Chapter 7

Work and Energy

7.1 Work and energy

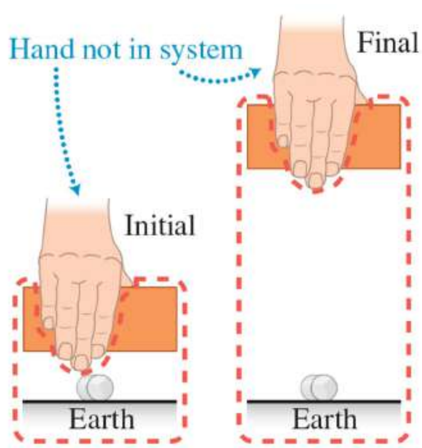
7.1.1 Observe and find a pattern

PIVOTAL ***Lab or class:*** *Equipment:* Brick, chalk, slingshot, cart. For experiment b. tape the chalk to a wall so that the cart hits it when it collides with the wall.

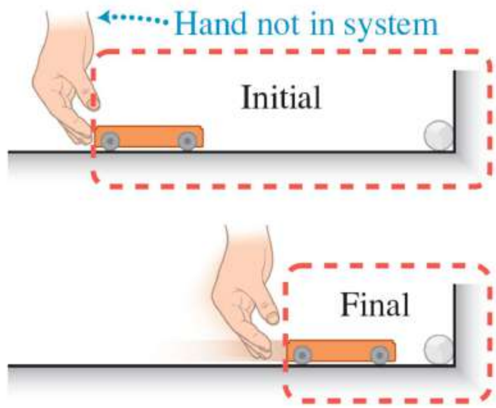
Below you see the description of several experiments. You can either repeat the experiments with your group members or read their descriptions here.

All three experiments involve a well-defined system and a process in which the system changes from an initial state to a final state. At the end of this process, we find that the system has the potential to do something it couldn't do before—to crush a piece of chalk into many pieces. In each chase, the chalk-crushing ability (CCA) increases due to the intervention of an external agent (you). For each situation, work with the members of your group to draw arrows indicating the direction of the external force that you (outside the system) exert on a system object and the displacement of the object while you exert the force in order to increase the CCA of the system. Fill in the table that follows to help you complete this activity.

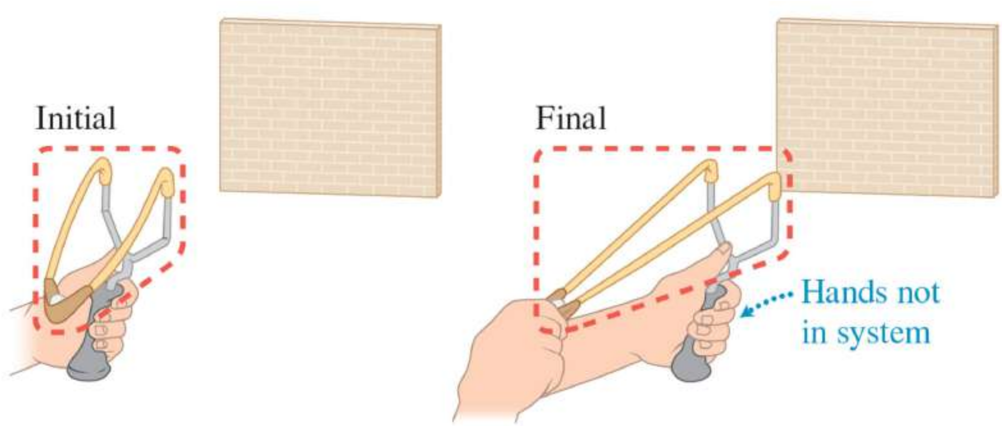
a. The system includes a brick with a flat bottom, Earth, and a piece of chalk. You (outside the system) pull up on brick so that slowly rises 0.5 m above the piece of chalk. After this lifting process, you release the brick. It falls and breaks the chalk.



b. The system includes a low friction cart that can roll on a floor and a piece of chalk that is placed by the vertical wall. You (outside the system) continuously push the cart over a set distance so that it rolls faster and faster toward the chalk on the wall and breaks the chalk when it hits it.



c. The system includes a slingshot that holds a piece of chalk. Your instructor (outside the system) slowly pulls back on the sling. When she/he releases the sling, the chalk shoots out at high speed and hits the wall, causing the chalk to break.



Experiment	**a.**	**b.**	**c.**
Draw arrows indicating the direction of the force you exerted on the system $\vec{F}_{\text{Y on S}}$ and the displacement $\vec{d}$ of the object in the system while you were exerting the force.			

d. Look for a pattern in what was done to the system to increase its chalk-crushing ability (CCA). Then, devise a new physical quantity to describe this pattern. Be explicit.

7.1.2 Observe and find a pattern

PIVOTAL ***Class or lab:*** *Equipment:* Brick, string, chalk, cart.

In Activity 7.1.1, you found that the external force you exerted on an object in a system *increased* the ability of the system to smash a piece of chalk. The force you exerted on the object in the system was always in the direction of the displacement of that object. Suppose that a friend outside the system decides to save the chalk in the first two experiments by exerting with her hand an opposing force on the brick or on the cart after they are released. In each case your friend pushes on the moving object opposite to the direction of its velocity. Describe for parts **a.** and **b.** of Activity 7.1.1 the direction of the force your friend exerts on the moving object relative to the displacement of the object as she stops it—in other words, what does she do to *reduce* the chalk-crushing ability of the system?

a. The system includes a 1-kg brick with a flat bottom and a string attached to the top, Earth, and a piece of chalk. You (outside the system) pull up on the string so that the 1-kg brick slowly rises 0.5 m above the piece of chalk. After this lifting process, you release the block, and it starts falling. Your friend then starts pushing upward on the falling brick, slows it down, and the brick does not break the chalk.

b. The system includes a 1-kg dynamics cart that can roll on a low-friction horizontal dynamics track and a piece of chalk that is taped to the fixed vertical end of the track. You (outside the system) push the cart so that it rolls faster and faster. Before the cart reaches the chalk, your friend pushes on it opposite its displacement. This causes the cart to slow down and stop so that it does not break the chalk.

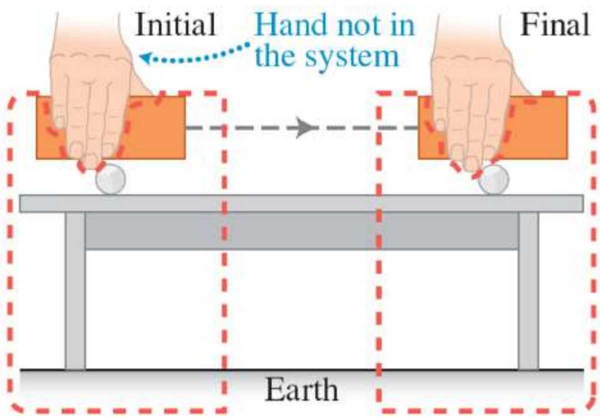
c. Discuss with your group: how could you modify the definition of the quantity you devised in Activity 7.1.1 to account for the system's loss of chalk-crushing ability thanks to your friend's intervention? Put your group's ideas down on a whiteboard.

7.1.3 Observe and find a pattern

PIVOTAL ***Class or lab:*** *Equipment:* Brick and chalk.

Consider a system that includes Earth and a 1-kg brick.

a. You (outside the system) hold the brick so that it stays about 2 cm above a table. A piece of chalk is placed on the table under the brick. If you release the brick and it falls on the chalk, the chalk does not break (it's too close to the chalk).



b. Next you slowly walk about 1 m beside the table, continually keeping the brick 2 cm above the surface. After you have walked the 1 m, the brick hangs over a second identical piece of chalk. Draw the force exerted by you on the brick and the displacement of the brick as you walked the 1 m.

c. Discuss with your group whether the *vertical* force that you exerted on the brick while moving it *horizontally* above the tabletop caused the system to have a better chance of crushing the second piece of chalk than the first piece. Work with your group members to revise the quantity you devised in the last two activities to account for this result. Your revision will involve the angle between the external force exerted on the object in the system and the object's displacement. We call this physical quantity *work.*

7.1.4 Construct a mathematical model

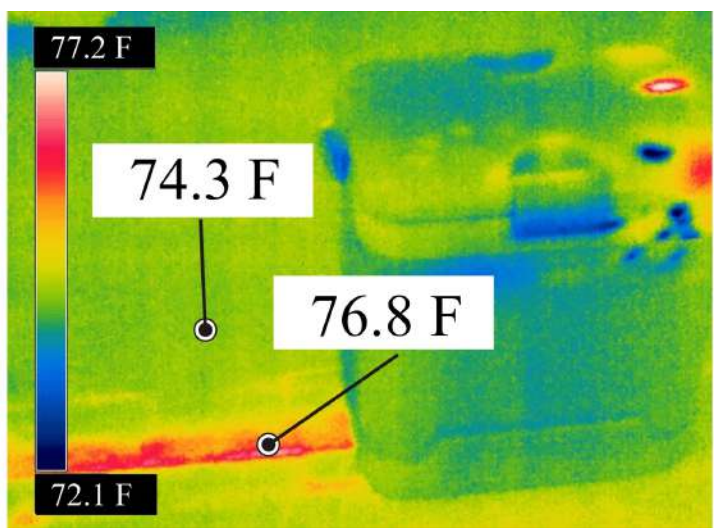
***PIVOTAL* Class:** *Equipment per group:* whiteboard and markers.

Work together with your group to construct a mathematical equation that relates changes in chalk-crushing ability, ΔCCA, to the new physical quantity you devised in Activities 7.1.1–7.1.3. Compare your equation with another group's equation and discuss and resolve any differences or inconsistencies.

7.1.5 Observe and find a pattern

***PIVOTAL* Class or lab:** *Equipment per group:* whiteboard and markers.

A system consists of a heavy cooler and a rough horizontal surface on which it sits. You (outside the system) pull on the handle of a cooler so that it moves slowly along the surface at constant velocity. You do positive work by pulling the cooler for about 5 m. Your friend takes a photo of the cooler and the surface using a thermal camera (see photo at right below). Describe how the system (cooler and surface) is different after you do the work than before the cooler started moving. Notice that the chalk-crushing ability of the cooler did not change (ΔCCA = 0), and yet you did work on it. Does this fit with the equation you just invented in Activity 7.1.4? Discuss with your group how you could modify the equation you came up with in Activity 7.1.4 to account for this anomalous result. *Hint*: You may need to invent a new physical quantity. Put your group's revised equation on a whiteboard and compare your ideas with those from another group.



7.1.6 Describe

PIVOTAL Class

You do work on a system to change its ability to do something—for example, to crush chalk or to make the touching surfaces of objects in a system warm. In Activities 7.1.1 through 7.1.5, the work done on the system by the external force caused different types of changes in the system. For each situation below, discuss with your group members how you can describe each type of change in the system as a result of the work done on it, and come up with a name for it.

a. The external force caused the block to move higher above Earth's surface.

b. The external force caused the cart to move faster and faster.

c. The external force caused the slingshot to stretch.

d. The external force caused the surfaces of the touching objects to warm.

7.1.7 Pose a problem

PIVOTAL Class

Describe with your group members a real-life situation in which an external force does the following:

a. positive work on a system;

b. positive work on a system, but with a value that is less than in part a.;

c. negative work on a system;

d. zero work even though an object in the system moves.

Note that your situations should be different from the scenarios we've already encountered in Activities 7.1.1–7.1.5

7.1.8 Apply

Lab: *Equipment:* children toys that allow for the processes described below.

For each item below, describe one real-life experiment that is consistent with the work–energy process.

a. Positive work causes an increase in the gravitational potential energy of the system.

b. Positive work causes an increase in the kinetic energy of the system.

c. Positive work causes an increase in the elastic potential energy of the system.

d. Kinetic energy in the system is converted to gravitational potential energy.

e. Kinetic energy in the system is converted to elastic potential energy.

f. Gravitational potential energy in the system is converted to internal energy.

g. Gravitational potential energy in the system is converted to elastic potential energy.

7.1.9 Reading exercise

Read Section 7.1 in the textbook and answer Review Question 7.1.

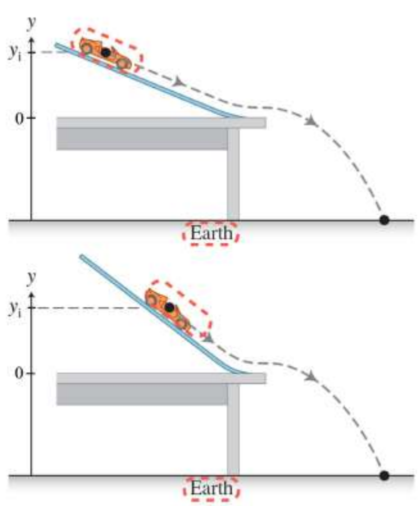
7.2 Energy is a conserved quantity

7.2.1 Observe and explain

Lab: *Equipment per group:* A track that that looks like the one in Testing Experiment Table 7.3, Hot Wheels car, meter stick, lab stand.

Work with your group members to put the experiment together, decide what data to record and how to record them.

a. Place the car on the track as shown in the figure below and note the height of the car above the table. Let the car go and record the location where it lands on the floor. Repeat the experiment to make sure the car lands reliably (same location) on the floor.



b. Change the steepness of the track but release the car from the same height above the table and record the location where it lands on the floor. Repeat the experiment to make sure the car lands reliably (same location) on the floor.

c. Repeat the experiment by changing the steepness again.

d. Present the results in a table. What pattern do you find? Discuss with your group members how you can explain this pattern using your knowledge of gravitational and kinetic energy. Provide your group's explanation.

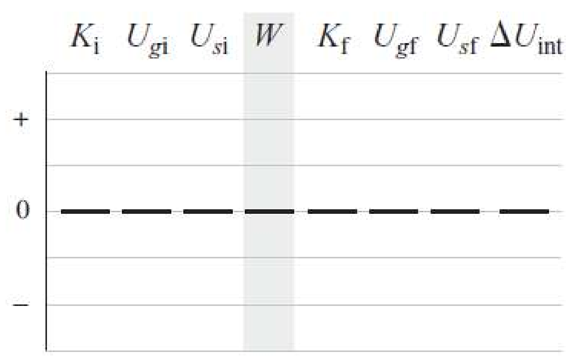
7.2.2 Represent and reason

PIVOTAL Class: *Equipment per group:* whiteboard and markers.

Work with your group member to analyze the following experiment: A rope pulls a skier, initially at rest, up a hill. *Initial state:* A skier is at rest at the bottom of the hill. *Final state:* The skier is moving at moderate speed at the top of the hill. *System:* Includes the skier, rope, and Earth but excludes the motor that pulls the rope up the hill. Ignore friction.

a. Draw a sketch showing initial and final states.

b. Construct a qualitative work–energy bar chart using the grid below, or on your whiteboard.



7.2.3 Represent and reason

PIVOTAL Class: *Equipment per group:* whiteboard and markers.

Experiment 1: A ball, initially at rest, is dropped from a certain height above the ground.

Initial state: Ball at rest *y* meters above the ground.

Final state: Ball is moving at speed *v,* just before it hits the ground.

System: Just the ball (Earth does work on the ball here).

Experiment 2: A ball, initially at rest, is dropped from a certain height above the ground.

Initial state: Ball at rest y meters above the ground.

Final state: Ball is moving at speed v, just before it hits the ground.

System: The ball *and* the Earth are the system.

For each experiment work with your group members to:

a. Draw a sketch showing initial and final states.

b. Construct a qualitative work-energy bar chart.

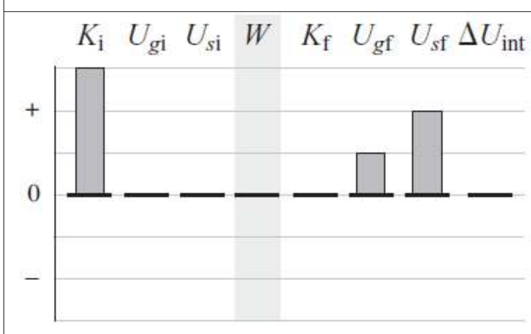
7.2.4 Bar-chart Jeopardy

PIVOTAL Class: *Equipment per group:* whiteboard and markers.

Work with your group members to

a. describe in words, and then

b. sketch a process (the system, its initial and final states, and any work done on the system) that is consistent with the qualitative work–energy bar chart shown below. There are many possible choices.

Bar chart for a process	**Describe in words one possible consistent process.**	**Sketch the process just described.**
		

7.2.5 Represent and reason

Class: *Equipment per group:* whiteboard and markers.

Devise a graphical method to determine the work done by an external force on a system object. *Note:* P = person and O = object. For each of the processes described below.

a. Sketch the process.

b. Draw $F_{\text{P on O}\,y}$ versus y (for vertical motion) or $F_{\text{P on O}\,x}$ versus x (for horizontal motion).

c. Describe how to use the graph to find the work done by the specified force.

Processes:

1. Shumaila lifts a backpack from the floor to the desk, exerting a constant upward force. The backpack and Earth (not Shumaila) are the system.

2. Kruti catches a medicine ball in the gym. The ball and Earth are the system but not Kruti. Her hands move back toward her body while stopping the ball. Assume that she exerts a constant force on the ball.

3. Ben stretches a horizontal rubber cord by 6.0 cm. To stretch the cord by 1.0 cm Ben exerts a 10.0-N force, and then to stretch the cord by 2.0 cm he exerts a 20.0-N force, to stretch the cord by 3.0 cm he exerts a 30.0 N force and so forth. The spring and Earth (not Ben) are the system.

7.2.6 Reading exercise

Read Section 7.2 in the textbook and answer Review Question 7.2.

7.3 Quantifying gravitational potential and kinetic energies

7.3.1 Find the relationships

PIVOTAL ***Class:*** *Equipment per group:* whiteboard and markers.

To develop mathematical expressions for gravitational potential energy and kinetic energy, we analyze the following situation: a cable lifts a block from vertical position y_i to vertical position y_f. When at position y_i, the block is moving up at speed $v_{y\text{i}}$, and when at position y_f it is moving up at greater speed $v_{y\text{f}}$.

a. Sketch the situation. Include a labeled vertical axis.

b. Write an expression for the work the cable does on the block during its displacement $y_\text{f} - y_\text{i}$.

c. Draw a force diagram for the block. Use it to find an expression for the force that the cable exerts on the block in terms of its mass *m*, acceleration *a*, and the gravitational constant *g*. Substitute this expression into the expression in part **b.**

d. Use a kinematics equation to convert the acceleration *a* in the equation from part **b.** into an expression involving the block's speeds v_i and v_f and its displacement $y_\text{f} - y_\text{i}$.

e. Substitute the expression from part **d.** into the expression for the force that the cable exerts on the block found in part **c**.

f. Substitute the new expression in part **e.** for the force that the cable exerts on the block into the expression for work in part **b**.

g. Examine the expressions that you derived in parts **e.** and **f**. Do you see that the work that the cable did on the block equals the sum of the changes of two quantities: $mgy_f - mgy_i$ and $\frac{1}{2}mv_{y\text{f}}^2 - \frac{1}{2}mv_{y\text{i}}^2$? Discuss how these expressions can be used to write an expression for the gravitational potential energy of the block–Earth system and an expression for the kinetic energy of the block.

7.3.2 Reading exercise

Read Section 7.3 in the textbook and answer Review Question 7.3.

7.4 Quantifying elastic potential energy

7.4.1 Test a relation

***PIVOTAL* Lab:** *Equipment per group:* Two different springs, ring stand, objects of known masses, force probes or spring scales.

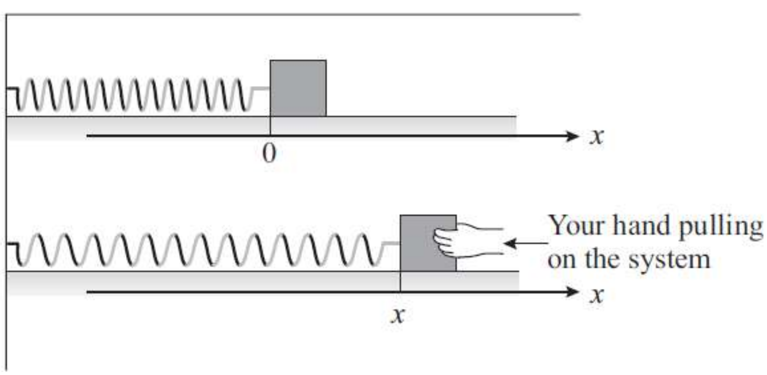
One of the famous mathematical relationships of physics is called *Hooke's law* in honor of a British physicist Robert Hooke (1635-1703). The mathematical relationship describes how springs respond to stretches or compressions and is written as $\vec{F}_{\text{S on O}} = -k\Delta\vec{x}$ where $\vec{F}_{\text{S on O}}$ is the force that the spring exerts on an object that is stretching or compressing it by a certain displacement Δx from its natural/unstretched length. k is a constant characterizing a specific spring. Work with your group members to design an experiment to test whether this mathematical relationship applies to the two springs that you have. Make sure that after you design the experiment, you make a prediction about its outcome based on Hooke's law before you conduct the experiment. Then conduct the experiment and compare the outcome to the prediction.

7.4.2 Find the relationship

PIVOTAL Class: *Equipment per group:* whiteboard and markers.

Work with your group members on a whiteboard to determine an expression for the elastic potential energy of a stretched spring. The spring is the system, and you stretch it from its equilibrium position ($x = 0$) to some final position x. You are outside the system. The spring is very easy to stretch at first but gets more and more difficult as you stretch it farther. (Remember that $F_{\text{Y on S}\,x} = kx$ i.e., the force that "you" exert on the "spring.")

a. Examine the sketch of the situation below, showing the system and the external object exerting a force on it. Note the origin of the x-axis at the end of the unstretched spring that you start pulling.



b. Draw a graph ($F_{\text{Y on S}\,x}$ versus x) and find the work that you did while stretching the spring from 0 to x as the area under the curve.

c. Examine the expression for work that you derived in part **b.** Do you see that the work that you did on the spring equals $\frac{1}{2}kx^2$? How does this expression relate to the elastic potential energy of the spring that you have stretched?

7.4.3 Reading exercise

Read Section 7.4 in the textbook and answer Review Question 7.4.

7.5 Friction and energy conversion

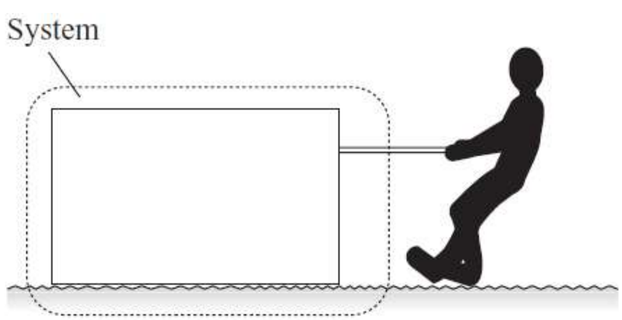
7.5.1 Reading exercise

Read Section 7.5 in the textbook and answer Review Question 7.5.

7.5.2 Represent and reason

PIVOTAL Class: *Equipment per group:* whiteboard and markers.

Work with your group, using a whiteboard, to determine an expression for the change in internal energy due to friction in a system that consists of a crate and a rough horizontal surface on which it slides. You, outside the system, pull on a rope attached to the crate so that it moves slowly at constant velocity (see figure below). At the end of the process, the bottom of the crate and the surface on which it was moving have become warmer.

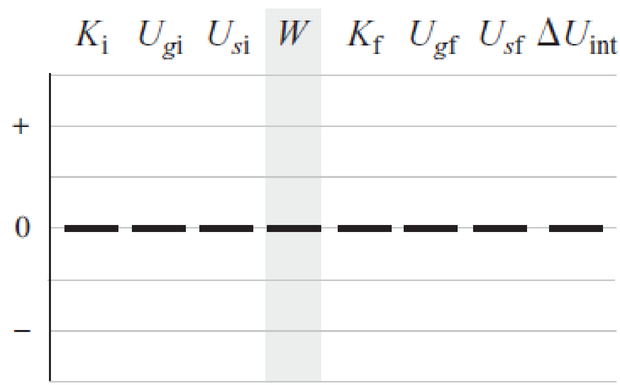


a. Write an expression for the work done on the system by the external force of the rope on the crate as the rope pulls the crate a distance s across the surface exerting a force $\vec{F}_{\text{R on C}}$.

b. Choose the crate alone as an object of interest and draw a force diagram for the crate. Apply Newton's second law for the horizontal x-axis. How are $\vec{F}_{\text{R on C}}$ (the force the rope exerts on the crate) and $\vec{f}_{\text{k S on C}}$ (the kinetic friction force that the surface exerts on the crate) related?

c. Now, combine parts **a.** and **b.** to write an expression for the work done by the force $\vec{F}_{\text{R on C}}$ on the system that consists of a crate and a rough horizontal surface on which it slides. Is it positive or negative?

d. Represent the process that you analyzed in part **c.** with a bar chart.



e. Examine the bar chart. Write an expression for the change in internal energy of the system and decide whether it increases or decreases.

7.6 Skills for analyzing processes using the work-energy principle

7.6.1 Reason

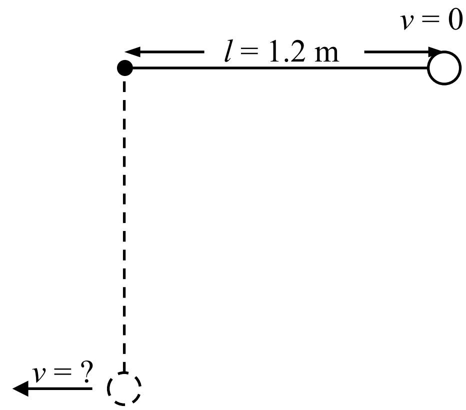
Class: *Equipment per group:* whiteboard and markers.

Summarize the results of the activities in Sections 7.3-7.5 to construct a generalized work–energy relationship—a relationship between the initial energy of a system, the external work done on the system, and the final energy of the system. Put your ideas on a whiteboard and compare with another group.

7.6.2 Reason

PIVOTAL Class

A pendulum consisting of a light string and an object (mass 2 kg) is released from a horizontal position and swings down (see the figure at right). Find the speed of the object when the pendulum reaches the bottom of its swing (as shown by the dashed lines in the figure) using your knowledge of energy.

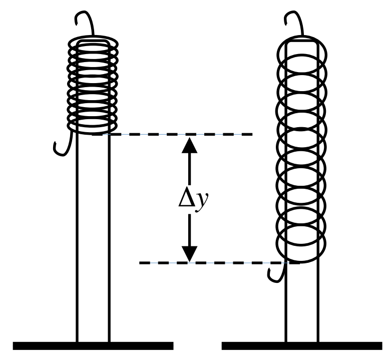


Discuss with your group: Could you figure this out using kinematics? Why, or why not?

7.6.3 Test your idea

PIVOTAL Lab: *Equipment per group:* Ring stand, a spring (bend one end in with pliers so that the hook is perfectly aligned with the side as shown in the diagram), meter stick, hanging objects, force probe and computer.

You have a vertical rod, a light spring with one end closed and the other end open so that it can slide down over the rod and then fly up when released, and various other pieces of equipment at your disposal. Use this equipment to design an experiment whose outcome you can predict using the generalized work-energy relationship and your newly discovered knowledge of how springs behave.



Work with your group to make sure you address the following points.

a. What hypothesis or model are you testing?

b. Describe your experiment clearly in words and diagrams.

c. What side experiment will you have to conduct to execute your testing experiment? Describe the side experiment, do it, and record your results with experimental uncertainties etc…

d. Show all mathematical steps to making a prediction. Be sure to include diagrams and bar charts as necessary in your explanation.

e. What assumptions are you making in your mathematical procedure? Also discuss how those assumptions might affect the outcome of the experiment when you do it.

f. Estimate your experimental uncertainties, both in measured values obtained to make your prediction *and* in your actual execution of the experiment.

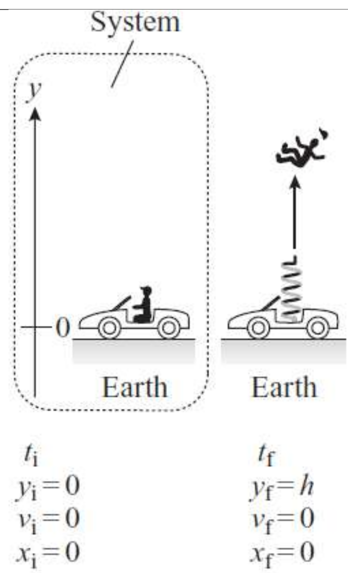
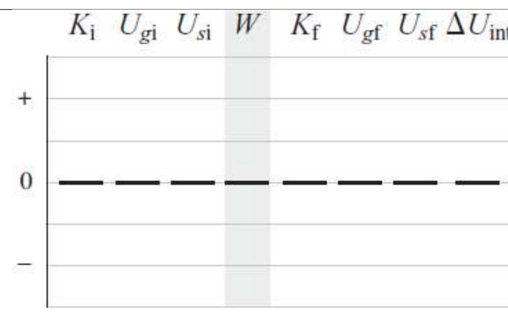
g. Run your experiment and record the results.

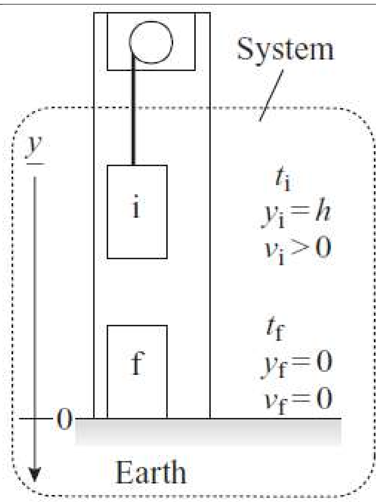
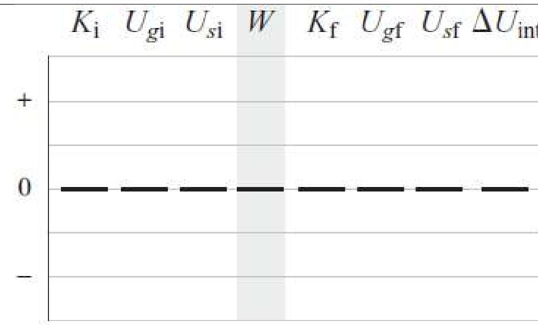
h. Make a judgment about the hypothesis you're testing including a discussion of whether your results agree with your prediction, and the role of uncertainties and assumptions and how those affected the results of your experiment.

7.6.4 Represent and reason

Class *Equipment per group:* whiteboard and markers.

Word descriptions and pictorial representations of two work–energy processes are provided in the table that follows. Work with your group to complete the last column of the table. Do not solve for anything.

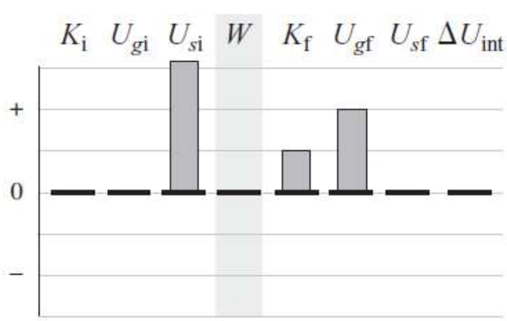
Word description	**Picture description**	**(i) Construct a bar chart for the process and apply in symbols the generalized work–energy principle.**
A stunt car has an ejector seat that rests on a vertical spring compressed a distance x_i .When the spring is released, the seat with its passenger is launched out of the car and reaches a		

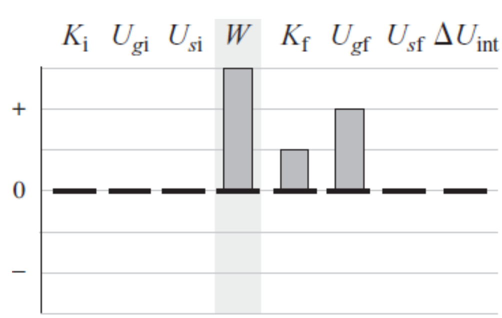
maximum height h_f above its starting position.		
Word description	**Picture description**	**Construct a bar chart for the process and apply in symbols the generalized work–energy principle.**
An elevator, while moving down at speed v_i, approaches the ground floor and slows to a stop in a distance h.		

7.6.5 Bar-chart Jeopardy

Class: *Equipment per group:* whiteboard and markers.

Each of the two bar charts below describes a real process. With your group members, answer the following questions to describe the possible processes. Do not solve for anything.





a. Draw an initial–final sketch of a process that might be described by the bar chart. Identify the system.

b. Describe the process in words.

c. Use the bar chart to apply in symbols the generalized work–energy equation to the process.

7.6.6 Application experiment

***PIVOTAL* Lab:** *Equipment per group:* Hot Wheels City Launch & Loop track set (key element: variable setting elastic launcher, the rest you can make out of whatever Hot Wheels parts you have),2-meter stick, stopwatch, ring stand, digital scale, 1-foot high wooden platform, computer with Microsoft Excel, masking tape.

Part I. Application experiment: Measure the energy stored in a Hot Wheels Launcher

The Hot Wheels car launcher has a plastic plunger that can be pulled back to latch at four different launch positions. As it is pulled back, it stretches a rubber band—a greater stretch for each of the four positions. Your task is to use the generalized work-energy principle to determine the elastic potential energy stored in the launcher in each of these positions.

Work with your group to address all of the following points:

Start by making a rough plan for how you will solve the problem. Make sure that you use two independent methods to determine the energy of each launch position. Note that we use two independent methods when we determine the value of a quantity in any application experiment.

Important: When you set up the rubber band(s) in the launcher, make sure it's strong enough to have a chance of getting the car around the loop-the-loop (part II). Don't test it in advance, but make sure it looks as if your car will be going fast enough.

For each of your independent methods:

a. Write a brief outline of your procedure. Include a labeled sketch of your setup.

b. In the outline of your procedure, identify the physical quantities you will measure and describe how you will measure each quantity. What steps will you take to minimize experimental uncertainty?

c. Construct force diagrams and energy bar charts wherever appropriate.

d. Devise the mathematical procedure you will use to solve the problem. Decide what assumptions you are making and specifically how each of them will affect the outcome.

e. Perform the experiment and record the data in an appropriate manner. *Hint:* You might find it useful to use Microsoft Excel to record and analyze your data. If you choose to do this though, make sure you format the spreadsheet so it is clear what everything is. Also, you need to provide an example of each type of calculation you are using Excel to automate so that it's clear what your mathematical procedure is.

f. Use the data collected to determine the elastic potential energy stored in the launcher at each position. Estimate the uncertainty in your results.

Once you have completed the two independent methods:

g. Combine the results of your two methods and determine final values for the elastic potential energy stored in the launcher at each setting. Be sure to include uncertainties for these quantities.

Part II. Application experiment: Getting the Hot Wheels car to successfully make a loop-the-loop

Your task now is to determine the lowest energy launching position so that the car will make it around the loop without losing contact with the loop—on the *first try* (do not use a trial and error method). If you use the next lower energy setting, the car should not make it around the loop. You may use the results you obtained from the previous experiment. Note: This is an application experiment, but given what the goal of the experiment is, doing a second independent experiment isn't applicable.

Start by connecting the car launcher to a level section of track, then connecting that to the loop-the-loop. Place a spin-out at the end of the track (to keep the car from leaving the table).

Include the following in your write-up:

a. Start by making a rough plan for how you will solve the problem. Identify which physics ideas you need for each part of what will happen. Identify the physical quantities you will measure, and describe how you will measure each of them.

b. Draw force diagrams and/or energy bar charts for each part of the problem. Think of what conditions are necessary for the car to make it through the loop successfully. *Hint:* Where is the car most at risk of leaving the track? What physics ideas can you use to describe mathematically the condition that will cause the wheels of the car to stay in contact with the track?

c. Use the force diagrams and bar charts you created to devise the mathematical procedure to solve each part of the problem. Combine the parts to create a complete solution to the problem.

d. List the assumptions you made in your mathematical procedure. Discuss specifically how each assumption could affect your result (By 'your result' we mean which launch position you will choose.)

e. Use the quantities that you measured and the mathematical procedure that you devised to decide which launch position to use. Make sure you consider experimental uncertainties. Why is it important to do this?

Perform the experiment and record the outcome. Also check to see if any lower energy launch positions also allow the car to make it around the loop-the-loop. If the launch position you decided on was not the minimum one that allowed the car to make it, suggest explanations for why this happened. *Hint:* What assumptions did you make?

7.6.7 Regular problem

Class *Equipment per group:* whiteboard and markers.

A 1000-kg elevator is moving downward. While moving down at 4.0 m/s, its speed decreases steadily until it stops in 6.0 m. Determine the magnitude of the tension force that the cable exerts on the elevator while it is stopping. Solve the problem with the help of your group members, following the problem solving strategy.

Sketch and translate • Sketch the initial and final states of the process, labeling known and unknown information. • Include the object of reference and the coordinate system. • Choose the system of interest. Sometimes you might need to go back and redefine your system after you draw a bar chart if you find that the chosen system is not convenient.	
Simplify and diagram • What simplifications can you make to the objects, interactions, and processes?	

• Decide which energy types are changing. • Are external objects doing work? • Use the initial-final sketch to help draw a work-energy bar chart. Include work bars (if needed) and initial and final energy bars for the types of energy that are changing. Specify the zero level of gravitational potential energy.	
Represent mathematically • Convert the bar chart into a mathematical description of the process. Each bar in the chart will appear as a single term in the equation.	
Solve and evaluate • Solve for the unknown and evaluate the result. • Does it have the correct units? Is its magnitude reasonable? Do the limiting cases make sense?	

7.6.8 Equation Jeopardy

Class: *Equipment per group:* whiteboard and markers.

Below you see the application of the generalized work–energy equation to two different processes (in fact, there are many possible processes described by each equation). For each mathematical description, collaborate with your group to

a. construct a sketch,

b. identify the system,

c. write a word description, and

d. draw a bar chart that is consistent with the equation.

Process 1:

$$\tfrac{1}{2}kx_i^{\,2} = \tfrac{1}{2}mv^2 + mgy$$

Process 2:

$$mgd\sin 20^\circ = \tfrac{1}{2}kx^2$$

7.6.9 Evaluate the solution

Class: *Equipment per group:* whiteboard and markers.

The problem: A 40,000-N/m spring cart launcher, initially compressed 0.50 m, is released and launches you and your cart (with total mass 100 kg) up a 30° incline. What distance along the incline do you travel before stopping?

Proposed solution: $\tfrac{1}{2}(40{,}000\ \text{N/m})(0.50\ \text{m}) = (100\ \text{kg})(9.8\ \text{m/s}^2)y \quad \Rightarrow \quad y = 10.2\ \text{m}$

a. Identify any missing elements or errors in the solution.

b. Provide a corrected solution if you find any missing elements or errors.

7.6.10 Evaluate the solution

Class *Equipment per group:* whiteboard and markers.

The problem: You are traveling in your 2000-kg pick-up truck at 20 m/s up a hill with a 6.0° incline when you see a goose crossing the road 24 m in front of you. You know from previous experience that when you hit the brakes, a 16,000-N friction force opposes the car's motion. Will you hit the goose?

Proposed solution:

$$\tfrac{1}{2}(2000\ \text{kg})(20\ \text{m/s})^2 = (16{,}000\ \text{N})x \Rightarrow$$
$$x = 25\ \text{m} \quad \text{Oops!}$$

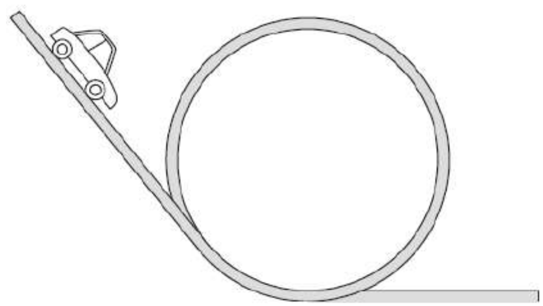
a. Identify any missing elements or errors in the solution.

b. Provide a corrected solution if you find any missing elements or errors.

7.6.11 Pose a problem

Class: *Equipment per group:* whiteboard and markers.

Pose a problem that can be solved with the generalized work–energy principle using the situation depicted in the illustration below. Make a list of necessary givens and assumptions to solve the problem. Put your problem on a whiteboard and trade your whiteboard with another group. See if you can solve each other's problem. Discuss with the other group any obstacles you ran into.



7.6.11 Real-world application: a climbing wind-up toy car

Class *Equipment per group:* whiteboard and markers.

The toy car on the figure at right is powered by a wind-up spring. The spring is exerting a force on one pair of wheels. In addition, the car has two magnets on the side that allow the car to climb up the vertical steel plate. In this case, the role of the magnets is to provide an attractive force between the steel and the magnets but the magnets are not touching the steel plate. Watch the video [https://mediaplayer.pearsoncmg.com/assets/_frames.true/sci-phys-egv2e-alg-7-6-11] and then work on the following activities. (Note: all activities refer to both experiments in the video.)

a. Draw a force diagram for the toy car when it is about half-way between the initial and the final state. Indicate any assumptions that you made.

b. Choose your system and represent the process with work-energy bar charts. Choose the initial and the final state as indicated in the video. Indicate any assumptions that you made.

c. What can you say about the work done on the car by the force that the table (or steel plate) exerts on the car while it is moving?

7.7 Collisions

7.7.1 Observe and find a pattern

Class *Equipment per group:* whiteboard and markers.

Study the slow-motion videos of several collisions of two carts [https://mediaplayer.pearsoncmg.com/assets/_frames.true/sci-phys-egv2e-alg-7-7-1]. The collisions where both the kinetic energy and the momentum are constant are called *elastic*; the collisions in which only momentum is constant are called *inelastic*. Classify all the videoed collisions into those two categories. The masses of the carts are given in the video.

Experiment	**Analysis**	**Decision (is the collision elastic or inelastic?)**

7.7.2 Reading exercise

Read Section 7.7 in the textbook and answer Review Question 7.7.

7.7.3 Application experiment: Measuring the coefficient of kinetic friction

PIVOTAL Lab: *Equipment per group:* Tissue box, soft pendulum, 2 meter sticks used as guide rails for the box, tape, spring scales, digital mass balance, stopwatch app (on your phone, the web, the lab computers, etc.)

The goal is to work with your group to use your knowledge of energy and momentum to measure the coefficient of kinetic friction between a tissue box and the table. You must come up with *two* independent methods. One of your methods must follow the same approach of whacking the tissue box with a sand-filled balloon as shown in the video [https://mediaplayer.pearsoncmg.com/assets/_frames.true/sci-phys-egv2e-alg-7-7-3].

Constructing the mathematical model

First, come to a consensus with your group members about the mathematical model you will use to represent the situation. Write up the results of your consensus. Include the following:

a. Start by making a rough plan for how you will solve the problem. In particular, divide the problem into two or more smaller problems that each involves one central physics idea. Write a brief outline of your mathematical procedure. Include a labeled sketch of your setup.

b. Identify which physics ideas you will use for each part. Describe how you will model the objects, interactions and processes you will use in your mathematical model.

c. Construct force diagrams, energy bar charts, and momentum bar charts as appropriate.

d. Identify the physical quantities you will need to measure in order to solve the problem.

e. Put together the complete mathematical method you will use to solve the problem. Do not plug in any numbers; express the method as an equation for the coefficient of kinetic friction written in terms of the physical quantities you will measure experimentally.

f. What assumptions did you make in your mathematical model? Specifically, what is the effect making each of them will have on the result produced by your mathematical model (will it make the calculated value smaller, larger, or randomly different than the real-world value)? Explain your reasoning.

Designing and performing the experiment

Now, design and perform an experiment that uses your mathematical model to determine the coefficient of kinetic friction between the box and the tabletop. Include the following in your write-up:

a. A detailed description of the experimental procedure. Include a labeled sketch of your setup.

b. Identify the physical quantities you will measure. Describe how you will measure each quantity. What steps will you take to minimize the uncertainty in your measurements?

c. Perform the experiment and record the data. Estimate the uncertainty in each measurement.

d. Use the mathematical procedure you devised to determine the coefficient of kinetic friction. Estimate the uncertainty in your result.

e. Design and use an independent method to determine the coefficient of kinetic friction. *Hint:* It can be a very simple experiment, but make sure you deal with assumptions and uncertainties properly and completely.

f. Decide if the results of the two experiments are consistent or not. What is your judgment about the model you used to represent the situation shown in the video?

Improving the mathematical model

a. Think about assumptions you made in your mathematical procedure. Which of your assumptions is least likely to be valid? Describe how you will revise part of your mathematical procedure to deal with this. What additional measurements will you need to make?

b. Construct force diagrams, and energy and/or momentum bar charts for your revised mathematical method.

c. Come up with a new equation for the coefficient of kinetic friction that incorporates your revisions. Again, do not plug in numbers.

d. Make any additional measurements you need, and then use your new equation to determine a revised value of the coefficient of kinetic friction.

e. Now are the results of the two independent methods consistent? Now what is your judgment about the mathematical model?

f. Are any of the other assumptions in your revised mathematical model questionable? How could you revise your mathematical procedure further to deal with these?

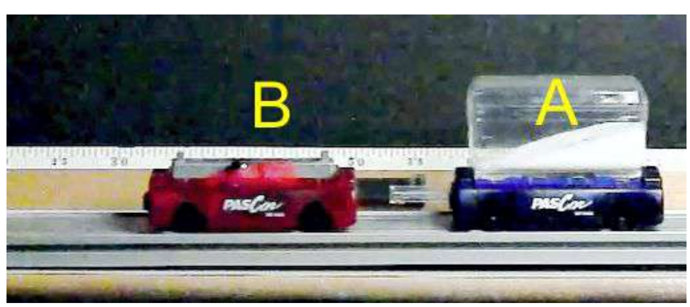
7.7.4 Regular Application

PIVOTAL Class: *Equipment per group:* whiteboard and markers.

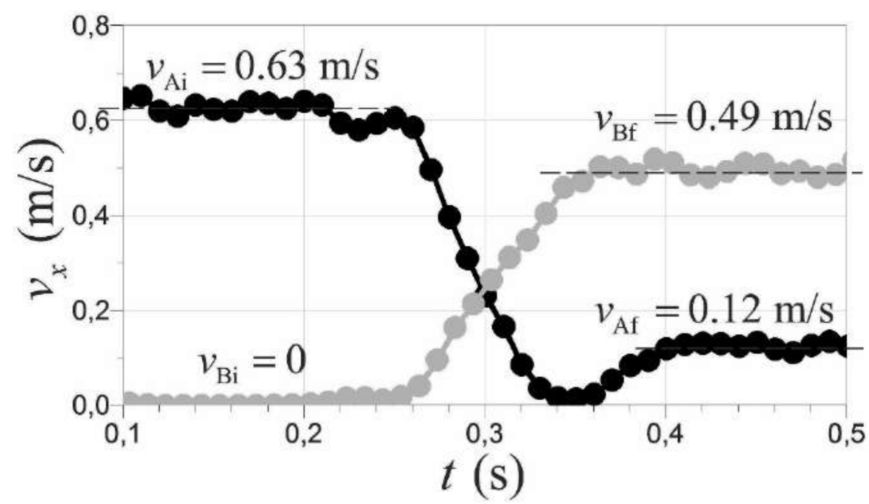
Solve the following problem on a whiteboard with your group. An 80-kg skier comes off a slope traveling at speed 15 m/s and moves onto a level snow-covered surface. The skier, wearing a Velcro-covered vest, runs into a padded 20-kg cart, also covered with Velcro. The skier and cart, now stuck together, compress a 1600-N/m spring on the other side of the cart (the spring's other end is mounted securely to a wall). The skier vibrates back and forth with the cart at the end of the spring. What maximum distance did the skier and cart compress the spring? Follow the problem-solving strategy.

7.7.5 Analyze

Class or Lab *Equipment per group:* whiteboard and markers.



You have two low-friction carts on a track. You fix a metal rod on cart B and a plastic box on cart A. Cart A has a spring bumper that compresses and expands elastically. You put some sugar mixed with a few steel ball bearings in the box on cart A (see figure on the right). Using modeling clay, you adjust the total mases of the carts so they are both equal to 565 g. You push cart A so that it starts moving towards cart B (which is initially at rest). The carts collide. You take a video of the experiment and analyze it (watch the video [https://mediaplayer.pearsoncmg.com/assets/_frames.true/sci-phys-egv2e-alg-7-7-5]). The figure below shows the velocity-versus-time graph for both carts during the collision. Average velocities of the carts before and after the collision are also shown on the graph.



In the following activities take both carts as the system.

a. Take the situation at $t = 0.20$ s as the initial state, and the situation at $t = 0.45$ as the final state. Can you say that the total momentum of the system is constant in this process? If not, explain what might have caused the total momentum change. Can you say that the total mechanical energy of the system is constant in this process? If not, estimate the change of the total mechanical energy and explain into what other forms of energy it was converted.

b. At $t = 0.35$ s, cart A almost stopped moving but after that time it sped up and continued moving with constant speed. Explain the mechanism that made the cart speed up. (*Hint*: watch the video again and carefully observe what is going on during the collision.)

c. Represent the processes in the experiment using impulse-momentum and work-energy bar charts. Take both carts as the system. Draw bar charts with three states. Take the situation at time $t = 0.2$ s as the initial state, the situation at time $t = 0.35$ s as the intermediate state, and the situation at time $t = 0.45$ s as the final state. If several parts of the system are moving, use additional bar charts to account separately for each part.

d. Your friend notices that the magnitudes of the slopes of the velocity-versus-time curves for the carts during the collision are different. He argues: "Different slopes mean different accelerations during the collision. Because the masses of the carts are equal, this means that the magnitudes of the forces exerted by one cart on another are not equal during the collision. I think that, based on this experiment, we can reject Newton's third law." Do you agree or disagree with your friend? If you disagree, find the mistakes in your friend's reasoning and show that the outcome and the data obtained in this experiment are consistent with Newton's laws.

e. Make a list of the physical quantities that you can determine based on the graph and data given above, and determine their values.

7.8 Power

7.8.1 Reason

Class *Equipment per group:* whiteboard and markers.

Matt, a weightlifter, can bench press 100 kg (220 lbs). He can lift the 100 kg from a height of 0.8 m above the ground to a height of 1.3 m in 0.2 seconds. Matt wants to determine the rate at which work is done on the barbell and weights. What should he do to determine the rate at which he does work on the barbell and weights?

7.8.2 Develop a relation

Class *Equipment per group:* whiteboard and markers.

Your friend Jade brings in two motors to class. She wants to install one of them in her lawnmower but wants to make sure that she picks the best motor. Your job is to determine a rating system for the motors to determine which one will cut her grass most efficiently.

You set the motors on top of a table and attach a string and 10.0 kg bob to each of them. You see that Motor 1 pulls the bob from the floor to the top of the table in 0.032 seconds. You see that Motor 2 pulls the bob from 5 cm above the floor to the top of the table in 0.020 seconds.

a. Estimate the height of the table. Devise a physical quantity that can describe the motor's ability to pull the bob up. What is its value for the first motor? The second motor? Are the numbers reasonable? How do you know?

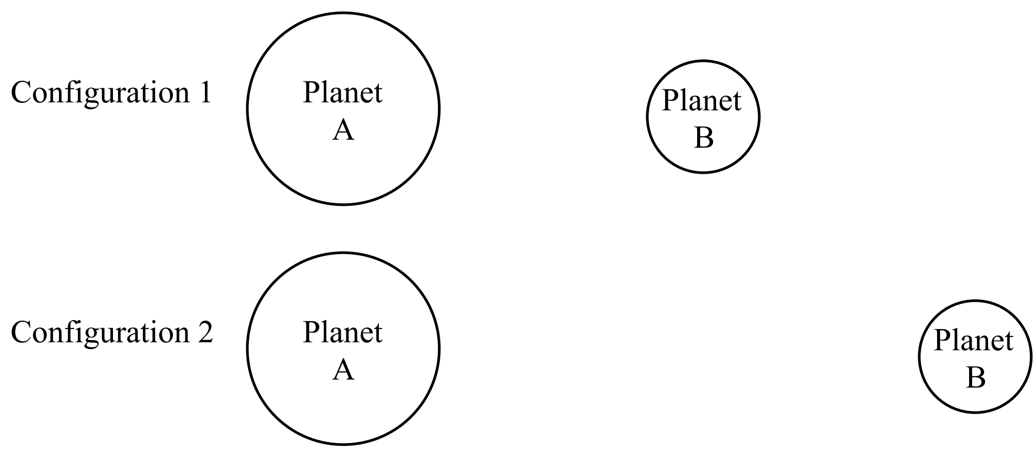
b. Read the definition of power in Section 7.8 in the textbook. How did your definition compare?

7.9 Improving our model of gravitational potential energy

7.9.1 Reason

PIVOTAL Class

In the diagram below, two different configurations of planets A and B are shown. In configuration 2, the two planets are further apart than in configuration 1. If we consider planets A and B as a system together, which configuration (1 or 2) has more gravitational potential energy? Justify your reasoning by using physics you already understand.



7.9.2 Reason

PIVOTAL Class: *Equipment per group:* whiteboard and markers.

Imagine that a 'space elevator' has been built to transport supplies from the surface of Earth to a spaceship that is located infinitely far away from Earth (so far that the gravitational interaction of the ship and Earth can be neglected). The elevator moves at constant velocity, except for the very brief acceleration and deceleration at the beginning and end of the trip.

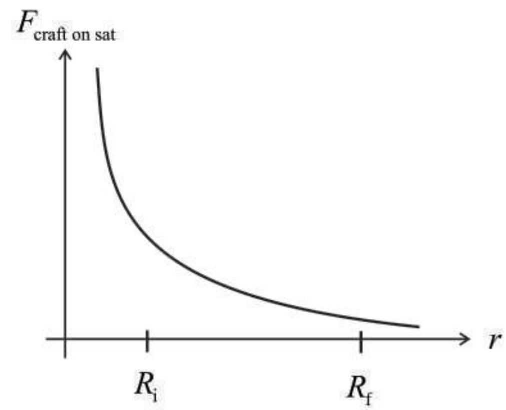
a. How much work must be done to transport the supplies from Earth's surface to the ship? To answer this question draw an $F_{\text{Elevator on Supplies}}(r)$ graph.

b. Draw a bar chart representing the process. What is the gravitational potential energy of the supplies-Earth system at the infinite distance from Earth? What sign should the gravitational potential energy of interaction have when the supplies are on Earth?

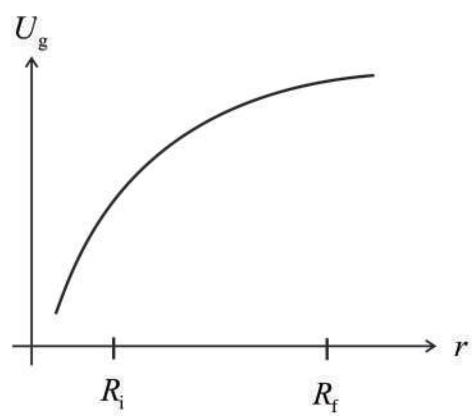
7.9.3 Represent and reason

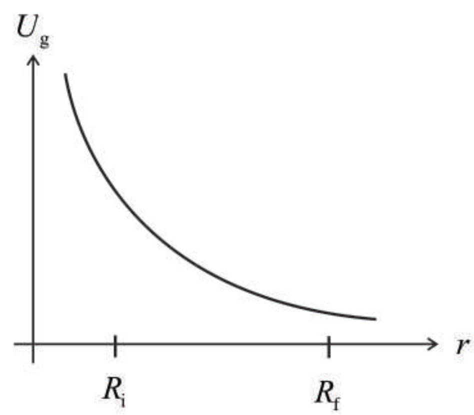
Class

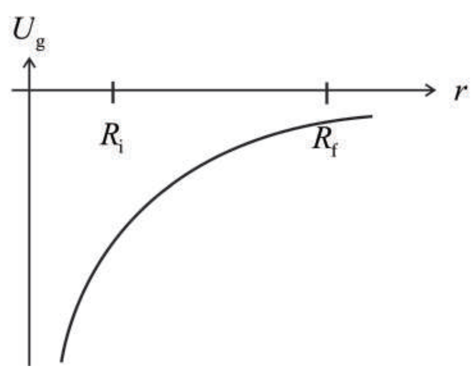
Imagine that you are trying to move a satellite very far away from Earth by using another spacecraft. The graph on the right shows how the force exerted by the spacecraft on the satellite depends on the separation between Earth and the satellite, as the satellite is pulled very slowly into space.



a. Which of the graphs below could represent the gravitational potential energy of the Earth-satellite system as a function of the separation between the two? If none of the graphs can represent U_g, explain why not.







b. Consider the interaction between the satellite and Earth. At what separation should the gravitational potential energy of the Earth-satellite system be equal to zero?

c. Which of these mathematical expressions could represent the gravitational potential energy of the Earth-satellite system? If none of them can, explain why not.

$$U_g = -\frac{GM_E m_s}{r} \qquad U_g = -\frac{Gm_s}{r} \qquad U_g = GM_E m_s r$$

7.9.4 Reason

PIVOTAL Class *Equipment per group:* whiteboard and markers.

The two expressions for gravitational potential energy look very different. The first one ($U_g = mgy$) was developed for processes with elevation changes on or near Earth's surface. Does the new expression $U_g = -\frac{GM_E m_S}{r}$ produce a similar result for such a change? Explain. Do not forget that $g = G\frac{M_E}{r^2}$.

7.9.5 (Speculative) Real-life application

Class: *Equipment per group:* whiteboard and markers.

People have considered making a satellite launch system using a rail gun, driven by a capacitor bank. (Don't worry about the technical details of this: basically imagine a big gun on Earth's surface shooting the satellite into space.) How much energy do you need to store in the capacitor bank to place a satellite in a circular orbit 200 km above Earth? What assumptions are you making in this estimate? At what speed would the satellite leave the rail gun? Does this explain why this launch system is not feasible?

a. Decide how you're going to model this system.

b. Draw an energy bar chart of the initial and final states.

c. Decide what objects are in your system and what is going to do work on your system (i.e., what's external to the system).

d. Perform relevant calculations to answer the questions.

e. What assumptions are you making? Discuss how those assumptions may affect the outcome of the launch.

7.9.6 Reason

Class: *Equipment:* Whiteboard and markers.

Talk with your group and work on a whiteboard to explain why NASA launches spacecraft from Florida rather than from (for example) Maine. Give a physics reason. (*Hint*: Spacecraft are always launched eastward. Why?)

7.9.7 Application

Class

Work with your group to estimate the Schwarzschild radius of the Sun. That is, if you wanted to turn the Sun into a black hole, what radius would you have to compress it to so that light could not escape? [*Hint*: One of the core assumptions of general relativity is that nothing can travel faster than light. The speed of light $c = 3 \times 10^8$ m/s .]

7.9.8 Reading exercise

Read Section 7.9 in the textbook and answer Review Question 7.9.