

Chapter 4

Applying Newton's Laws

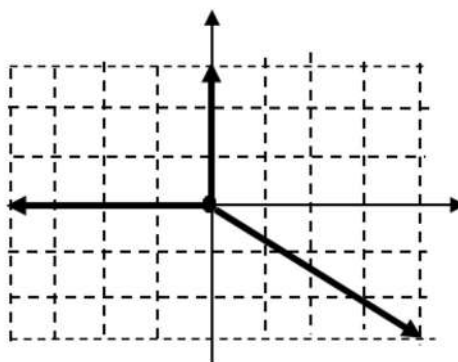
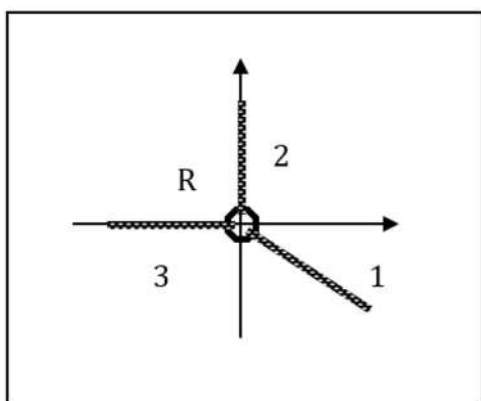
4.1 Vectors in two dimensions and force components

OALG 4.1.1 Components of force vectors

The sketch below shows three strings pulling in different directions in a horizontal plane on a small ring (R) at the center. The corresponding force diagram for the ring is also shown on a grid.

a. Based on what you see in the force diagram, explain why the ring does not accelerate in the positive or negative x -direction. Be explicit.

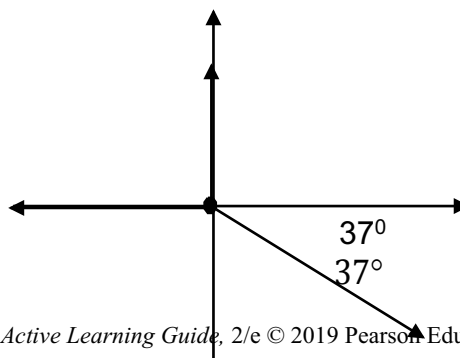
b. Repeat the same for the y -direction.



Comment Notice that string 1 exerts a 4-N force on the ring toward the right, which balances the 4-N force exerted by string 3 toward the left. Similarly, string 2 exerts a 3-N force on the ring upward, which is balanced by the 3-N downward pull exerted by string 1. If you don't see this, go back to the force diagram and try to visualize it. You should be able to realize that string 1 pulls in both the horizontal x -direction and the vertical y -direction. We say that $\vec{F}_{1 \text{ on } R}$ has an x -component of $F_{1 \text{ on } R, x} = +4 \text{ N}$, and a y -component of $F_{1 \text{ on } R, y} = -3 \text{ N}$. Normally, we don't have force diagrams on grids that allow us to visualize the components so explicitly in this way. In the next activity, we will do the same analysis using trigonometry.

OALG 4.1.2 Test your ideas

The sketch on the right shows the same three strings pulling on the ring as in the previous activity. However, an angle is now shown for the pulling direction of string 1 relative to the x -axis.



- a. How could you calculate the effect of string 1 pulling in the x -direction?
- b. How could you calculate string 1's effect pulling in the y -direction? That is, how could you calculate the x - and y -components of $\vec{F}_{1 \text{ on } R}$ if you know only the magnitude of the force (5 N) and the direction of the force relative to the x -axis (37° below the positive x -axis)? What are the magnitudes of the other two forces?
- c. Watch the video of this experiment [<https://youtu.be/Rous6rCYYRQ>] and check whether the forces that you found keep the ring in the equilibrium.

OALG 4.1.3 Read and interrogate

Read and interrogate Section 4.1 in the textbook and answer Review Question 4.1.

OALG 4.1.4 Practice

Solve problems 1, 3 and 5 on page 111 in the textbook.

4.2 Newton's second law in component form

OALG 4.2.1 Real-world application: Accelerometer

A string with a 10-gram decoration on the end is attached to the rear-view mirror of your friend's father's Ferrari. You're curious about how fast this fancy sports car can accelerate. You decide to measure the acceleration by measuring the angle that the string makes with the vertical when the car is accelerating. Your friend's father puts the pedal to the metal... Using a protractor, you measure that the angle the string makes with the vertical is 40° while the car is accelerating. What is the acceleration of the Ferrari? [Hints: Choose the decoration as the system for your force diagram. Use the vertical-component equation of Newton's second law to find the magnitude of the force that the string exerts on the decoration. Then continue with the horizontal-component equation.]

OALG 4.2.2 Represent and reason

The x - and y -components of Newton's second law for a specific scenario are:

$$x: 1.0 \text{ m/s}^2 = (60 \text{ N}) \times (\cos 30^\circ) / (50 \text{ kg})$$

$$y: 0 \text{ m/s}^2 = [(50 \text{ kg}) \times (9.8 \text{ N/kg}) + (60 \text{ N}) \times (\sin 30^\circ) + (-520 \text{ N})] / (50 \text{ kg})$$

Draw a force diagram for the system and a sketch of a possible scenario. How many scenarios can you come up with?

OALG 4.2.3 Read and interrogate

Read Section 4.2 in the textbook and carefully study Physics Tool box 4.2 along with the Tip that follows.

4.3 Friction

OALG 4.3.1 Observe and find a pattern

a. Watch the video [<https://mediaplayer.pearsoncmg.com/assets/frames.true/secs-experiment-video-7>] and analyze the experiments using force diagrams. The system is the block. Describe the patterns that you find.

Observational experiments	Force diagram for the block Remember that each object interacting with the block exerts one force on it
A block is at rest on the horizontal surface of a desk.	
A spring scale pulls lightly on the block that is at rest on a horizontal surface; the block does not move.	
The spring scale pulls harder on the block at rest on the horizontal surface; the block still does not move.	
The spring scale pulls even harder on the block at rest on the horizontal surface, right at the instant it starts to move.	
The spring scale pulls the block at a slow constant velocity across the horizontal surface.	
Patterns	

b. What is the direction and magnitude of the force that the desk exerts on the block in the experiments described above? Does the force have a constant magnitude? Constant direction?

c. Resolve the force that the desk exerts on the block into two components: one perpendicular to the interacting surfaces and one parallel. The perpendicular vector component is called the **normal force** (normal is the term for “perpendicular” in mathematics) and the parallel vector component is called the **friction force**.

OALG 4.3.2 Observe and find patterns

- a. Watch the experiments in the video [<https://youtu.be/5zOxH7CMIlg>] to investigate what physical quantities or other factors affect the maximum static friction force component of the force that the surface exerts on the object pulled across it.
- b. What patterns did you find? Make a list.

OALG 4.3.3 Observe and find a pattern

Find a pattern from the following experimental data. A spring scale pulls a 1-kg block over a medium smooth surface. The reading of the scale can be used to determine the magnitude of the maximum static friction force—in this instance, the force when the block starts to slide. In some experiments, a compressible spring also pushes vertically down on the block (see the second block).



- a. Use the data in the table to draw a graph of the maximum static friction force versus the normal force exerted by the surface on the block.

Extra downward force exerted on the 1-kg block	N exerted by the surface on the block, $N_{s \text{ on } b}$	Maximum static friction force, $f_{s \text{ on } b}$
0 N	10 N	3 N
5 N	15 N	4.5 N
10 N	20 N	6 N
20 N	30 N	9 N

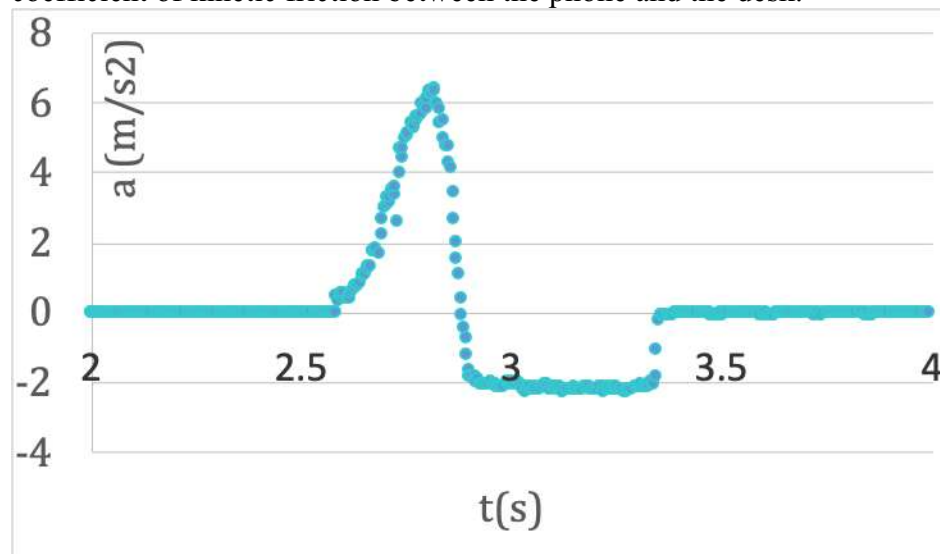
- b. Devise a mathematical relationship between the normal force exerted by the surface and the maximum static friction force exerted by the surface. If you are having difficulties, read and interrogate Section 4.3 in the textbook, paying attention to Testing Experiment Table 4.3 and Equation 4.3 on page 93. Figure 4.6 is especially important for understanding the nature of the force that the surface exerts on an object on top of it.
- c. Use the video in Activity 4.3.2 to estimate the coefficient of static friction in each of the experiments. The mass of the wooden block is 154 g (do not forget to convert to kg).

OALG 4.3.4 Observe, represent, and explain

- a. Watch the video of an object on a platform scale [<https://youtu.be/R8lXd5Un8dA>]. Note the reading, then record the reading when the scale is tilted 10° or so and the object does not slide. Draw two force diagrams for the object on the scale: One for the case when the scale is level, and one for the case when the scale is tilted.
- b. How are the different forces (or their components) exerted on the object related to each other when the scale is tilted? What coordinate system might you want to choose to best show the key relationships?

OALG 4.3.5 Observe and analyze

- a. Fix your phone on top of a textbook and turn on the Phyphox application “acceleration without g ”. Push the book across the table, let it slide and come to a stop. Record the acceleration-vs time graph that the Phyphox application created.
- b. Write down everything you can determine from the graph. Where on the graph is the time when you released the book?
- c. Use the graph that we collected from a similar experiment (we just pushed the phone - it was not on top of a textbook) to determine (1) the time when the hand stopped pushing the phone, (2) the largest speed of the phone, (3) how far the phone went while slowing down, and (4) the coefficient of kinetic friction between the phone and the desk.



OALG 4.3.6 Observe and explain

- a. Watch the video of Eugenia walking on sand [<https://youtu.be/faEMgURzNfc>].
- b. Draw force diagrams for Eugenia (consider the person to be a point like object) when she starts a step and when she finishes the step. What object is exerting a force that makes her accelerate?
- c. How does the shape of the sand help us determine the direction of the force that the sand exerts on the person?

OALG 4.3.7 Read and interrogate

Read and interrogate Section 4.3 in the textbook and answer Review Question 4.3.


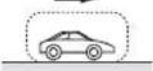
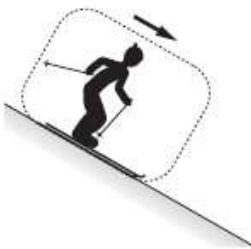

OALG 4.3.8 Practice

Solve problems 15, 18, 19, 20, 23, 25, 26, and 31 on pages 112-113 in the textbook.

4.4 Skills for analyzing processes involving forces in two dimensions



OALG 4.4.1 Reason

The table below shows sketches of several situations. Draw a force diagram for the system for their respective case. Do not forget to label each force arrow in each diagram with two subscripts —the external object causing the force and the system. [**TIP** When choosing a coordinate system for your force diagram, start by identifying the direction that the object of interest is accelerating (or the direction of motion if there is no acceleration) and align one of your coordinate axes with that direction. If you do this, you can always draw the force vectors perpendicular to the direction of acceleration so they add to zero, because the object is not accelerating in this direction.] If you are having trouble, work through Examples 4.3, 4.4, and 4.6 on pages 96 – 100 in the textbook.

Sketch of the situation.	a. Person pulling a sled (smooth snow). 	b. Car skidding to a stop. 	c. Person skiing (smooth snow). 	d. Person pushing a box. Arms point 30° below the horizontal. 
Force diagram for the system.				

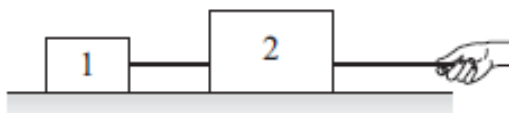
OALG 4.4.2 Diagram Jeopardy

Unlabeled force diagrams for objects moving on an inclined surface are shown below. For each case, sketch and describe in words a process for which the diagram might represent the forces that other objects exert on the object of interest.

Force diagram (label the force arrows).	Sketch a situation consistent with the diagram.	Describe a process consistent with the diagram.
a. 		
b. 		

OALG 4.4.3 Represent and reason

As shown below, two blocks on a table are connected with a light rope. Another light rope is connected to block 2 and you pull it horizontally, exerting a force $\vec{F}_{Y \text{ on } R2}$. The table and the blocks are smooth.



a. Fill in the table that follows.

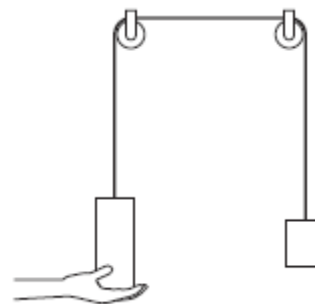
Experiment	Draw a motion diagram.	List objects that interact with the system.	Draw a force diagram and find the direction of the sum of the forces.
You pull the rope to the right. Your system is block 2.			
You pull the rope to the right. Your system is block 1.			

b. Are your force diagrams consistent with the motion diagrams? How do you know?

- c. Are your force diagrams consistent with Newton's third law? How do you know?
 d. What assumptions did you make about the force that the connecting rope exerts on each box?

OALG 4.4.4 Represent and reason

Two blocks are connected with a light string that runs over two light pulleys. A hand initially supports the left block, and then the hand is removed and the system is allowed to accelerate.



- a. Fill in the table below. If you are having trouble analyzing the systems, consult Example 4.7 on page 101 in the textbook.

Experiment	Draw a motion diagram.	List objects that interact with the system.	Draw a force diagram and find the direction of the sum of the forces.
The left block is at rest while held by the hand. The left block is the system.			
The right block is at rest while the hand supports the left block. The right block is the system.			
The left block (the system) after the hand is removed.			
The right block (the system) after the hand is removed.			

- b. List any assumptions that you made about the magnitude of the force that the string exerts on both objects. How can you test whether those assumptions are valid?

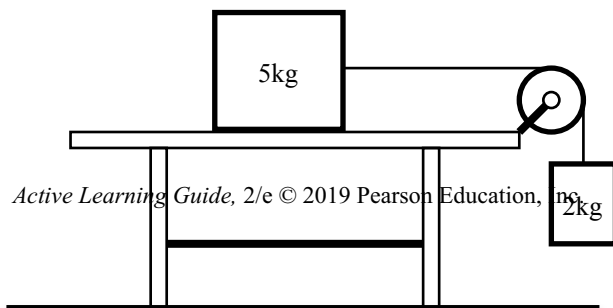
OALG 4.4.5 Test your ideas

The video [<https://youtu.be/sUQPIAGbyMo>] shows the following experiment:

The object on the left has a mass of 112 g and the object on the right, 101 g. The experimenter holds the left object on a table; the right one is about 1 meter above the table. Use your knowledge of Newton's laws and kinematics to predict how far the object on the left will move in the first second after letting go of the object on the table. After you have made that prediction, watch the video and use the frame count to determine the time. The video is digitized at 30 frames per second. Did the outcome match that prediction? Consider the assumptions that you made about the motion of the objects, the pulley, and the uncertainties in the data.

OALG 4.4.6 Application

Find the acceleration of this system and the force exerted by the rope on either the 5 kg



block or the 2 kg block. There is a coefficient of kinetic friction between the 5 kg block and the table: $\mu_k=0.1$. Evaluate your answer. What do you need to assume to solve this problem? If you are having trouble, consult Example 4.7 in the textbook on page 101.

OALG 4.4.7 Equation Jeopardy

Think of one physical process that the equations below might describe (there are many possibilities). Assume that $g = 10 \text{ N/kg}$. (Note: Here, “N” stands for the unit Newton and “N” stands for the normal force.) The object is on an incline.

$$x: a_x = [0 + (-(100 \text{ kg}) \times (10 \text{ N/kg}) \times \cos 20^\circ)] / (100 \text{ kg})$$

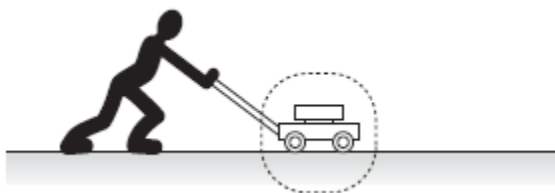
$$y: 0 = [N + (-(100 \text{ kg}) \times (10 \text{ N/kg}) \times \sin 20^\circ)] / (100 \text{ kg})$$

$$0 - (16 \text{ m/s}) = a_x t$$

Construct a sketch of a process that the equations might describe and write a problem for which the equations might be used. Evaluate whether the sketch and words are consistent with the given equations. If you are having trouble, consult Example 4.4 on page 98 in the textbook.

OALG 4.4.8 Evaluate the solution

Identify any errors in the solution to the following problem and provide a corrected solution if needed.



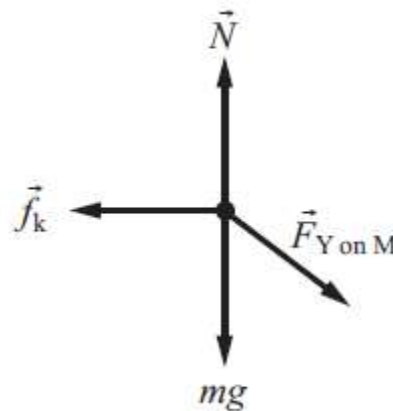
The problem: You push a 20-kg lawn mower, exerting a 100-N force on it. You push 37° below the horizontal. The coefficient of kinetic friction between the grass and mower is 0.60. Determine the acceleration of the lawn mower. Assume that $g = 10 \text{ N/kg}$.

Proposed solution: The situation is pictured at the right. The mower is the object of interest and is considered a point-like object. The forces that other objects exert on the mower are shown in the force diagram. The magnitude of the kinetic friction force is:

$$f_k = \mu_k N = 0.60 \times (20 \text{ kg}) \times (10 \text{ N/kg}) = 120 \text{ N}$$

The acceleration of the mower is:

$$a_x = (F - f_k) / m = [(100 \text{ N}) - (120 \text{ N})] / (20 \text{ kg}) = -1.0 \text{ m/s}^2$$



OALG 4.4.9 Design an experiment

Watch the videos to determine the coefficient of static friction between the shoe and the board
[<https://youtu.be/772zpP3cGkc>]

a. Take your measurements, devise a mathematical procedure to determine the coefficient of static friction, record your results, and estimate the uncertainty in your determined value of μ_s for each method.

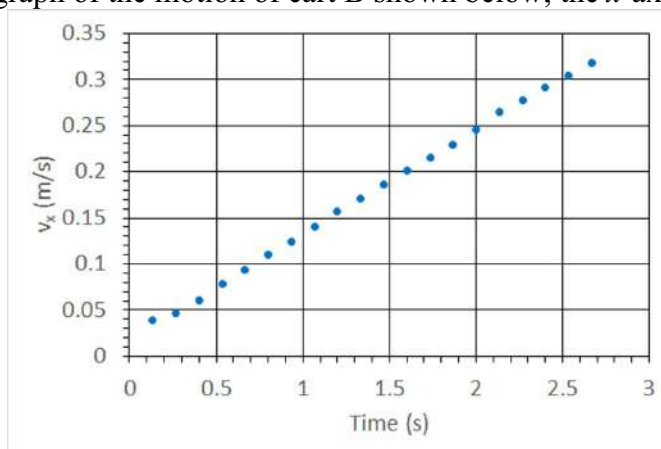
c. Compare the outcome of the two methods. Do your two measurements agree within expected uncertainties? Explain. Discuss what assumptions you made to implement each mathematical method and how these assumptions might impact the results you found.

OALG 4.4.10 Explain

Imagine that you have two low friction carts to carry out the following three experiments. You fix a vertical board on cart A and a battery-operated fan on cart B. As the fan blades on cart B rotate, they exert a constant force on the air.

Experiment 1: You put cart B on the track and switch on the fan. While the cart is moving to the right, you record the following video

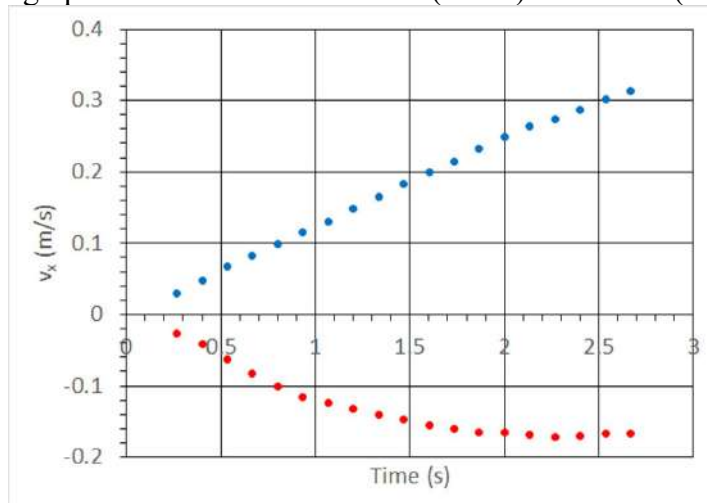
[<https://mediaplayer.pearsoncmg.com/assets/frames.true/sci-phys-egv2e-alg-4-4-16a>]. Analyze the video frame by frame and produce the velocity-versus-time graph of the cart's motion (alternatively use the graph of the motion of cart B shown below; the x -axis points to the right.).



Experiment 2: You put both carts on the track, connect cart A and cart B together (using Velcro) and switch on the fan (see the figure below). The air pushed by the fan on cart B is blowing toward the vertical board on cart A, but both carts remain at rest.



Experiment 3: You repeat Experiment 2 but this time you separate the carts so they can move independently. While the carts are moving, you record the following video [<https://mediaplayer.pearsoncmg.com/assets/frames.true/sci-phys-egv2e-alg-4-4-16b>]. Analyze the video frame by frame and produce the velocity-versus-time graph of the carts' motions (alternatively, use the graphs of the motions of cart A (in red) and cart B (in blue) below).



- Explain the outcome of each experiment using Newton's laws. Show how the outcomes of the first two experiments help you explain the outcome of the third experiment.
- Draw qualitative force-versus-time curves that show the time dependence of the sum of the forces exerted on each cart (draw both curves on the same graph).
- Eugenia says: "The outcome of Experiment 3 violates Newton's third law." What might have led Eugenia to this conclusion? Do you agree or disagree with her? If you disagree, what would you say to Eugenia to convince her that Newton's third law is not violated in this experiment?

OALG 4.4.11 Practice

Solve problems 34, 38, 39 40 and 49-54 on pages 113 – 114 in the textbook.

4.5 Projectile motion

OALG 4.5.1 Observe and find a pattern

[<https://mediaplayer.pearsoncmg.com/assets/frames.true/sci-phys-egv2e-alg-4-5-1>]

If you have a ball (or any object you can throw up and catch) at home, try to do this experiment yourself first. The goal of the experiment is for you to run at constant speed and throw the ball up while running so that the ball lands in your hands when it comes back down. If you have a friend at home, they can take a video of the experiment (as soon as you do it successfully.) Then, carefully analyze the motion of the ball. If you do not have a ball or room to run, use the video above.

- Observe the motion of the ball and the person, and describe what you observe in simple words.

- b.** Observe the motion of the person with respect to the floor. Draw a motion diagram representing the motion of the person. Describe the motion of the person relative to the floor.
- c.** Observe the motion of the ball with respect to the person. (It is helpful if you can view your video frame by frame.) Draw a motion diagram representing the motion of the ball with respect to the person. Describe the motion of the ball relative to the person.
- d.** Observe the motion of the ball with respect to the floor. What pattern do you see? What can you say about the motion of the ball and the person with respect to each other that is always true? Draw a motion diagram representing the motion of the ball relative to the floor.
- e.** How is the motion diagram you constructed in part d. related to the motion diagrams in parts b. and c.? Is there a relationship? What is it? Come up with an explanation for the direction of the throw that lets the ball land successfully in the runner's hands.

OALG 4.5.2 Test an idea

Your friends came up with an idea to explain the patterns in their experiments in Activity 4.5.1. They said that the runner needed to throw the ball exactly upwards with respect to herself *because the vertical and horizontal motions of the ball are independent of each other*. Use this explanation (the hypothesis) to predict the outcome of the following experiment. Then watch the video, describe the outcome, and make a judgment about the explanation.

Testing experiment	Prediction	Outcome
At time zero, ball 1 is dropped. Simultaneously, ball 2 is shot horizontally when a compressed spring is released. Both balls start from the same height. Which ball hits the surface first? [See the figure in the Testing Experiment Table 4.6, page 103 in the textbook.]	Make sure you draw motion diagram(s) on your whiteboard to justify the prediction in terms of the idea you're testing.	[https://mediaplayer.pearsoncmg.com/assets/frames.true/sci-phys-egv2e-alg-4-5-3]
Conclusion		

Read and interrogate the subsection “Qualitative analysis of projectile motion” in Section 4.5 on pages 102-104 in the textbook.

OALG 4.5.3 Represent and reason

You throw a tennis ball as a projectile. Draw an arrow (or arrows) representing the ball's instantaneous velocity, acceleration, and the force (or forces) exerted on the ball by other objects when at the following four positions: (1) before it reaches the top of its flight, (2) right at the top, (3) falling back down, and (4) landing (at the moment it touches the ground). If you are having difficulties, read and interrogate Conceptual Exercise 4.8 on page 104 in the textbook.

OALG 4.5.4 Real-life application

Read and interrogate the subsection "Quantitative analysis of projectile motion" on page 105 in the textbook. Then proceed to solving the following problem.

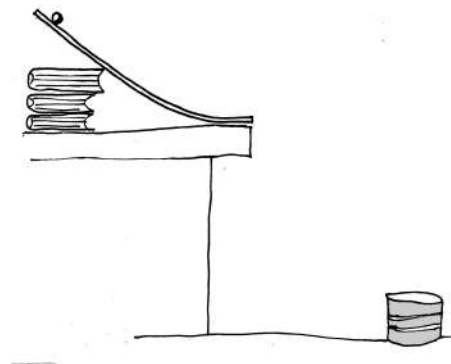
A stunt motorcyclist plans to ride horizontally off a 30 m (90 ft) high cliff at a speed of 10 m/s. She plans to place a pool of water on the flat ground beneath the cliff to land in.

- a. Where should she place the pool so that she lands in it?
- b. Make a list of assumptions you made for your calculation in part a.
- c. Give her advice on which way she should move the position of the landing pool if any of the assumptions you made were not valid.

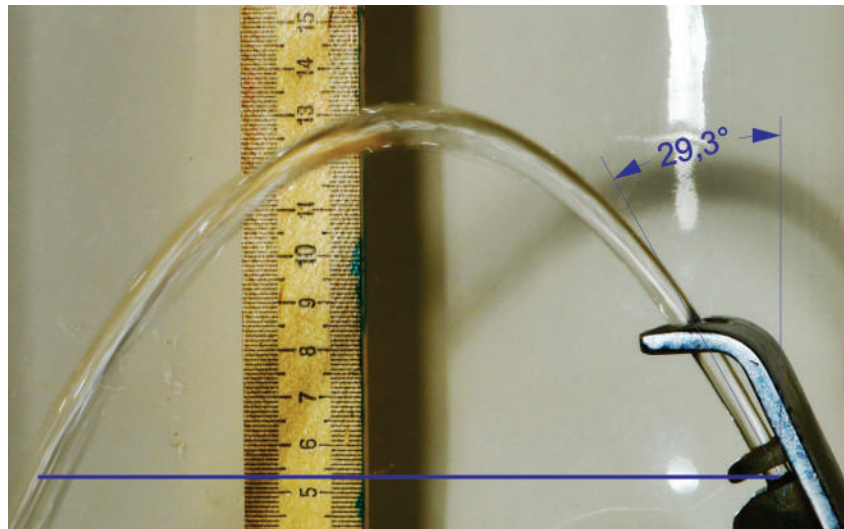
OALG 4.5.5 Application experiment

For this experiment, you will need a marble, an incline that ends horizontally (you can make it from a soft board), a tape measure or a meter stick, and a container with some cloth inside to prevent the marble from bouncing.

- a. Think of how you can predict where the marble will land on the floor if you let it go from the top of the incline plane and let it roll off the incline as shown in the figure on the right
- b. Take the necessary measurements to do the calculation, predict the distance, and place your container at that location. Do not forget to take into account the uncertainty!
- c. Run the experiment. Did the marble land in the bucket? If not, reexamine your procedure and assumptions.

**OALG 4.5.6 Real-life application**

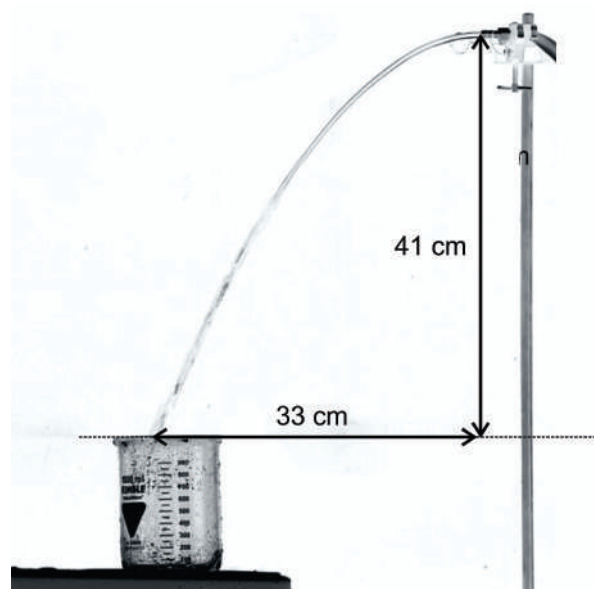
Obtain a photo of a steady jet stream from a water fountain or use the photo below. Estimate the speed of the water. Indicate any assumptions that you made and evaluate the result.



Units on the meter stick are centimeters.

OALG 4.5.7 Real-life application

The photo below shows a steady water jet coming from a 5.5-mm diameter nozzle. The water from the jet fills a 900-mL beaker in 35 seconds. Estimate the speed of the water at the nozzle. Indicate any assumptions that you made.

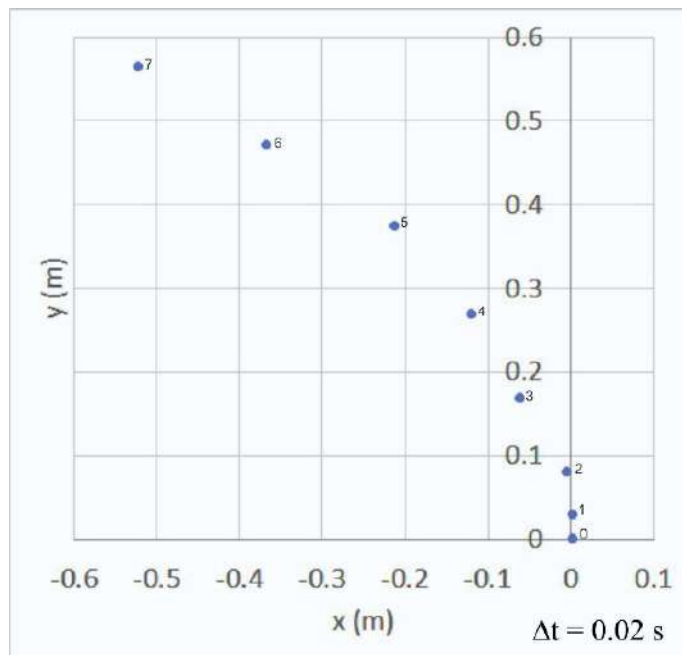


OALG 4.5.8 Real-life application

A small version of a trebuchet (see the photo on the right) was used to launch a ball (view the slow motion video



[<https://mediaplayer.pearsoncmg.com/assets/frames.true/sci-phys-egv2e-alg-4-5-9>]). The figure below shows a two-dimensional motion diagram of the ball during the launch (the data were obtained from the video). The coordinate system is shown in the photo. The time interval between two successive points is 0.020 s.



- Based on the motion diagram, estimate the what time when the trebuchet arm stopped exerting a force on the ball (at $t = 0$, the ball was at the 0 mark). Include uncertainty in your answer. Explain how you made the estimate.
- Estimate the launch speed of the ball (the speed of the ball when it loses contact with the arm) and the angle above the horizontal at which the ball was launched.
- Estimate the distance from the trebuchet at which the ball landed on the ground. Indicate any assumptions that you made.
- Your friend says that the trebuchet arm exerts a constant force on the ball during the launch. Can you reject his hypotheses based on the data provided above? Explain.

OALG 4.5.9 Read and interrogate

Read and interrogate Section 4.5 in the textbook and answer Review Question 4.5.

4.5.10 Practice

Solve Problems 56, 61, 63, and 69 on pages 114-115 in the textbook.

4.6 Starting and stopping a car

OALG 4.6.1 Read and interrogate

Read and interrogate Section 4.6 in the textbook and answer Review Question 4.6.

OALG 4.6.2 Real-life application

If you have a bicycle at home, it will help you with this activity. If you do not have one, the activity will still work. Imagine your professor is pedaling their bicycle from a standstill and progressively going faster and faster (i.e., they are speeding up). Consider your professor and their bicycle as the system.

a. Compare the *magnitude and direction* of the frictional force exerted by the road on (i) the back wheel of the bicycle, and (ii) the front wheel of the bicycle.

b. Is the frictional force exerted by the road on the front and/or back wheel static or kinetic? Discuss. How can you tell whether the friction is static or kinetic?

c. Explain which object is exerting an unbalanced force on what part of the system that allows the system to accelerate. (Remember, you should treat your professor and their bicycle as your system.)

OALG 4.6.3 Real-life application

If you have a bicycle at home, it will help you with this activity. If you do not have it, the activity will still work. Imagine your professor is freewheeling (not pedaling) on their bicycle at 20 mph when they pull the front brake and slow down to a stop. Consider your professor and their bicycle as the system.

a. Compare the *magnitude and direction* of the frictional force exerted by the road on (i) the back wheel of the bicycle, and (ii) the front wheel of the bicycle.

b. Is the frictional force exerted by the road on the front and/or back wheel static or kinetic? Discuss. How can you tell whether the friction is static or kinetic?

c. Explain which object is exerting an unbalanced force on what part of the system that allows him/her to slow down. (Remember, treat your professor and their bicycle as your system.)

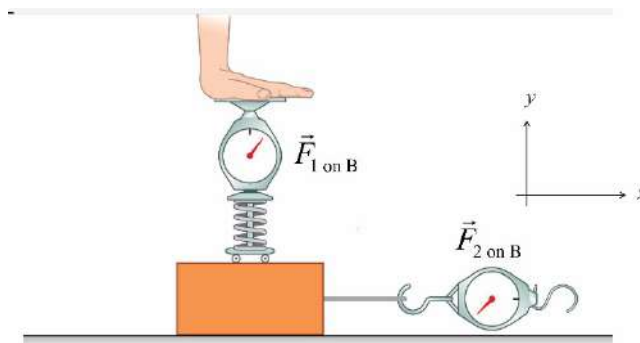
OALG 4.6.4 Linearize

We use a string attached to a spring scale to pull a wooden block. The mass of the block is unknown and it does not change during the experiment. We push down on the block with a spring that exerts a series of downward forces $\vec{F}_{1 \text{ on } B}$ on it. For each of these downward forces, we use the pulling string and spring scale to determine the magnitude of the maximum force $\vec{F}_{2 \text{ on } B}$ that we can exert on the block in the horizontal direction before the block starts sliding.

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The downward pushing spring is on wheels to prevent it from exerting horizontal forces on the block.

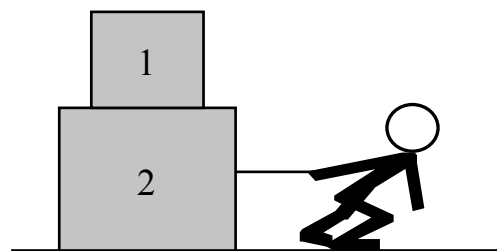


Using the measurements in the table below, determine the coefficient of static friction between the block and the table, and the mass of the block.

$F_{1 \text{ on } B} \text{ (N)}$	$F_{2 \text{ on } B} \text{ (N)}$
0.0	1.2
1.0	1.8
2.0	2.2
3.0	2.6

OALG 4.6.5 Regular problem

Block 1 has a mass of 8 kg and block 2 has a mass of 12 kg. The coefficient of static friction between block 1 and block 2 is $\mu_s=0.4$. The coefficient of kinetic friction between block 2 and the ground is $\mu_k=0.2$. A light rope is attached to block 2 and a person pulls the rope.



What is the maximum possible force that the person can exert on block 2 so that block 1 does not slip off block 2? Hints: (i) which way will block 1 slip relative to block 2, if it slips? (ii) Draw separate force diagrams for blocks 1 and 2 and be *very careful* to identify all objects interacting with the object of interest.

OALG 4.6.6 Practice

Solve problems 78, 81, 82, 84, 86, and 93 on page 116 in the textbook.