Chapter 8

Extended Bodies at Rest

8.1 Extended and rigid bodies

8.1.1 Observe and find a pattern

Use a pencil eraser to push at several points on the edge of a thin, flat, irregularly-shaped piece of plywood or cardboard that you put on the smooth surface. Work with your group members to identify a pattern in the direction of the forces that do not cause the object to rotate. *Hint:* Draw lines on the object in the direction of the forces.

The lines drawn in the directions that do NOT cause the object to rotate (only translate) all pass through a common point.

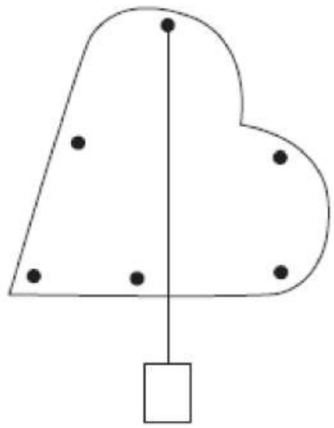
8.1.2 Observe and find a pattern

Repeat Activity 8.1.1, only this time place a 500-g block on top and near one side of the plywood or cardboard. Does the pattern change in the direction of the forces that do not cause the object to rotate?

The pattern remains the same, but the common point through which the lines pass is at a different location.

8.1.3 Test your idea

Work with your group members and use your knowledge of the gravitational force exerted by Earth and your knowledge of the center of mass to predict the outcome of the following experiment. Do not perform it yet. Imagine that you take the same irregularly-shaped board as in Activity 8.1.1 and hang it on a nail going through one of the holes drilled at its edges; the board should hang freely. You then attach the end of a string to the nail and hang a 100-g block at the string's other end so that the string hangs vertically (see the figure below). Next, suppose you hang the plywood and string on the nail through other holes. Predict where the lines (along which the string is oriented) intersect when you hang the board from different positions.



a. Write down your prediction.

The lines will all intersect at one point.

b. Explain how you arrived at your prediction.

That point is the center of mass of the board and it is oriented always below the point of suspension.

c. Perform the experiment; record the outcome.

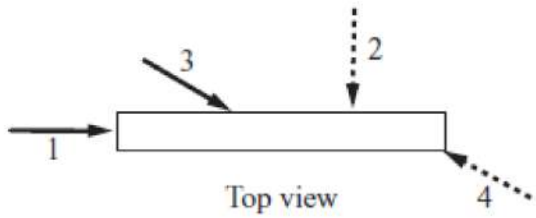
It agreed with prediction.

d. Reconcile the results with your prediction.

8.1.4 Represent and reason

Imagine that you place a board on your desk and push it in different directions, as illustrated. Forces 1 and 3 cause the board to slide, and forces 2 and 4 cause it to slide and rotate. Collaborate with your group to find the center of mass of the board.

The center of mass is at the location is lines along 1 and 3 intersect.



8.1.5 Reading exercise

Review Question 8.1:

One way is to lay the framed painting face down on a smooth surface. Push it from the sides and draw light lines in the back that point in directions that cause the painting to not rotate—only translate. Locate the center of mass. Draw a line from the center of mass to the top of the painting. Hang the painting from that point.

8.2 Torque: a new physical quantity

8.2.1 Observe and find a pattern

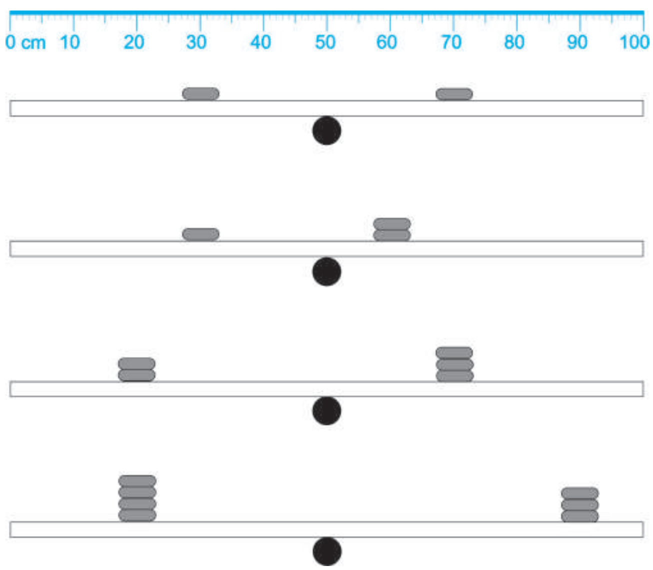
***PIVOTAL* Lab or class:** *Equipment per group:* meter stick, play-dough, whiteboard marker, several identical-mass washers (or nuts).

Work with your group to conduct the following observational experiments: Place the whiteboard marker horizontally on the table and secure it in place with some play-dough so it cannot roll. Place the meter stick on top of the marker so that it balances at the 50 cm mark and doesn't touch the surface of the table. Note: If your meter stick doesn't balance at 50 cm exactly, add either a paper clip or some play-dough to a suitable place on it so that it does. Now place different numbers of washers on the left and on the right of the balance point. Figure out where you need to place the washers in order for the system to balance and complete the following table:

Number of washers on the left	Distance of left washer group from the middle	Number of washers on the right	Distance of right washer group from the middle
1	20 cm	1	*20 cm*
1	20 cm	2	*10 cm*
2	30 cm	3	*20 cm*

4	30 cm	3	*40 cm*

a. For each situation, draw a picture of the meter stick showing all of the forces exerted on it. In other words, sketch the apparatus and draw an extended-body force diagram for the meter stick, showing forces and points where those forces are exerted on the meter stick.

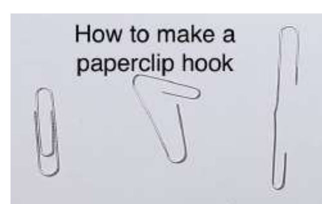


b. Find a pattern that relates the distances of the washers from the balance point and the magnitudes of the forces the washers exert on the meter stick. What is the relationship between the positions (as measured from the middle) of the left and right groups of washers, and the force that each group exerts on the ruler?

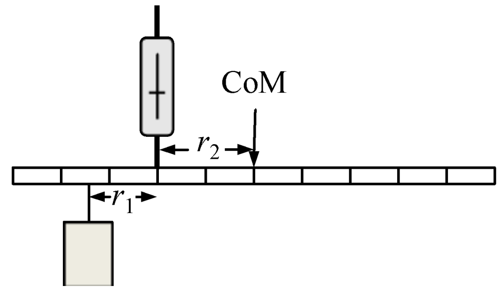
The product of number of washers on a side and the distance from the middle should be the same for each side.

8.2.2 Observe and Explain

Using the same meter stick as in Activity 8.2.1, suspend the meter stick 20 cm to the left of the center of the stick—at the 30 cm mark, using a paper clip (see photo at right). Suspend the meter stick from a spring scale and hang the spring scale on a ring stand. Using a paper clip to hang an object, find the point where you need to hang a 150-g object for the system to balance.

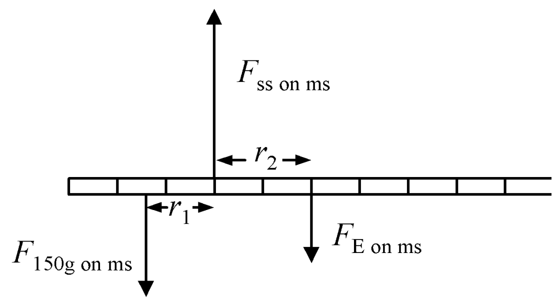


a. Find a pattern between the distances from the suspension point and magnitudes of the forces exerted on the meter stick by the scale, by the hanging object, and by Earth. (You need to measure the mass of the meter stick.)



This is what the setup should look like:

To find the pattern, draw an extended-body force diagram showing all the forces exerted on the meter stick by other objects and the points at which those forces are exerted. Calculate and fill in the numerical values of the force magnitudes onto your force diagram. The force diagram should look like this:



b. Describe the pattern you found, both in words and mathematically, and put it on a whiteboard. *The product of the force and its distance left from the suspension point must be the same as the product of the force and its distance to the right of the suspension point. In other words:*

$r_1 \times F_{\text{150g on ms}} = r_2 \times F_{\text{E on ms}}$

c. Explain the scale reading (both in words and mathematically) using your knowledge of Newton's laws.

Because the meter stick is not accelerating up or down, the sum of the forces in the y direction must add up to zero:

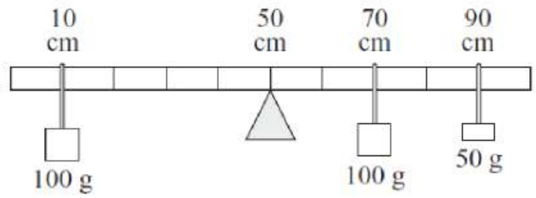
$+ F_{\text{ss on ms}} + (- F_{\text{150g on ms}}) + (- F_{\text{E on ms}}) = 0$, or, $F_{\text{ss on ms}} = F_{\text{150g on ms}} + F_{\text{E on ms}}$

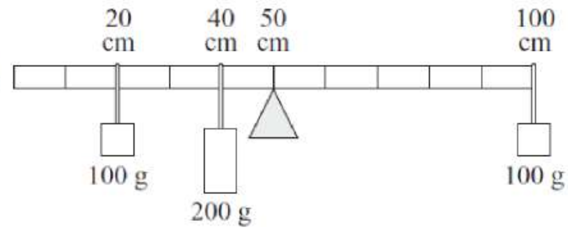
8.2.3 Observe and find a pattern

A meter stick is balanced at its center. When you hang different mass blocks from different positions on the stick, as shown in the illustrations that follow, the stick remains balanced. Work together to devise a rule to explain this behavior (or extend the rules developed in the previous activities). Be sure that the rule is compatible with all of the experiments.

Draw all *forces exerted on the stick by other objects. Remember the force exerted by Earth on the stick.*

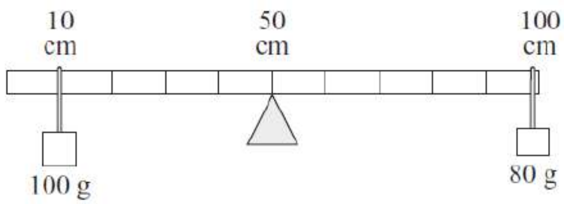
$(100g)(40cm)=(100g)(20cm)+(50g)(40cm)$

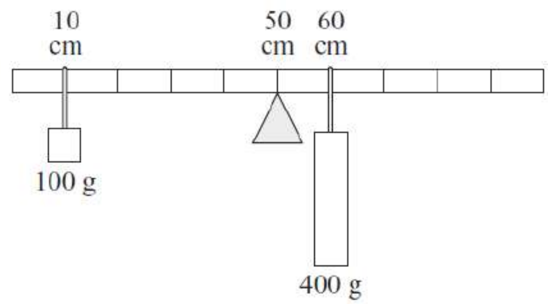
a.


b.


$(100g)(30cm)+(200g)(10cm)=(100g)(50cm)$

$(100g)(40cm)=(80g)(50cm)$

c.


d.


$(100g)(40cm)=(400g)(10cm)$

Describe the rule you devised, both in words and mathematically.

The sum of the products of the forces and their distances from the suspension point for forces on the left of the suspension point equals the sum of the products of forces on the right and their distances to the right of the suspension point. $\Sigma_{LeftSuspensionPoint} Fl = \Sigma_{RightSuspensionPoint} Fl$

8.2.4 Test an idea

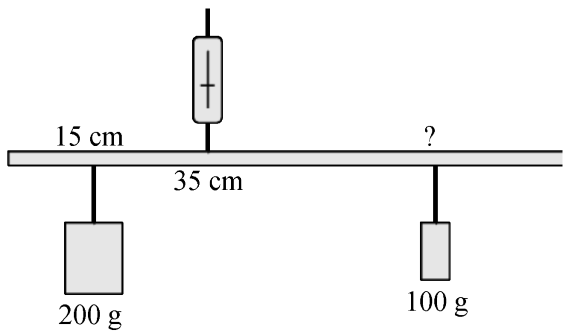
***PIVOTAL* Lab:** *Equipment per group:* meter stick, an assortment of hanging objects, paper clips, spring scale, ring stand, whiteboard, markers.

Your task is to test whether the conditions of equilibrium are applicable to the behavior of this situation by predicting:

1. The scale reading of the spring scale the whole apparatus is suspended from.

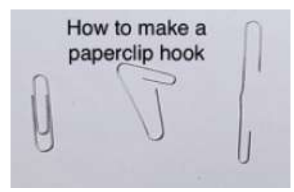
2. The position of the 100-g block if the 200-g block and the spring scale are arranged as shown in the figure below. The indicated distances are relative to the left end of the meter stick.

Remember, since this is a testing experiment you need to make these predictions *before* you perform the experiment.



In your write-up, be sure to pay attention to the following points:

a. What is the hypothesis that you are going to test in this experiment?
Conditions of static equilibrium. I.e., when an object is in equilibrium the sum of the forces exerted on it and the sum of the torques exerted on it must both (separately) add up to zero.

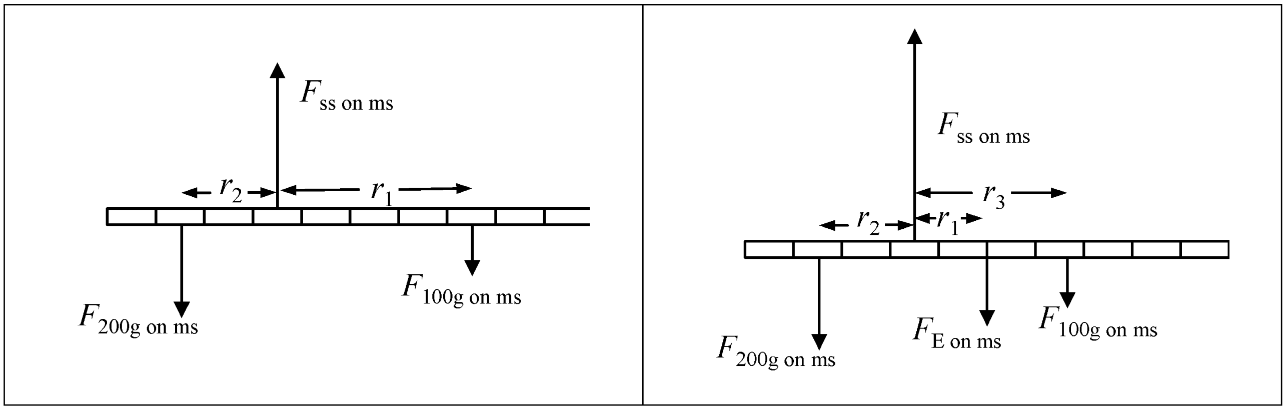


b. Draw a labeled sketch of the experimental set-up.

See above.

c. Construct a force diagram using the meter stick as the system of interest.

Ignoring the mass of the meter stick	*Including the mass of the meter stick*



d. Devise a mathematical procedure that you can use to make your prediction. State specifically how the prediction is based on the hypothesis being tested. What assumptions are you making in your mathematical procedure? Remember that an assumption is a simplification that you are making in the way you are thinking about the situation.

Because this experiment happens so early on, it is likely that some students will ignore the mass of the meter stick. That is okay. If they ignore the mass of the meter stick then, taking the spring scale attachment as the origin of the coordinate system:

$$(0.200\text{ kg})(10\text{ N/kg})(0.2\text{ m}) = (0.100\text{ kg})(10\text{ N/kg})r_1$$
$$\Rightarrow r_1 = 0.4\text{ m or }40\text{ cm}$$

The 100-g block is hanging from the 75 cm position on the stick. They will find that the meter stick does not balance. Ask them to re-examine their assumptions and revise their calculation.

Including the center of mass of the meter stick and taking the spring scale attachment as the origin of the coordinate system:

$$(0.200\text{ kg})(10\text{ N/kg})(0.2\text{ m}) = (m\text{ kg})(10\text{ N/kg})(0.15\text{ m}) + (0.100\text{ kg})(10\text{ N/kg})r_3$$

I left the mass of the meter stick as an unknown variable m in the equation. Students should measure the mass of the meter stick on a scale in order to solve for r_3.

The key assumption made in this calculation was that I assumed that the center of mass of the meter stick was exactly at 50cm, in the middle of the meter stick. It is quite likely that if you're using a wooden meter stick the center of mass could be as much as 0.5cm away from the middle! If this is the case, the meter stick will not balance. Ask students to re-examine their assumptions *and actually find the center of mass of the meter stick. (Let them think how they would do it. One quick way is to balance the meter stick on a whiteboard marker.)*

e. What if the assumption of the location of the center of mass of the stick is not valid? How might it affect your prediction?

The torque due to the force that Earth exerts on the stick would be shifted slightly. How it exactly affects r_3 depends on whether the real CoM is to the left or the right of the 50cm mark. But students should redo the calculation for r_3 using the correct CoM.

f. Do you need to assume that the meter stick is horizontal in this experiment? How can you evaluate whether or not this assumption is reasonable?

It would be ok if tilted about 10 degrees or less.

g. Can you assume that the meter stick is massless? How would making this assumption affect the prediction?

No you can't. Note the failure of the initial calculation in part d.

h. Predict the reading of the scale and the position of the 100-g block using your mathematical

procedure. First choose the origin of the coordinate system at the location where the spring scale is attached.

See d. To predict the scale reading, the forces in the y direction must add up to zero:

$$F_{\text{ss on ms}} = F_{\text{200g on ms}} + F_{\text{E on ms}} + F_{\text{100g on ms}}$$

Then repeat the analysis only this time place the origin of the coordinate system at the 15-cm

position where the 200-g block hangs from the stick. Do you get the same prediction for the scale reading and the desired location of the 100-g block?

This is a key exercise for students. Do not let them skip over it. They have a hard time understanding this. Their immediate reaction is that they must move the spring scale so that it *attaches to the ruler at 15cm instead of 35cm. You will need to help them understand the conceptual idea, which is that "we're not changing the set-up of the experiment at all, we are moving the coordinate system so that the origin is at a different point. We are re-doing the calculation from a different point of reference." In this calculation they need the force exerted by the spring scale because now it is exerting a torque. Results of the torque calculation should be consistent irrespective of where the origin of the coordinate system is chosen to be.*

i. What are your sources of experimental uncertainty? What steps will you take to minimize them?

Measuring lengths with a meter stick and mass with a spring scale

j. Build the apparatus and perform the experiment. Record the data in an appropriate format.

k. What is the outcome of the experiment? Is it consistent with the prediction within experimental uncertainty? If not, how might the assumptions contribute to the difference between the prediction and the outcome?

Once all assumptions (mass of meter stick, and the location of the center of mass of the meter stick) are taken care of, they should find that outcome of the experiment (the physical location of r_3*) is consistent with the predicted location of* r_3 *to within a very tiny experimental uncertainty.*

l. Improve how you model your system so that you are making as few assumptions as possible. Use this improved method to make new predictions. Repeat part k.

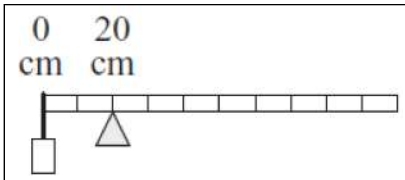
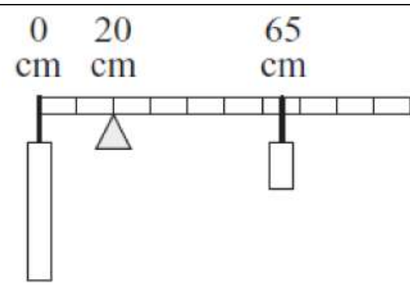
See all the discussions above.

m. Given the consistency of the prediction and outcome, and the effects of the assumptions you made, make a judgment about the hypothesis you are testing.

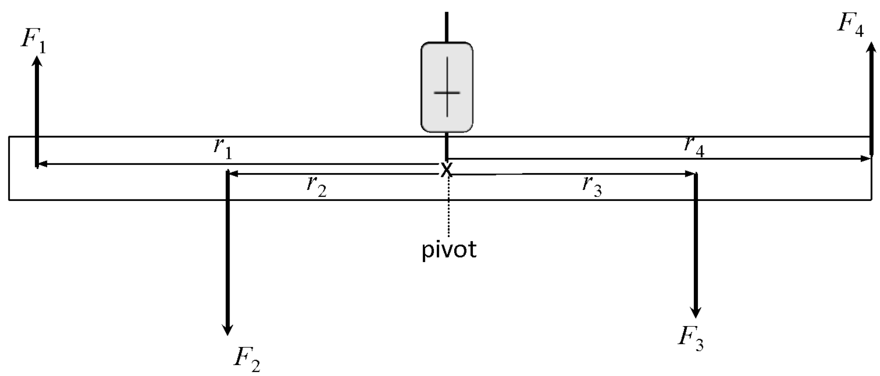
The conditions of equilibrium are valid physical principles.

8.2.5 Test your idea

A fulcrum supports a uniform meter stick as shown. The center of mass of the meter stick is at its middle. Use the rule or rules you developed earlier to predict the mass of the meter stick in the following two experimental illustrations. In each picture, the available objects cause the meter stick to be balanced.

 m_{object} = 130 g (130 g)(20 cm) = m(30 cm) or m = 87 g.	 $m_{\text{left object}}$ = 300 g, $m_{\text{right object}}$ = 70 g or (300 g)(20 cm) = (70 g)(45 cm) + m(30 cm) m = 95 g. The two results are not completely consistent.

8.2.6 Represent and reason

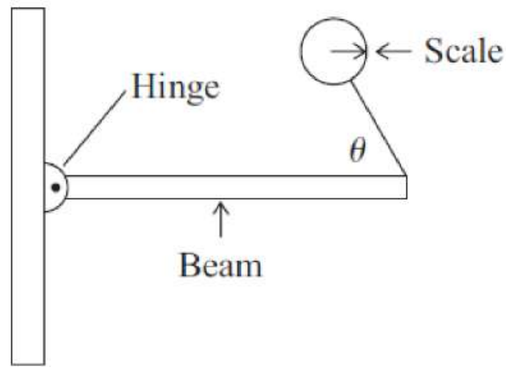


$$-F_1r_1 + F_2r_2 - F_3r_3 + F_4r_4 = 0$$

Four forces are exerted at different points on a meter stick (as shown in the diagram) that is held up at its center by a spring scale. The meter stick is in static equilibrium (not moving, not rotating). Work with your group to write a mathematical expression (in terms of F_1 to F_4 and r_1 to r_4) for the rotational condition for static equilibrium about the pivot point shown in the figure. Discuss with your group how you might develop a rule about the "direction" of torque and incorporate that into your equation for rotational equilibrium.

8.2.7 Observe and find a pattern

Working with your group members, measure and record the mass of the beam. Then, assemble the experiment as shown. Pull on the beam at the angles listed in the table and record the scale reading.



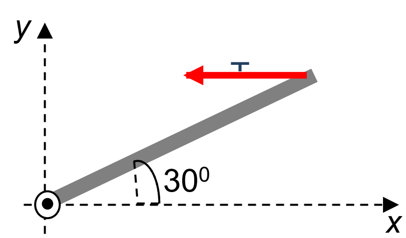
Angle (degrees)	90°	60°	45°	37°	30°
Force exerted by the string on the beam (N)					

Use the data that you collected to modify any rules you have made for the effect of a force on the rotation of a rigid body. In particular, discuss with your group members how the rotational effect of a force depends on its magnitude, direction, and the distance from the pivot point.

$\tau = \pm Fl\sin\theta$

We will call lsinθ the lever arm

8.2.8 Practice

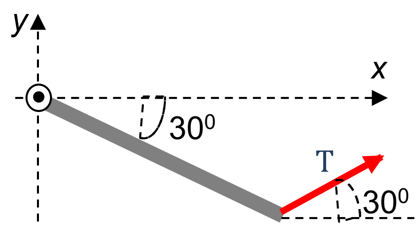


Three situations are shown on the right. Work with your group members to determine the torque caused by the force in each case:

a. A rope exerts a 100-N tension force on the end of the 5.0-m long beam shown at the right. Determine the torque caused by this force.

+250 Nm

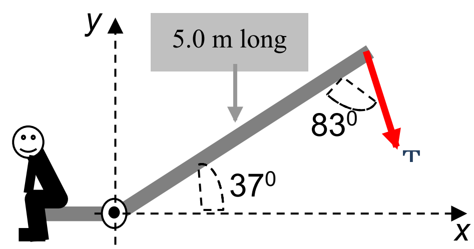
b. A rope exerts a 100-N tension force on the 5.0-m long beam shown at the right. Determine the torque caused by this force.



+433 Nm

$$\tau = +(100\text{ N})(5.0\text{ m})\sin 60^o = +433\text{Nm}$$

c. A rope exerts a 100-N tension force on the end of the 5.0-m long beam shown at the right. Determine the torque caused by this force.



497.5 Nm

8.2.9 Reading exercise

(a) Two ropes pull up on the ends of a beam. The torque due to the left rope is zero about a coordinate system centered on the place where the left rope connects to the beam but is not zero about a coordinate system where the right rope connects to the beam. (b) The force is along a line that passes through the axis of rotation. (c) The torques must add to zero.

8.3 Conditions of equilibrium

8.3.1 Explain

a. Use the patterns and rules that you devised in the previous activities to summarize what you know about the sum of the forces exerted on an object that is in equilibrium and the sum of the torques caused by these forces.

The x and y components of forces must both add to zero and the sum of the torques due to these forces must also add to zero; plus torques tend to rotate the object counterclockwise and negative torques tend to rotate the object clockwise.

b. $\Sigma F_x = 0$ and $\Sigma F_y = 0$ and $\Sigma \tau = 0$ *(See above for the signs of the torques.)*

d. Use the ideas you discussed in part **b** to explain why the placement of the bob did not matter for the balancing of the board.

Moving the bob down did not change the lever arm – the distance between the line of force and the axis of rotation. The force exerted on it by Earth also did not change, and thus the force it exerted on the board remained the same. As the force and the lever arm did not change, the torque produced by the bob remained the same.

8.3.2 Design an experiment

Design an experiment to test the second condition of equilibrium.

A 40-cm long beam is supported at the middle while hanging from a string. A1.0-kg block hangs from a string that is connected to the left end of the beam. How large of a mass should hang from a string connected 10 cm from the right end of the beam? After making your prediction, do the experiment to compare the outcome to the prediction

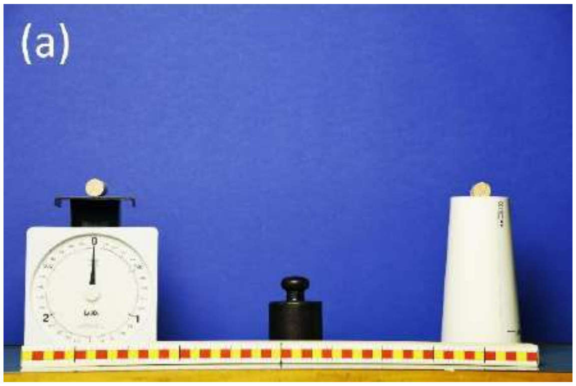
$\Sigma\tau = +(1.0 \text{ kg})g(20 \text{ cm}) - (x)g(10 \text{ cm}) = 0$ or $x = 2.0$ kg.

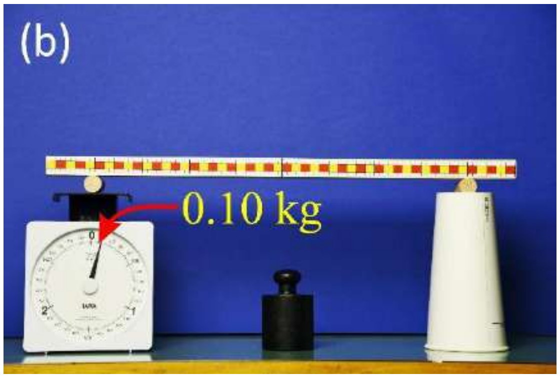
d. *Use the conditions of equilibrium to make your prediction. (see above).*

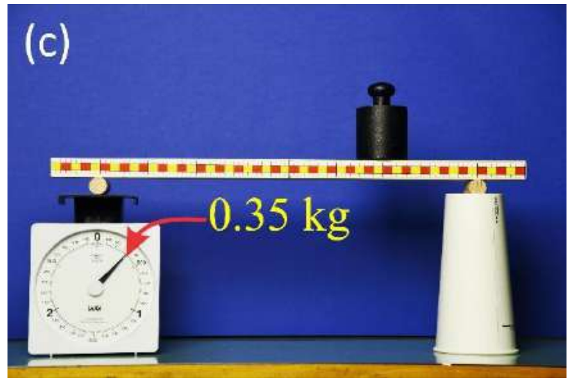
e. *Conduct the experiment and record the outcome. The outcome of the experiment matches the t*

8.3.3 Apply

We have a scale, a ruler with an unmarked scale, a paper cup and an object of unknown mass (figure (a)). If we place the ruler with one end on the scale and the other end on the paper cup (figure (b)), the scale reads 0.10 kg. If we then place the object on the ruler, the scale shows 0.35 kg (figure (c)).





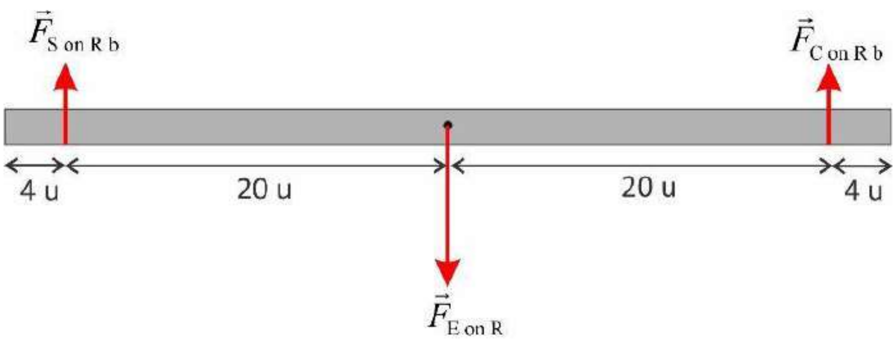


a. Draw a force diagram for the ruler for the case shown in figure b and for the case shown in figure (c). Indicate any assumptions that you made.

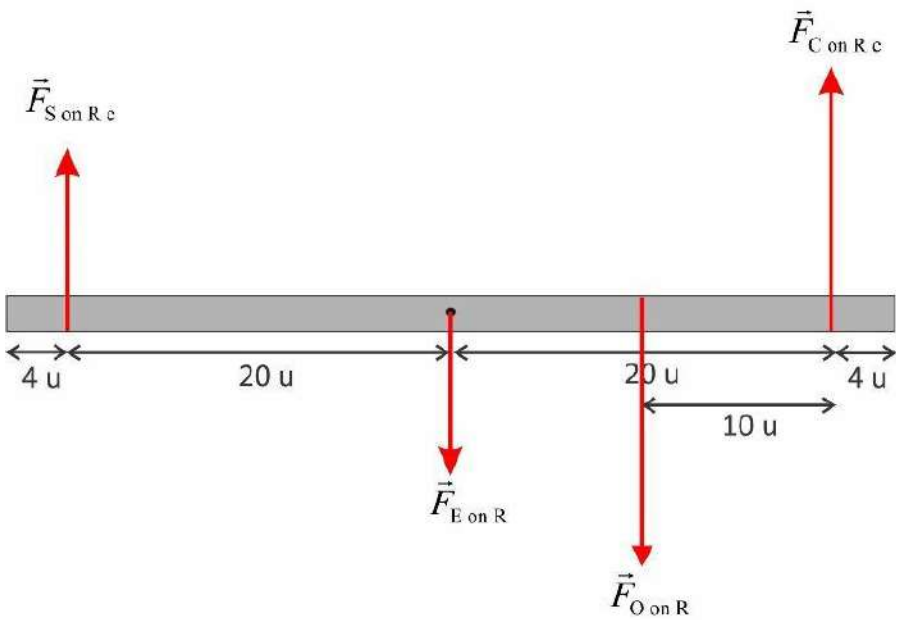
b. Using the force diagrams and the information above, calculate the mass of the object and the mass of the ruler.

SOLUTION:

a) Force diagram for the case in figure b. We assumed that the stick is made of the material with uniform density.



Force diagram for the case in figure c.



b) Applying the equilibrium conditions to the case in figure b, we get mR = 0.20 kg. Using this information and applying the equilibrium conditions to the case in figure c, we get mO = 1.0 kg.

8.3.4 Reading exercise

The torque due to that force is then zero. You can then usually use the second condition of equilibrium to determine some other unknown quantity.

8.4 Center of mass

8.4.1 Design an application experiment

You have a meter stick of unknown mass and a small 100-g object. Design an experiment to determine the mass of the meter stick using your knowledge of static equilibrium.

a. Draw a picture of the experimental set-up.

b. Describe the procedure in words.

c. Apply the concepts of equilibrium to develop equations that can be used to predict the mass of the meter stick. Then predict the mass.

d. Use a scale to measure the mass and compare the result to the predicted value.

e. How can you explain the difference between the predicted and the measured value?

There are many possible experiments that one can design. One experiment can be first finding the location of the center of mass of the meter stick by balancing it on some pivot and then placing the object at the end of the meter stick and balancing the meter stick again. Then use conditions of the equilibrium to determine the mass of the meter stick

8.4.2 Test an idea

Your friend says that the mass of any object is distributed evenly around its center of mass. Design an experiment to test your friend's idea. You have a meter stick, a set of small objects of different masses, masking tape, and a mass measuring scale.

Finding one experiment that is consistent with the friend's idea does not prove it correct. Instead, I will try to disprove the friend's idea with an experiment. Place a 5-g object on left end of a 12-in ruler or 6 in from its center. The ruler will balance if place a 30-g object at the 7-in mark or 1-in from the center of the ruler. The center of torque is now at the center of the ruler. However, the mass is not distributed uniformly (5 g on the left and 30 g on the right).

8.4.3 Represent and reason

You balance a bread knife by laying the flat side across one finger. Where is the center of mass of the knife? How does the mass of the knife on the left side of the balance point compare to the mass of the knife on the right side of the balance point?

8.4.4 Reading exercise

Disagree. You could do the experiment in 8.4.2.

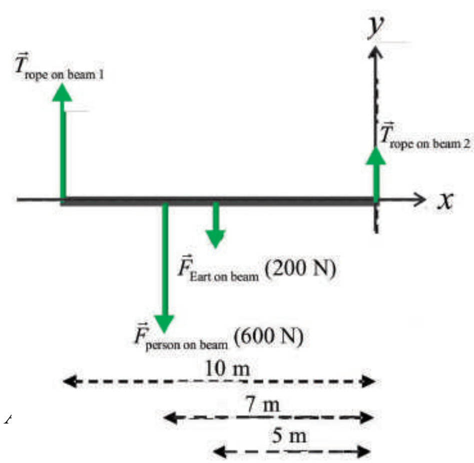
8.5 Skills for analyzing situations using equilibrium conditions

8.5.1 Practice problem solving strategy

This problem is worked in the book using the complete PS strategy on pages 234-235,

8.5.2 Represent and reason

A force diagram for a beam is shown at the right. Help your group members to apply the first and second conditions of



equilibrium for this situation and solve for the unknown forces. Draw a picture of a situation that the diagram might describe. The situation could be a 10-m long beam supported by ropes on each end with a person standing 3 m from the left end.

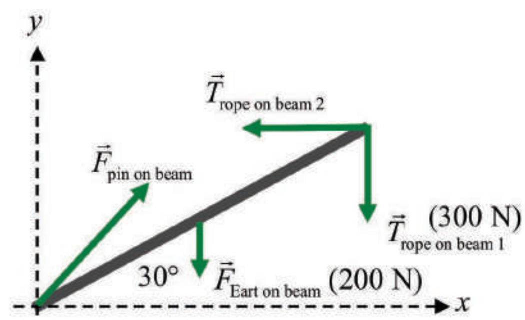
$$T_1 - F_{\text{PonB}} - F_{\text{EonB}} + T_2 = 0$$

$$-T_1(10\text{ m}) + F_{\text{PonB}}(7\text{ m}) + F_{\text{EonB}}(5\text{ m}) = 0$$

Substituting the known forces in the second equation above gives $T_1 = 520$ N. *Substituting this and the known forces in the first equation gives $T_2 = 280$* N.

8.5.3 Represent and reason

A force diagram for a 2.0-m long uniform beam is shown at the right. Apply the first and second conditions of equilibrium for this situation and solve for the unknown forces. Draw a picture of a situation that the diagram might describe.



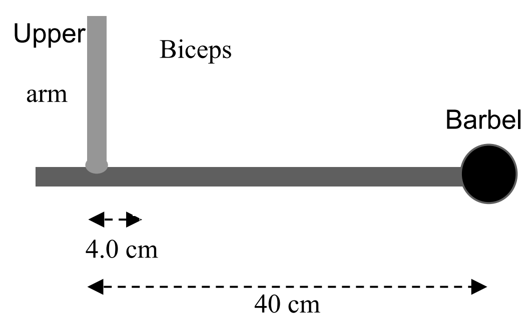
$$\Sigma F_x = -T_2 + F_{\text{PonB}x} = 0$$

$$\Sigma F_y = F_{\text{PonB}\,y} - 200\text{ N - }300\text{ N} = 0$$

$$\Sigma\tau = -(300N)(2.0m)\sin 60^o - (200\text{N})(1.0\text{m})\sin 60^o + T_2(2.0\text{m})\sin 30^o = 0$$

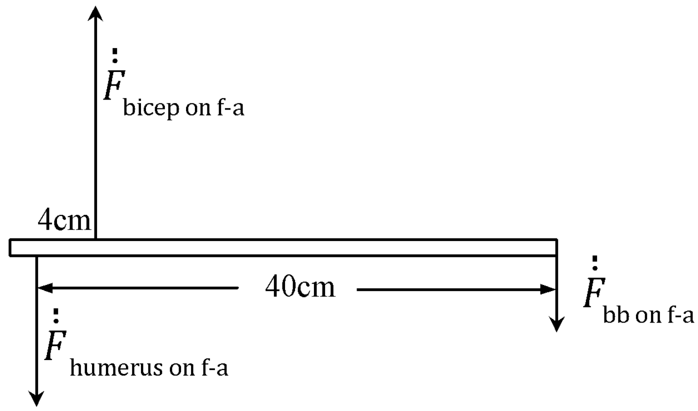
From the last equation, we find that T2 = 693 N*, From this, and the first equation, we find that this also equals the x component of the force that the pin exerts on the beam. From the second equation, we see that the y component of the force that the pin exerts on the beam is 500* N*. Thus, the magnitude of the pin force is* $F_{\text{PonB}} = \sqrt{693^2 + 500^2} = 855\text{N}$*. Since the x-component is 693 N and the y component is 500 N, we find that that the force that the pin exerts on the beam makes a 35.8-degree angle above the positive x axis.*

8.5.4 Real-world application

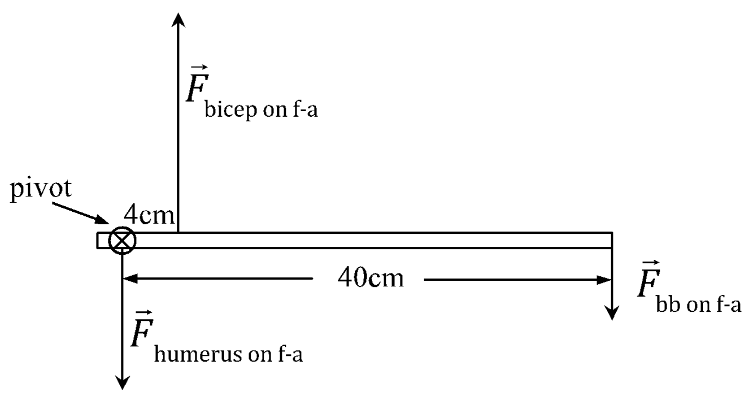


Your arm lifts an 8.0-kg barbell that is 40 cm from the elbow joint between the upper arm and the lower arm (forearm). Determine the force that the biceps muscle exerts on the forearm 4.0 cm from the joint.

Step 1: Draw a force diagram for the fore-arm. The trick here is: What is our object of interest? Why would we pick the forearm as our object of interest? (Because we want to know the force that the bicep exerts on the forearm so it makes sense that the forearm is the object of interest and we examine other objects exerting forces on the object of interest the forearm.) My concern with this step is that students recognize that the upper arm bone (humerus) exerts a downward force on the forearm bone. (Questions: How do you know there's a force there? How do you know it is down?) Make sure you can justify all that.



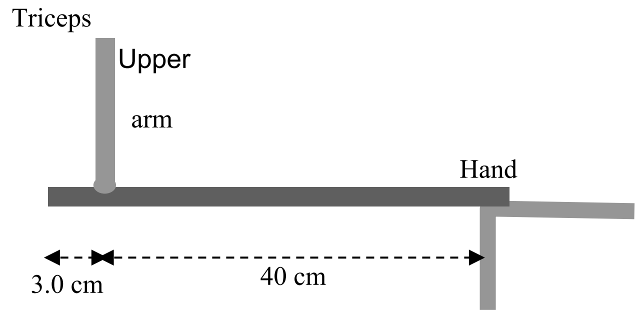
Step 2: The crucial step that students sometimes forget: You need to identify a pivot point: The origin or center of your coordinate system. The trick to remember is: Pick the pivot on a point where you don't want to deal with the force. That way you can eliminate one of the forces you don't know from your torque equation. If you look at this problem, the force exerted by the humerus is not asked for so we really don't care about it. It makes sense to put the origin of the coordinate system on the elbow joint where the humerus connects to the forearm and then the only unknown thing we're solving for is the force exerted by the bicep on the forearm.



Step 3: Apply the equation for rotational equilibrium about the chosen pivot point. Remember, all r's are measured relative to that pivot point to the point where the force is exerted. I'm not going to put the steps in here but the answer should be $F_{bicep} = 10F_{bb}$ because of the ratio of the two lengths. So if F_{bb} is 80N then F_{bicep} will be 800N. Note that the major assumption here is we didn't worry about the mass of the forearm itself. So we left out the force exerted by the earth on the forearm.

8.5.5 Real-world application

Your hand exerts a 80-N force pressing down on a table 40 cm from the elbow joint between the upper arm and the lower arm (forearm). Determine the tension in the triceps muscle pulling 3.0 cm from the joint. What assumptions did you make in working the problem? Ignored the mass of the forearm and the torque exerted by Earth on that arm. Also ignored on the muscle groups in the arm.

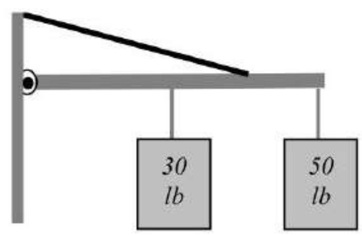


$$-T_{\text{triceps}}(3.0\text{cm}) + F_{\text{table}}(40\text{cm}) = 0$$

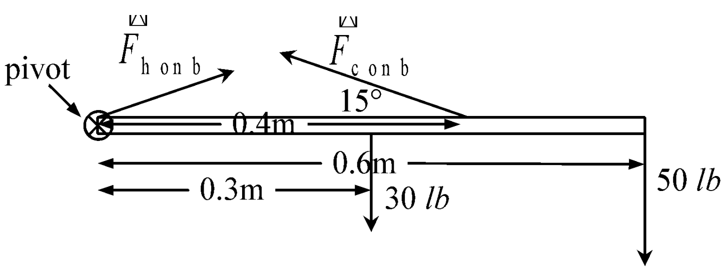
or T = 1067 N.

8.5.6 Real-world application

The woman at the right is lifting a 30 lb barbell. A mechanical model of her upper body is shown below the photo. The beam is her backbone and the cable is her back muscle (a complex set of muscles in the real back). Earth's gravitational force on her upper body at its center of mass is 30 lb. Earth's gravitational force on her head, arms, and the barbell is 50 lb at the end of the beam. The back muscle (cable) connects 0.20 m from the right end of the 0.60 m long beam (the backbone) and makes a 15° angle with the beam. Apply the conditions of equilibrium to the beam and use them to estimate the force that one primary back muscle (it is a complex system) exerts on the backbone and the force that the hinge (joint) exerts on the backbone on the left side. (Each year more than half a million Americans get serious back problems by lifting this way.)



I will start by drawing an extended force diagram for the beam/backbone:



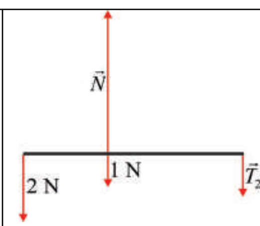
Because the beam is in static equilibrium I can say that the torques about the chosen pivot point must add up to zero:

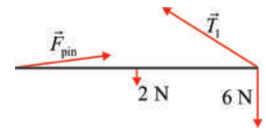
$$\sum \vec{\tau} = 0$$
$$\Rightarrow -(30\text{ lb})(0.3\text{ m}) + [-(50\text{ lb})(0.6\text{ m})] + F_{\text{c on b}}\sin 15°(0.4\text{ m}) = 0$$
$$\Rightarrow F_{\text{c on b}} = \frac{(30\text{ lb})(0.3\text{ m}) + (50\text{ lb})(0.6\text{ m})}{\sin 15°(0.4\text{ m})}$$
$$\Rightarrow F_{\text{c on b}} = 377\text{ lb or }1676\text{ N}$$

The backbone is being compressed against the "hinge" by the x component of $F_{c\ on\ b}$ (or the component of the force that is aligned with the beam. So to estimate how much force is compressing the spine I'd take $F_{c\ on\ b}\cos 15° = 364$ lb or 1619 N, which is a lot of force!

8.5.7 Equation Jeopardy

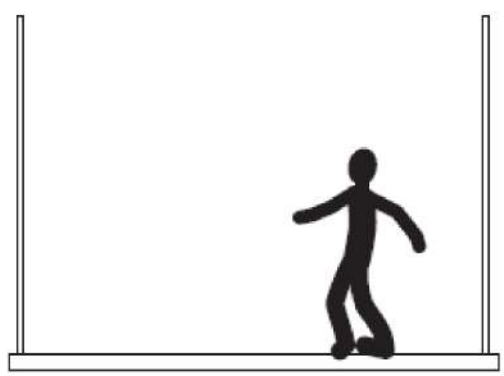
The conditions of equilibrium applied to two processes are shown below. Fill in the table that follows using the information in the equations. Remember that *N* stands for normal force and N for Newton, the unit of force.

a. Equations	$+(2.0\text{ N})(0.25\text{ m}) + N(0) + (1.0\text{ N})(0) - T_2(0.40\text{ m}) = 0$ $-(2.0\text{ N}) + N - (1.0\text{ N}) - T_2 = 0$		
Determine the unknown quantities in the equations.	$T_2 = 1.25\text{ N}$ $N = 4.25\text{ N}$	**Draw an extended-body force diagram consistent with the equations.**	
Describe a possible situation in words.	A massless 0.65-m long beam supports a 2.0N object on its left end and a string with a 1.25-N object attached on the right end, A 1-0 N object sits 0.25-m from the left end. The beam is supported by a 4.25-N normal force exerted 0.25 m from the left end.	**Sketch the situation.**	*Sketch*
b. Equations	$+F_{\text{pin}}(0) - (2\text{ N})(0.5\text{ m}) - (6\text{ N})(1.0\text{ m}) + T_1(1.0\text{ m})\sin 37° = 0$		

	x equation: $F_{\text{pin}}\cos\theta - T_1\cos37° = 0$ y equation: $F_{\text{pin}}\sin\theta - (2\text{ N}) - (6\text{ N}) + T_1\sin37° = 0$		
Determine the unknown quantities in the equations.	$F_{\text{pin}} = 11.24\text{ N}$ oriented $5.1°$ above x axis $T_1 = 14\text{ N}$	**Draw a force diagram consistent with the equations.**	
Describe a possible situation in words.	A rope pulls up and in at a 37degree angle on the right end of a horizontal 2.0-N 1.0-m long beam held by a pin in the wall on the left end of the beam. A 6-N plant hangs from the middle of the beam.	**Sketch the situation.**	*Sketch*

8.5.8 Evaluate the solution

The problem: An 800-N painter stands 3.0 m from the right side of a 10.0-m-long beam of a scaffold, which is connected to cables at each end (see diagram below). The force exerted by Earth on the uniform beam is 200 N. Determine the forces that the cables exert on each end of the beam. Assume that $g = 10\text{N/kg}$.

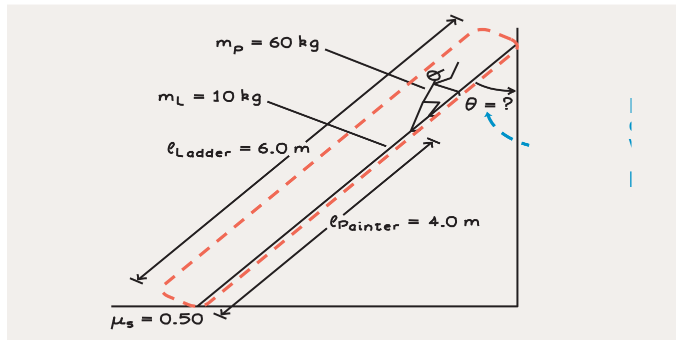


Proposed solution:

$$+T_{\text{left}}(10.0\text{ m})+(200\text{ N})(5.0\text{ m})+(800\text{ N})(3.0\text{ m})+T_{\text{right}}(0)=0$$

a. Work with your group to identify any missing elements or errors in the solution.

The answers for the forces are actually correct assuming that the solver expects some torques and some force components to come out as negative. But there is no justification for the mathematical representation – no force diagram and no explanation where to choos the axis of rotation.



b. Provide a corrected solution or missing elements if there are errors.

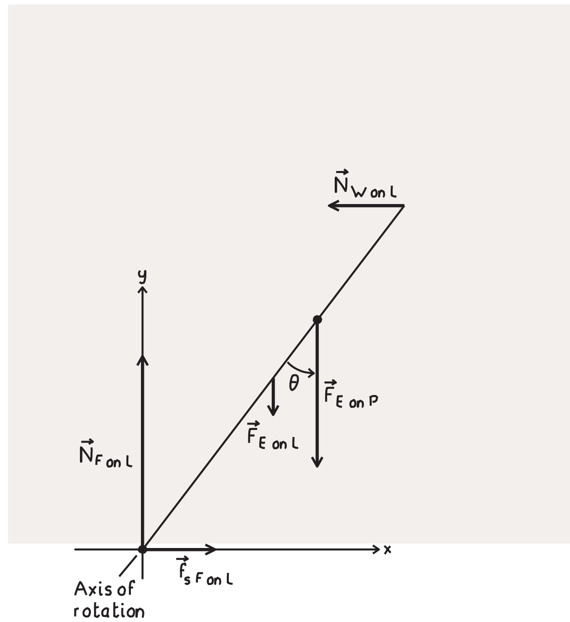
We need to specify the axis of rotation – the rope on the right.
We also need to specify that the downward y direction is positive and upward is negative, and the clockwise torque is negative. Once these issues are settled, we can draw the force diagram for the beam as our system. The force diagram needs to have the positive y-axis pointing down.
We need to force diagram here

8.5.9 Regular problem

Any time you have to climb a ladder, you want the ladder to remain in static equilibrium. At what angle should a 60-kg painter place his ladder against the wall in order to climb two-thirds of the way up the ladder and have the ladder remain in static equilibrium? The ladder's mass is 10 kg and its length is 6.0 m. The exterior wall of the house is very smooth, meaning that it exerts a negligible friction force on the ladder. The coefficient of static friction between the floor and the ladder is 0.50.

Sketch and translate *We've sketched the situation below. If the ladder is tilted at too large an angle from the vertical, everyday experience indicates that it will slide down the wall. The static friction force exerted by the floor on the ladder is what is preventing the ladder from sliding, so one way to look at the situation is to ask, What is the maximum angle relative to the wall that the ladder can have before the static friction force is insufficient to keep the ladder in static equilibrium? We choose the lad- der and the painter together as the system of interest.*

Simplify and diagram *Model the ladder as a rigid body and the painter as a point-like object. A force diagram for the ladder-painter system includes the following forces: the gravitational force exerted by Earth on the ladder $F_{E\ on\ L}$, the gravitational force that Earth exerts on the painter $F_{E\ on\ P}$, the normal force of the wall on the top of the ladder $N_{W\ on\ L}$, the normal force of the floor on the bottom of the ladder $N_{F\ on\ L}$, and the static friction force of the floor on the bottom of the ladder $f_{sF\ on\ L}$. We place the axis of rotation where the ladder touches the floor. Rather than determine the center of mass of the sys- tem, we have kept the gravitational forces exerted by Earth on the ladder and painter separate.*



Represent mathematically and solve

Use the force diagram to apply the conditions of equilibrium. Since the forces in this situation point along more than one axis, we sue both components of the force conditions of equilibrium.

y-component force equation $\Sigma F_y = N_{\text{F on L}} + (-m_{\text{Ladder}}g) + (-m_{\text{Painter}}g) = 0$

x-component force equation $\Sigma F_x = (-N_{\text{W on L}}) + (f_{\text{s F on L}}) = 0$

We can insert the expression for the maximum static friction force ($f_{\text{s Max F on L}} = \mu_{\text{s}} N_{\text{F on L}}$ *) into the force component force equation to get* $N_{\text{W on L}} = \mu_{\text{s}} N_{\text{F on L}}$.

From the y-component we get $N_{\text{F on L}} = m_{\text{Ladder}}g + m_{\text{Painter}}g$.

Combining the two equations we get $N_{\text{W on L}} = \mu_{\text{s}}(m_{\text{Ladder}} + m_{\text{Painter}})g$.

The torque equation is:

$$[-F_{\text{E on L}}(\frac{l_{\text{Ladder}}}{2})\sin\theta]+[-F_{\text{E on P}}(\frac{2l_{\text{Ladder}}}{3})\sin\theta]+[N_{\text{W on L}}l_{\text{Ladder}}\sin(90°-\theta)]=0$$

$$\Rightarrow -m_{\text{Ladder}}g\frac{l_{\text{Ladder}}}{2}\sin\theta-m_{\text{Painter}}g\frac{2l_{\text{Ladder}}}{3}\sin\theta+\mu_{\text{s}}(m_{\text{Ladder}}+m_{\text{Painter}})gl_{\text{Ladder}}\cos\theta=0$$

.

Solve and evaluate *We can cancel* gl_{Ladder} *out of each term in the torque equation and then solve what remains for* θ *:*

$$\Rightarrow \frac{m_{\text{Ladder}}}{2}\sin\theta+\frac{2m_{\text{Painter}}}{3}\sin\theta-\mu_{\text{s}}(m_{\text{Ladder}}+m_{\text{Painter}})\cos\theta=0$$

$$\Rightarrow (\frac{m_{\text{Ladder}}}{2}+\frac{2m_{\text{Painter}}}{3})\frac{\sin\theta}{\cos\theta}-)(1+6)\mu_{\text{s}}(m_{\text{Ladder}}+m_{\text{Painter}})=0$$

$$\Rightarrow (\frac{m_{\text{Ladder}}}{2}+\frac{2m_{\text{Painter}}}{3})\tan\theta=\mu_{\text{s}}(m_{\text{Ladder}}+m_{\text{Painter}})$$

$$\Rightarrow \tan\theta=\frac{\mu_{\text{s}}(m_{\text{Ladder}}+m_{\text{Painter}})}{\frac{m_{\text{Ladder}}}{2}+\frac{2m_{\text{Painter}}}{3}}=\frac{6\mu_{\text{S}}(1+m_{\text{Painter}}/m_{\text{Ladder}})}{3+4m_{\text{Painter}}/m_{\text{Ladder}}}=\frac{6(0.5)}{3+4(6)}=\frac{7}{9}$$

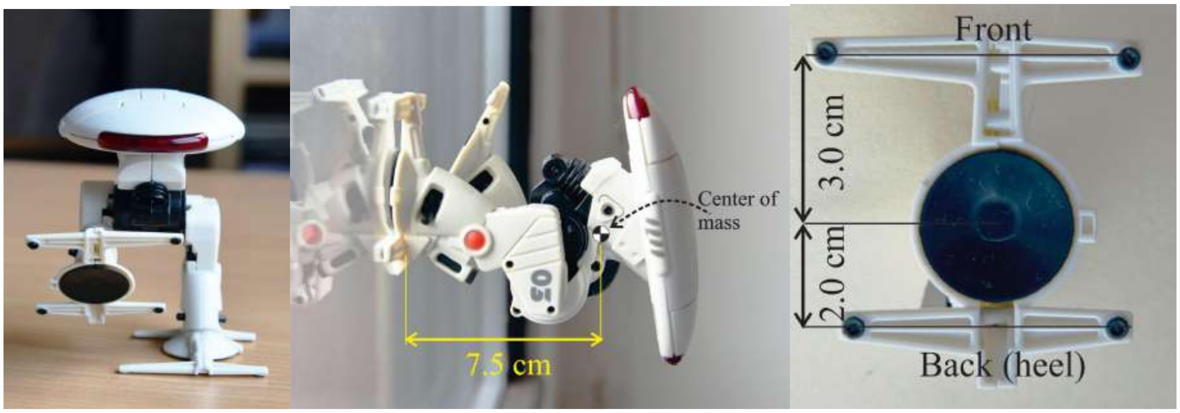
$$\Rightarrow \theta=38°$$

This seems reasonable and similar to what we observe in real life phenomenon when somebody uses a ladder. Some ladders come with a warning not to have the angle exceed 15 degrees. As a limiting case, if coefficient of static friction were zero, then our result says that the angle should also be zero. This makes sense: without friction between the ground and the ladder, the ladder would need to be perfectly vertical or it would slip.

8.5.9 Real-world application

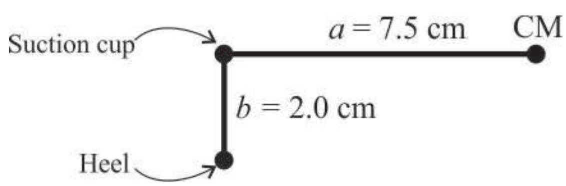
Class or lab *Equipment per group:* whiteboard and markers.

Widow climbing robot A 0.150-kg toy robot can climb up the window (see the figures below) using suction cups on its feet. The center of the mass of the robot is shown on the photo in the middle, and the details of its foot on the photo on the right.



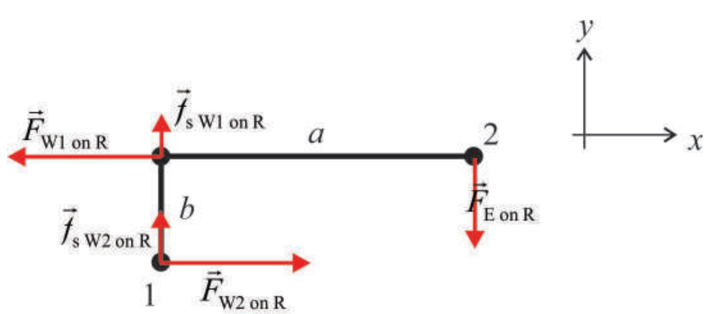
a. Propose a simplified model of the robot that will allow you to determine the forces that other objects exert on the robot when it is in the position shown on the photo in the middle (the robot is at rest). Draw a sketch of the model. (*Hint*: Note that the robot's right foot is not touching the window; also note that the front part of the left foot is not touching the window; replace the forces exerted on the suction cup with a single force in the center of the cup).

We can model the robot as an L shaped object, which mass is concentrated in the robot's center of mass



b. Determine the forces that other objects exert on the robot when it is in the position shown on the photo in the middle. Indicate any assumptions that you made.

The sketch bellow shows the force diagram for the robot. Two vector components, one perpendicular to the window and one parallel to the window replace each force exerted by the window on the robot.



We choose the coordinate system as indicated. Because the robot is in static equilibrium, the following conditions apply:

Force condition:

$$F_{\text{W1 on R}\,x} + F_{\text{W2 on R}\,x} = 0$$

$$f_{\text{s W1 on R}\,y} + f_{\text{s W2 on R}\,y} + F_{\text{E on R}\,y} = 0$$

or using the magnitudes of the forces

$$(-F_{\text{W1 on R}}) + F_{\text{W2 on R}} = 0$$

$$f_{\text{s W1 on R}} + f_{\text{s W2 on R}} + (-F_{\text{E on R}}) = 0$$

Torque condition:

Axis in 1: $F_{\text{W1 on R}} \cdot b + (-F_{\text{E on R}} \cdot a) = 0$

Axis in 2: $F_{\text{W2 on R}} \cdot b + (-f_{\text{s W1 on R}} \cdot a) + (-f_{\text{s W2 on R}} \cdot a) = 0$

Combining these equations and knowing that $F_{\text{E on R}} = 1.5\text{ N}$ *we find:*

$$F_{\text{W1 on R}} = F_{\text{W2 on R}} = \frac{a}{b} F_{\text{E on R}} = \frac{7.5\text{ cm}}{2.0\text{ cm}} 1.5\text{ N} = 5.6\text{N}$$

We do not know how the coefficient of static frictions between the suction cup and the glass compares to the coefficient of static friction between the heel and the glass. We also do not know the normal force exerted by the suction cup on the window (there is additional force exerted on the cup due to difference in air pressure inside and outside the cup). Therefore, we cannot determine forces of static friction separately but we can determine their sum:

$$f_{\text{s W1 on R}} + f_{\text{s W2 on R}} = F_{\text{E on R}} = 1.5\text{ N}$$

c. The robot on the middle photo is facing up. Discuss how the forces exerted on the robot will change if the robot is facing down and then determine the forces.

If the robot is facing down then the distance b becomes 3.0 cm but the analysis that we did above is still valid. Changing b from 2.0 cm to 3.0 cm will result in smaller horizontal forces exerted by the window on the robot: $F_{\text{W1 on R}} = F_{\text{W2 on R}} = \dfrac{7.5\text{ cm}}{3.0\text{ cm}} F_{\text{E on R}} = 3.8\text{ N}$

d. In which case it is more likely that the suction cups will detach from the window: when the robot is facing up or when it is facing down? Explain.

The suction cups will more likely detach when the robot is facing up because the force exerted by the robot leg on the suction cup is larger in this case.

8.5.10 Reading exercise

With the arm straight out, the dumbbell is far from the body and must exert a large force on the dumbbell to support it. With the arm bent at the elbow, the dumbbell is only about half as far from the body and less force is needed to support it.

8.6 Stability of equilibrium

8.6.1 Observe and explain

You have probably observed that it is easier to balance and avoid falling while standing in a moving bus or subway train if you spread your feet apart in the direction of motion. By doing this you are increasing the area of support, the area of contact between an object and the surface it is supported by.

a. *Draw force diagrams and consider the torques exerted on two people by the gravitational force and normal forces at their feet when they are on a slowing-down train. Person A is standing with their feet close together and person B is standing with their feet wide apart.*

b. *Person A with feet together is more likely to fall.*

c. *Equilibrium is stable if a line down from the center of mass passes in the area of support—for standing humans, between the feet.*

8.6.2 Test your idea [See Testing Experiment Table 8.4.]

a. Place a full box of crackers on a flat but rough surface, such as a wooden desk. Its center of mass is at its geometric center. If the full box is 13 cm wide and 20 cm tall, the center of mass is at its center. If tilted and released, the box will return to its upright position if the center of mass of the full tilted box is above the flat bottom of the box. We predict that the box will not tip over if the tilt angle is less than 33 degrees (see 4 in Table 8.4).

b. The experiment matched the prediction.

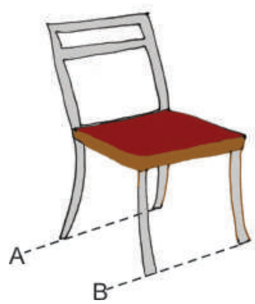
c. Then remove some crackers from the box and make a new prediction. You need to be able to decide the location of the center of mass with some of the crackers removed. See Testing Experiment 8.4. The experiment matches the prediction.

d. The equilibrium is stable if a downward line from the center of mass passes through the bottom of the box.

8.6.3 Apply your knowledge I

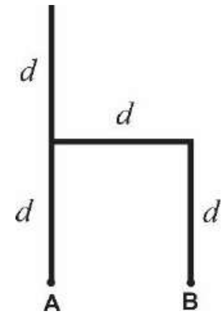
a. Obtain a simple chair (see the figure below) and try to tip it over by slowly tilting it either around axis A or around axis B. You need to tilt the chair at a larger angle to tip it if tilt over B. Estimate both angles.

$$\theta_A \approx 20^\circ - 30^\circ \text{and } \theta_B \approx 30^\circ - 40^\circ$$



b. Explain the outcome of your experiment qualitatively. The center of mass of the chair is near the back of the seat of the chair. The chair tilts when that center of mass is in front of line A or in front of line B.

c. Assume the following simplified model of a chair (the figure below shows a side view). All parts of the chair are made of the same board. Determine the tipping angle for tilting the chair around A and the tipping angle for tilting the chair around B.



First determine the location of the center of mass of this chair. We assume that the mass density of the wood is ρ.

$$\text{cm}_x = \frac{0+0+d\rho\frac{d}{2}+d\rho d}{4d\rho} = \frac{3d}{8}$$

$$\text{cm}_y = \frac{2d\rho d + d\rho d + d\rho\frac{d}{2}}{4d\rho} = \frac{7d}{8}$$

The angle when the chair tilts about A is:

$$\theta = \tan^{-1}\frac{3d/8}{7d/8} = 23.2^o.$$

The angle where the chair tilts about B is:

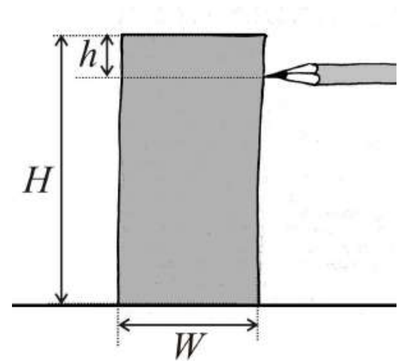
$$\theta_B = \tan^{-1}\frac{5d/8}{7d/8} = 35.5^o$$

d. *The differences are due to the shape of the chair and inaccurate estimates of angles.*

8.6.4 Apply your knowledge II.

While searching in the literature for simple experiments that will allow you to determine the coefficient of static friction, you came across the following method:

Using a pencil, exert a force in a horizontal direction on a wooden block (height *H*, width *W*) and try to tip the block over. Start at the top of the block and move down in small steps (see the figure on the right and the video [https://mediaplayer.pearsoncmg.com/assets/_frames.true/sci-phys-egv2e-alg-8-6-4]). At some point you will reach a distance *h* where the block will not tip but rather slide forward. Repeat the procedure several times to determine the distance *h* where this transition happens, as precisely as possible. The coefficient of static friction between the block and the supporting surface is determined by the following expression:



$$\mu_s = \frac{W}{2(H-h)}$$

a. Obtain a block and try the experiment described above.

b. Evaluate the equation given above. Then derive the equation.

Evaluation:

The units are correct: both sides of the equation have no units (or unit 1).

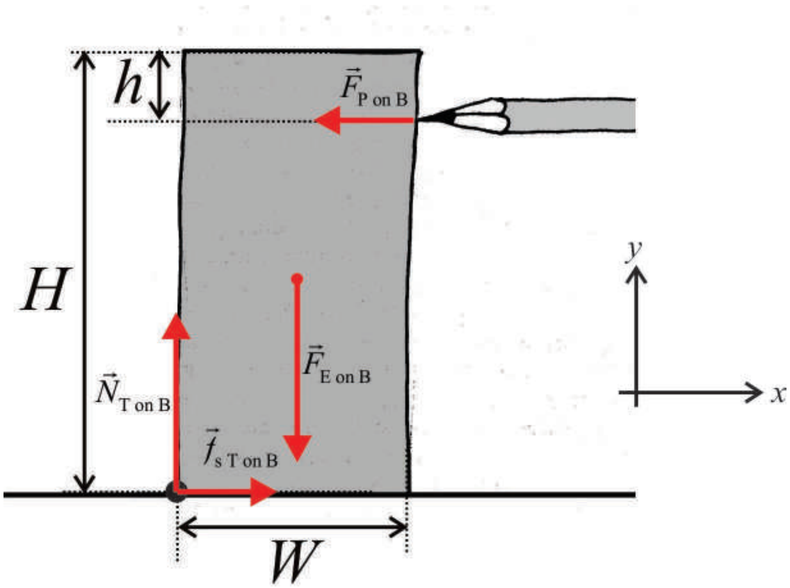
Based on experiences from pushing different boxes or pieces of furniture on the rough floor we can say:

For very large coefficients of static friction (the block "sticks" to the table) we expect to be able to push on the block closer to the top (smaller h) before the block starts sliding. This agrees with the equation: smaller h => larger μ.

If the box is wider (larger W) it is less likely that it will tip over when pushed (assuming all other parameters remain constant). In other words, we can push a wider box at the point that is closer to the top and still not tipping the box. To see that this agrees with the equation, we rearrange the equation to obtain: $h = H - \frac{W}{2\mu}$ *; larger W => smaller h.*

Derivation:

We draw a force diagram for the box just before it starts turning around the bottom left edge.



Because the box is in a static equilibrium, the following conditions apply:

Force component condition:

$$f_{\text{s T on B}\,x} + F_{\text{P on B}\,x} = 0$$

$$N_{\text{T on B}\,y} + F_{\text{E on B}\,y} = 0$$

or using the magnitudes of the forces

$$f_{\text{s T on B}} + (-F_{\text{P on B}}) = 0 \;\Rightarrow F_{\text{P on B}} = f_{\text{s T on B}}$$

$$N_{\text{T on B}} + (-F_{\text{E on B}}) = 0 \;\Rightarrow N_{\text{T on B}} = F_{\text{E on B}}$$

Torque condition:

$$F_{\text{P on B}} \cdot (H - h) + (-F_{\text{E on B}} \cdot \frac{W}{2}) = 0 \;\Rightarrow f_{\text{s T on B}} \cdot (H - h) = F_{\text{E on B}} \cdot \frac{W}{2}$$

In addition, we know that when the static friction force reaches its maximum (just before the box starts sliding) we have:

$$f_{\text{s T on B}} = \mu_s N_{\text{T on B}} = \mu_s F_{\text{E on B}}$$

Combining the last two equations, we finally get

$$\mu_s(H-h)=\frac{W}{2}\Rightarrow\mu_s=\frac{W}{2(H-h)}$$

c. Why does the result not depend on the mass of the block? Explain.

The mass of the block affects the experiment in two ways that cancel. The larger the mass the larger the force you need to exert on the box to tip it. However, the larger mass also results in the larger maximal static friction force that opposes the force that you exert when you are pushing the box.

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