-2x)} & \text{Need to switch subtraction order, use } -5 \text{ in middle} \\ \boxed{4y(2y-3)}\boxed{-5(2x-3)} & \text{Now } 2x-3 \text{ match on both! Factor out this GCF} \\ (2x-3)(4y-5) & \text{Our Solution} \end{array}$$

World View Note: Sofia Kovalevskaya of Russia was the first woman on the editorial staff of a mathematical journal in the late 19th century. She also did research on how the rings of Saturn rotated.

6.2 Practice - Grouping

Factor each completely.

1) $40r^3 - 8r^2 - 25r + 5$

2) $35x^3 - 10x^2 - 56x + 16$

3) $3n^3 - 2n^2 - 9n + 6$

4) $14v^3 + 10v^2 - 7v - 5$

5) $15b^3 + 21b^2 - 35b - 49$

6) $6x^3 - 48x^2 + 5x - 40$

7) $3x^3 + 15x^2 + 2x + 10$

8) $28p^3 + 21p^2 + 20p + 15$

9) $35x^3 - 28x^2 - 20x + 16$

10) $7n^3 + 21n^2 - 5n - 15$

11) $7xy - 49x + 5y - 35$

12) $42r^3 - 49r^2 + 18r - 21$

13) $32xy + 40x^2 + 12y + 15x$

14) $15ab - 6a + 5b^3 - 2b^2$

15) $16xy - 56x + 2y - 7$

16) $3mn - 8m + 15n - 40$

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Math

Factoring Trinomials Using the "AC" Method

<u>The "AC" Method (Factoring Trinomials)</u>

The "AC" method or factoring by grouping is a technique used to factor trinomials. A trinomial is a mathematical expression that consists of three terms ($ax^2 + bx + c$).

Example of "AC" method:

1.	$\overset{a}{6x^2} + \overset{b}{7x} + \overset{c}{2}$		
2.	a(c) = __ 6(2) =12	b = __ b = 7	First, find the product of **a** and **c.**
3.	12 = 12(1) 12 = 6(2) 12 = 4(3)	$12 + 1 \neq 7$ $6 + 2 \neq 7$ $\mathbf{4 + 3 = 7}$	A. List all the products that equal ($a \cdot c$). B. Check to see if numbers listed equal b, when added.
4.	$6x^2 + 3x + 4x + 2$		Rewrite trinomial with new numbers taking the middle term's place. 7x is now 3x + 4x.
5.	$(6x^2 + 3x) + (4x+2)$ $3x(2x + 1) + 2(2x + 1)$		Isolate similar terms and factor out the greatest common factor (GCF).
6.	$(3x + 2)(2x+1)$		Factor out 2x + 1 and rewrite.

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7.8 Practice A

Factor the polynomial.

1. $2x^2 + 14x + 20$
2. $5s^2 + 10s - 40$
3. $6q^2 - 24q + 18$
4. $2n^2 - 11n - 21$
5. $8p^2 + 10p - 3$
6. $12z^2 + 40z + 25$
7. $2w^2 - 5w - 18$
8. $10t^2 + 29t - 21$
9. $3v^2 - 17v + 10$

10. Describe and correct the error in factoring the polynomial.

$$\boxed{\times \quad 2y^2 - y - 6 = (y + 3)(2y - 2)}$$

11. The area (in square feet) of a rectangular rug can be represented by $6x^2 + 7x + 2$. Write the expressions that represent the dimensions of the rug.

Solve the equation.

12. $4z^2 - 4z - 48 = 0$
13. $3b^2 - 5b - 2 = 0$

Factor the polynomial.

14. $-2x^2 + 9x - 10$
15. $-20r^2 - 16r + 21$

7.9 Lesson

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You can use special product patterns to factor polynomials.

Key Idea

Difference of Two Squares Pattern

Algebra

$a^2 - b^2 = (a + b)(a - b)$

Example

$x^2 - 9 = x^2 - 3^2$

$= (x + 3)(x - 3)$

EXAMPLE 1 Factoring the Difference of Two Squares

Factor each polynomial.

a. $x^2 - 25$

$x^2 - 25 = x^2 - 5^2$ — Write as $a^2 - b^2$.

$= (x + 5)(x - 5)$ — Difference of Two Squares Pattern

b. $64 - y^2$

$64 - y^2 = 8^2 - y^2$ — Write as $a^2 - b^2$.

$= (8 + y)(8 - y)$ — Difference of Two Squares Pattern

c. $4z^2 - 1$

$4z^2 - 1 = (2z)^2 - 1^2$ — Write as $a^2 - b^2$.

$= (2z + 1)(2z - 1)$ — Difference of Two Squares Pattern

Remember

You can check your answers using the FOIL Method.

On Your Own

Now You're Ready
Exercises 4–8

Factor the polynomial.

1. $x^2 - 36$ **2.** $100 - m^2$ **3.** $9n^2 - 16$ **4.** $16h^2 - 49$

Key Idea

Perfect Square Trinomial Pattern

Algebra

$a^2 + 2ab + b^2 = (a + b)^2$

$a^2 - 2ab + b^2 = (a - b)^2$

Example

$x^2 + 6x + 9 = x^2 + 2(x)(3) + 3^2$

$= (x + 3)^2$

$x^2 - 6x + 9 = x^2 - 2(x)(3) + 3^2$

$= (x - 3)^2$

Algebra 1
Factoring Worksheet

Factoring Difference of Squares Worksheet

Name__________________

Factor the following Difference of Perfect Squares.

1. $x^2 - 25$
2. $x^2 - 9$
3. $x^2 - 81$
4. $x^2 - 49$
5. $x^2 - 36$
6. $9x^2 - 16$
7. $x^2 - 1$
8. $64x^2 - 9$
9. $144x^2 - 169$
10. $16x^2 - 9$
11. $x^2 - 121$
12. $25x^2 - 9$

Factor out a GCF if necessary. Then determine if this is a Difference of Perfect Squares and factor accordingly.

13. $8x^2 - 12$
14. $x^2 + 81$
15. $16x^2 - 25$

7.5 Lesson

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Key Vocabulary
factored form, *p. 358*
Zero-Product Property, *p. 358*
root, *p. 358*

A polynomial is in **factored form** when it is written as a product of factors.

Standard form	*Factored form*
$x^2 + 2x$	$x(x + 2)$
$x^2 + 5x - 24$	$(x - 3)(x + 8)$

When one side of an equation is a polynomial in factored form and the other side is 0, use the **Zero-Product Property** to solve the polynomial equation. The solutions of a polynomial equation are also called **roots.**

Key Idea

Zero-Product Property

Words If the product of two real numbers is 0, then at least one of the numbers is 0.

Algebra If a and b are real numbers and $ab = 0$, then $a = 0$ or $b = 0$.

EXAMPLE 1 Solving Polynomial Equations

Solve each equation.

a. $x(x + 8) = 0$

$x(x + 8) = 0$ — Write equation.

$x = 0 \quad or \quad x + 8 = 0$ — Use Zero-Product Property.

$x = -8$ — Solve for x.

The roots are $x = 0$ and $x = -8$.

Check

Substitute each solution in the original equation.

$0(0 + 8) \stackrel{?}{=} 0$

$0(8) \stackrel{?}{=} 0$

$0 = 0$ ✓

$-8(-8 + 8) \stackrel{?}{=} 0$

$-8(0) \stackrel{?}{=} 0$

$0 = 0$ ✓

b. $(x + 6)(x - 5) = 0$

$(x + 6)(x - 5) = 0$ — Write equation.

$x + 6 = 0 \quad or \quad x - 5 = 0$ — Use Zero-Product Property.

$x = -6 \quad or \quad x = 5$ — Solve for x.

The roots are $x = -6$ and $x = 5$.

On Your Own

Now You're Ready
Exercises 4–9

Solve the equation.

1. $x(x - 1) = 0$

2. $3t(t + 2) = 0$

3. $(z - 4)(z - 6) = 0$

4. $(b + 7)^2 = 0$

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EXAMPLE 2 Solving a Polynomial Equation

What are the solutions of $(2a + 7)(2a - 7) = 0$?

(A) -7 and 7 (B) $-\frac{7}{2}$ and $\frac{7}{2}$

(C) -2 and 2 (D) $-\frac{2}{7}$ and $\frac{2}{7}$

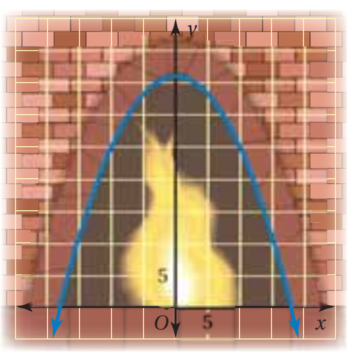
$$(2a + 7)(2a - 7) = 0 \qquad \text{Write equation.}$$

$$2a + 7 = 0 \quad or \quad 2a - 7 = 0 \qquad \text{Use Zero-Product Property.}$$

$$a = -\frac{7}{2} \quad or \quad a = \frac{7}{2} \qquad \text{Solve for } a.$$

∴ The correct answer is (B).

EXAMPLE 3 Real-Life Application



The arch of a fireplace can be modeled by $y = -\frac{1}{9}(x + 18)(x - 18)$, where x and y are measured in inches. The x-axis represents the floor. Find the width of the arch at floor level.

Use the x-coordinates at floor level to find the width. At floor level, $y = 0$. So, substitute 0 for y and solve for x.

$$y = -\frac{1}{9}(x + 18)(x - 18) \qquad \text{Write equation.}$$

$$0 = -\frac{1}{9}(x + 18)(x - 18) \qquad \text{Substitute 0 for } y.$$

$$0 = (x + 18)(x - 18) \qquad \text{Multiply each side by } -9.$$

$$x + 18 = 0 \quad or \quad x - 18 = 0 \qquad \text{Use Zero-Product Property.}$$

$$x = -18 \quad or \quad x = 18 \qquad \text{Solve for } x.$$

The width is the distance between the x-coordinates, -18 and 18.

∴ So, the width of the arch at floor level is $18 - (-18) = 36$ inches.

On Your Own

Now You're Ready
Exercises 10–15

Solve the equation.

5. $(3p + 5)(3p - 5) = 0$

6. $(12 - 6x)^2 = 0$

7. The entrance to a mine shaft can be modeled by $y = -\frac{1}{2}(x + 4)(x - 4)$, where x and y are measured in feet. The x-axis represents the ground. Find the width of the entrance at ground level.

Name ______________________ Date __________

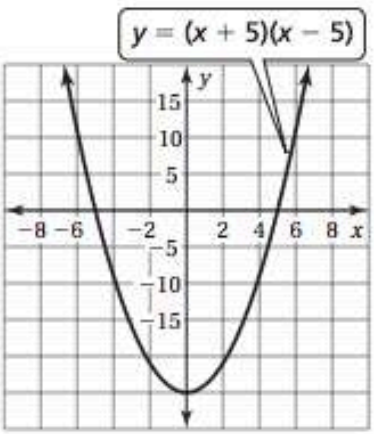
7.5 Practice A

Solve the equation.

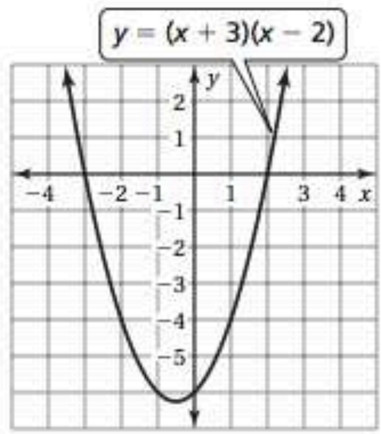
1. $y(y-2)=0$
2. $3m(m+5)=0$
3. $(p-3)(p+10)=0$
4. $(z+4)(z-4)=0$
5. $(h+3)^2=0$
6. $(s-9)^2=0$
7. $(2+x)(2-x)=0$
8. $(5-d)(4+d)=0$
9. $(7x+2)^2=0$

Find the *x*-coordinates of the points where the graph crosses the *x*-axis.

10.



11.



Solve the equation.

12. $(z+30)(z-30)=0$
13. $q(q-10)^2=0$
14. $x(x+1)(x+3)=0$
15. $(z+7)(z-7)(z-8)=0$

Elementary Algebra Skill

Solving Quadratic Equations by Factoring

Solve each equation by factoring.

1) $x^2 - 9x + 18 = 0$

2) $x^2 + 5x + 4 = 0$

3) $n^2 - 64 = 0$

4) $b^2 + 5b = 0$

5) $35n^2 + 22n + 3 = 0$

6) $15b^2 + 4b - 4 = 0$

7) $7p^2 - 38p - 24 = 0$

8) $3x^2 + 14x - 49 = 0$

9) $3k^2 - 18k - 21 = 0$

10) $6k^2 - 42k + 72 = 0$

11) $x^2 = 11x - 28$

12) $k^2 + 15k = -56$

13) $3m^2 = -16m - 21$

14) $8x^2 = 30 + 43x$

15) $x^2 + 17x + 49 = 3x$

16) $m^2 = 2m$

17) $2k^2 - 14 = -3k$

18) $3v^2 + 36v + 49 = 8v$

19) $10x^2 - 26x = -12$

20) $15p^2 + 80 = -80p$