Chapter 5

Circular Motion

5.1 Qualitative dynamics of circular motion

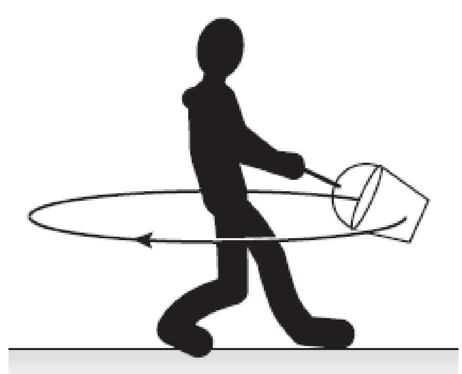
5.1.1 Observe and find a pattern

PIVOTAL ***Lab or class:*** *Equipment per group:* Bowling ball, mallet, backpack or bucket, rope or strong string

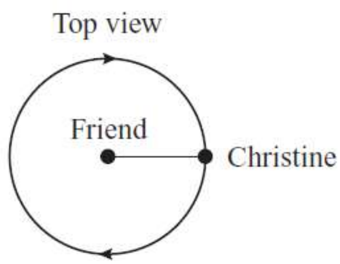
Together with your group, perform or observe the following three experiments. For each experiment, fill in the blanks in the table that follows.

a**.** Let one person in your group roll a bowling ball along a smooth floor. As the ball moves, tap it with a rubber mallet, trying to make it move in a circle. In what direction did you need to tap to make it move in a circle?

b**.** Swing a bucket attached to a rope and filled with water or sand at constant speed in a horizontal circle.



c. Imagine that Christine is wearing rollerblades; she holds one end of a rope and a friend securely holds the other end. A third person pushes strongly on Christine and she moves in a straight line. When the rope becomes taut, Christine starts moving in a circle. As Christine moves at constant speed in the circle, her friend holding the rope turns, always pulling in on her.



Fill in the table below; assume that the friction forces exerted on all three objects are negligible.

Experiment; the circling object is in bold.	**List objects that interact with the circling object.**	**Draw a top view force diagram for the circling object.**	**List forces or force components that add to zero.**	**Indicate the direction of the sum of the forces exerted on the object.**
a. Tapping a **bowling ball.**				
b. Swinging a **bucket** in a horizontal circle.				
c. Pulling a rope attached to a moving **rollerblader** so she moves in a circle.				

5.1.2 Find a pattern

PIVOTAL ***Lab or class:*** *Equipment per group:* whiteboard and markers

With your group members, review your analysis recorded in the table for Activity 5.1.1. Based on your observations and on the analysis, find a pattern for the direction of the sum of the forces exerted on an object moving at constant speed in a circle. Summarize your pattern on a whiteboard and compare what you found with another group.

5.1.3 Explain

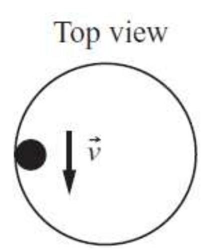
PIVOTAL ***Lab or class:*** *Equipment per group:* whiteboard and markers

Work with your group members to devise one or more explanations for the pattern in the direction of the sum of the forces exerted on an object moving at constant speed in a circle. Write your group consensus on your whiteboard.

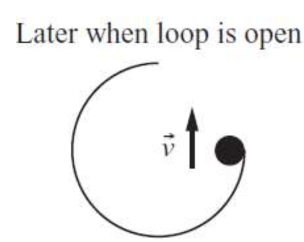
5.1.4 Test your explanation

PIVOTAL ***Lab or class:*** *Equipment per group:* whiteboard and markers, ring with removable piece and small ball [Alternative: Video experiment at https://mediaplayer.pearsoncmg.com/assets/_frames.true/sci-phys-egv2e-alg-5-1-4]

For the following testing experiment, use the pattern that you formulated in Activity 5.1.2 and the explanation you formulated in Activity 5.1.3 to predict the outcome of the experiment.



a. Inside a metal ring, roll a small ball or a marble on a smooth horizontal surface. Is the motion of the ball consistent with the pattern formulated in Activity 5.1.2? Explain.



b. Predict what will happen to the ball if after the ball rolls for a couple of turns, you remove a quarter of the ring, as shown in the figure. Justify your prediction in words and with a force diagram. Put your prediction and justification (including force diagram) on a whiteboard and compare with another group before you conduct the experiment.

c. After you make your prediction, perform the experiment to check the outcome. Discuss with your group: What judgment can you make about the idea that you're testing? Does the outcome support, prove, or disprove the idea you're testing?

5.1.5 Reading exercise

Read Section 5.1 in the textbook and answer Review Question 5.1.

5.2 Analyzing velocity change for circular motion

5.2.1 Represent and reason

***Class:** Equipment per group:* whiteboard and markers

In the activities in the previous section you learned that the sum of the forces exerted on an object moving in a circle at constant speed is pointed toward the center of the circle. Why is that? Think of the motion of the object. The speed is constant but is the velocity constant? How can you find the direction of the velocity of such an object at every instant?

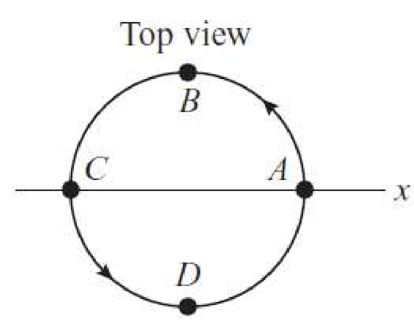
a. Work together with your group members to draw on a whiteboard the velocity vectors for such an object at four different points of the circle. What is the direction of the velocity vector? What is its magnitude?

b. What can you say about the motion of the object? Is it motion with constant velocity? If not, how can you determine the acceleration at each point in the motion? Think of the definition of acceleration ($\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$) and how you determined the direction of the acceleration in Chapter 2 for objects moving in a straight line.

c. Read Physics Toolbox 5.1 in Section 5.2 of the textbook to learn the technique for determining the direction of acceleration of an object that is not moving along a straight line.

5.2.2 Represent and reason

***PIVOTAL Class:** Equipment per group:* Whiteboard and whiteboard markers



An object moves at constant speed in a circle.

a. The task of your group is to determine the direction of its acceleration at each of the four positions shown in the illustration. Split the work among group members so that each member is responsible for one point. Work on a shared whiteboard. Use what you learned in Physics Toolbox 5.1. Make sure you take a point right before the point of interest and right after, and use a ruler to make sure the lengths of the velocity vectors remain the same and their directions are tangent to the circle.

b. Examine the findings of other members of the group. Can you agree on a pattern in the directions of the acceleration vectors? If so, what is it? Summarize your pattern on your whiteboard and compare what you found with the findings of another group

5.2.3 Explain

PIVOTAL Class: *Equipment per group:* whiteboard and markers

Have a discussion with your group: Explain how the pattern you found in Activity 5.2.2 is connected with the pattern you found in Activity 5.1.2. Does your explanation for why the sum of the forces exerted on an object moving in a circle at constant speed points toward the center of the circle match the one you constructed in Activity 5.1.3? If not, which one needs to be revised? Put your revised ideas on a whiteboard and discuss with another group.

5.2.4 Represent and reason

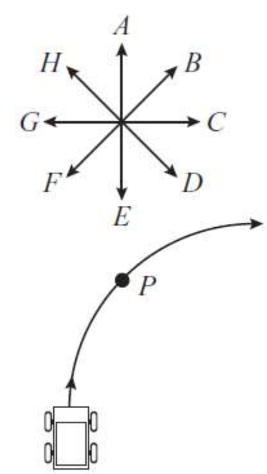
Class: *Equipment per group:* whiteboard and markers

Imagine that a golf cart moves on a level path around a curve shown in the top-view diagram to the right. For each situation below, use the graphical velocity subtraction method to estimate which arrow is closest to the direction of the cart's acceleration when passing the midpoint *P*.

a. The cart moves at constant speed.

b. The cart's speed is decreasing as it goes around the curve.

c. The cart's speed is increasing as it goes around the curve.



5.2.5 Observe and find a pattern

PIVOTAL Lab or class: *Equipment per group:* Pendulum with a very heavy bob (0.5 or 1 kg) on a string

Wrap the pendulum's string around your finger and let it swing in a vertical arc. Do you feel any change in how the string presses on your finger when the pendulum is at different points of the swing? Think about why this could be. To help answer the question, work with your group members to answer the following questions.

a. Consider different points along the arc, including the very bottom. Draw velocity vectors at those points. What can you say about the lengths of the velocity vectors at different points?

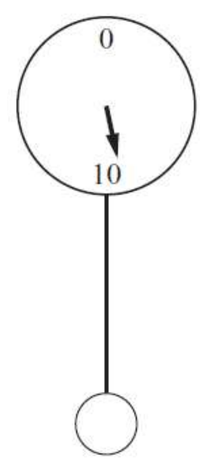
b. Use the velocity technique shown in Physics Toolbox 5.1 in the textbook to determine the direction of the acceleration of the pendulum bob at four locations: at the top of the swing on the right; at some intermediate point when the pendulum is moving down; at the bottom of the swing; and at some point on the left before the pendulum reaches the highest point.

c. What patterns do you notice? Does the acceleration of the pendulum bob always point toward the center of circle? If not, can you explain why it does not?

d. Explain why the acceleration points straight up at the bottom of the swing.

5.2.6 Test your ideas

PIVOTAL Lab or class: *Equipment per group:* Pendulum with a 1 kg bob on a string attached to a 20 N spring scale calibrated in newtons. [Note: equipment is flexible: if using a 500 g bob, use a 10 N spring scale.]



Now that you have figured out how to determine the direction of the acceleration of the pendulum bob at the bottom of its swing, use everything you have learned about circular motion so far to make a prediction of the outcome of the following experiment:

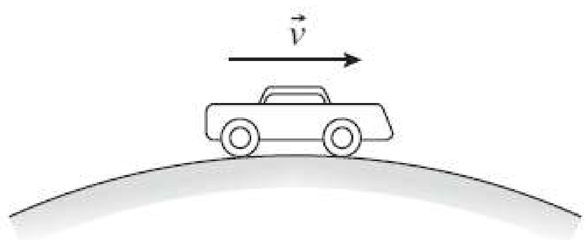
a. A 1.0-kg ball hangs from a 1.0-m long string. The other end of the string is attached to a Newton force measuring scale. The string pulls up on the ball exerting a 9.8-N force and the string and ball in turn pull down on the scale exerting a 9.8-N force—the scale reads 9.8 N. Imagine that you pull the ball to the side and release it so that the ball swings like a pendulum at the end of the string. Predict the scale reading as the ball passes directly under the scale (i.e., is it more than, less than, or equal to 9.8 N?).

b. Perform the experiment; record the outcome and compare it to the prediction. Did the outcome support the pattern?

5.2.7 Represent and reason

PIVOTAL Class: *Equipment per group:* whiteboard and markers

A battery-powered toy car moves at constant speed over the top of a circular hump, as shown in the illustration below. Working with your group, answer the questions that follow.



a. Indicate the direction of the acceleration of the car at the top of the hump.

b. Draw a force diagram for the car when passing over the top of the hump. Make sure the force arrows have the correct relative lengths.

c. Explain in words whether the results of parts a. and b. are consistent with Newton's second law.

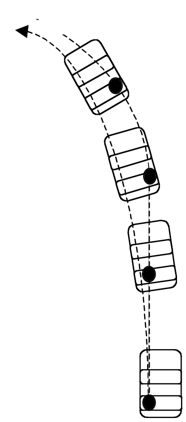
5.2.8 Reading exercise

Review Sections 5.1 and 5.2 in the textbook and answer the following question: Is the following statement true: Because an object moving in a circle at constant speed needs to change the direction of its velocity, and therefore have acceleration, the sum of the forces exerted on that object needs to be in the direction of the required acceleration. Explain your opinion with examples from the textbook sections.

5.2.9 Discuss

PIVOTAL Class: *Equipment per group:* whiteboard and markers.

Talk with your group about the following question, and draw diagrams on your whiteboard as needed: How can you reconcile the dynamics of circular motion with a feeling of being "thrown outward" when a car moves around a curved road?

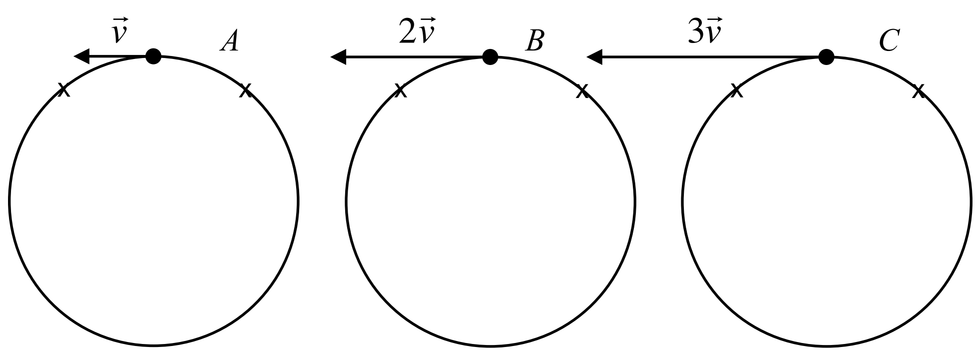


5.3 Radial acceleration and period

5.3.1 Observe and find a pattern

Class: *Equipment per group:* whiteboard and markers.

Imagine three small toy cars travel at constant speed in identical-radii horizontal circular paths (a top view is shown below). Car *A* moves at speed *v*, car *B* at speed 2*v*, and car *C* at speed 3*v*. Use the velocity technique (Physics Toolbox 5.1 in the textbook) to determine how the magnitude of the acceleration of the cars depends on their speeds. Remember that acceleration is $\Delta\vec{v}/\Delta t$ and that you need to compare the velocity change $\Delta\vec{v}$ vectors for the three speeds and also the time interval Δt needed for the velocity changes in each of the three cases.

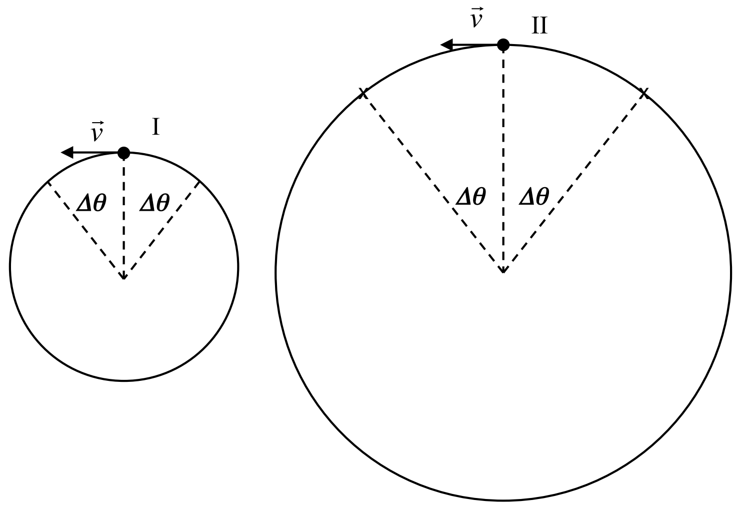


5.3.2 Observe and find a pattern

Class: *Equipment per group:* whiteboard and markers

Two small toy cars travel at the same constant speed in horizontal circular paths (a top view is shown below). Car I moves in a circle of radius *r* and car II in a circle of radius 2*r*.

a. Use the velocity technique (Physics Toolbox 5.1 in the textbook) to determine how the magnitudes of the accelerations of the cars depend on the radii of the circles. Do not forget to consider the time intervals needed for the velocity changes.



b. Combine the results from Activities 5.3.1 and 5.3.2a to write a general expression for the magnitude of the acceleration during constant-speed circular motion.

5.3.3 Test the relation

PIVOTAL Lab: *Equipment per group:* whiteboard and markers, ruler, string, objects, ring stand, arms and rods, clamp, digital scale, stopwatch, protractor.

Learning goal: Use the equipment to construct a conical pendulum and use that set-up to test Newton's second law for circular motion, namely $\frac{v^2}{r} = \frac{\sum F}{m}$.

a. First brainstorm with your group members. What physical quantities can you measure? What physical quantity could you predict with the equation in order to test it? Describe how you will model the objects, interactions, and processes you will use in your mathematical model. Construct force diagrams as appropriate.

b. Describe your experimental procedure. Include a sketch of your experimental design. Explain what steps you will take to minimize experimental uncertainty.

c. Decide what assumptions about the objects, interactions, and processes you need to make to solve the problem. How might these assumptions affect the result? Be specific.

d. What are the sources of experimental uncertainty? Which measurement is the most uncertain? How did you decide?

e. Make a numerical prediction. Be sure to show your mathematical procedure. Show your work to an instructor.

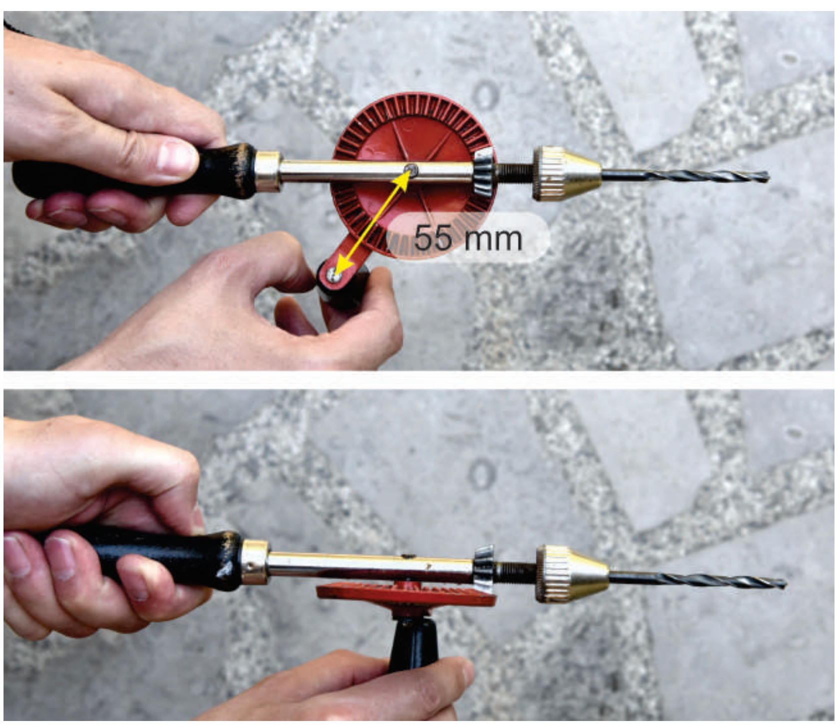
f. Perform the experiment. Record your results in an appropriate format. What is the outcome of the experiment?

g. Make sure to compare your experimentally measured and predicted values. Taking into account experimental uncertainties and the assumptions you made, decide if these two values are consistent or not. If they are not consistent, explain possible reasons for how this could have happened.

5.3.4 Observe and find a pattern

Class: *Equipment per group:* whiteboard and markers

The figure below shows a hand-held drill and the video [https://mediaplayer.pearsoncmg.com/assets/_frames.true/sci-phys-egv2e-alg-5-3-4] shows how the device is used. The video is recorded at 30 images per second. Using the video and the data in the figure, complete the following activities (you will need a ruler to solve this problem).



a. Estimate from the video the period of the crank T_{crank} and the period of the drill bit T_{drill}. Estimate from the figure dimensions such as the radius of the large toothed wheel, the radius of the small toothed wheel, and the diameter of the drill bit. Compare the ratio $T_{\text{crank}} / T_{\text{drill}}$ to the ratios between the different dimensions (including the dimension shown in the figure). Which ratio is equal to the ratio between the periods? Does it make sense? Explain.

b. Based on the quantities that you determined in **a.**, determine the speed of the teeth on the large wheel and the speed of the teeth on the small wheel. How do they compare? Does it make sense? Explain.

5.3.5 Reading exercise

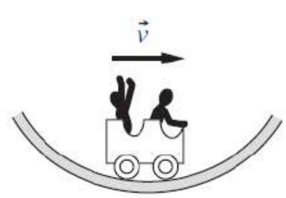
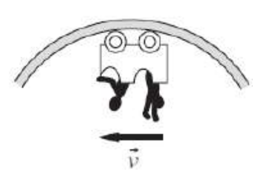
Read Section 5.3 in the textbook. Explain in your own words why the expression for radial acceleration $a_r = \dfrac{v^2}{r}$ makes sense.

5.4 Skills for analyzing processes involving circular motion

5.4.1 Represent a process in multiple ways

PIVOTAL Class: *Equipment per group:* whiteboard and markers

Work together with your group on a whiteboard. Below are three scenarios involving a rollercoaster car running on a smooth nearly-frictionless track.

I	II	III
The roller coaster car glides at constant speed along a frictionless, level track.	The roller coaster car moves along a frictionless circular dip in the track.  	The roller coaster car moves inverted along the top of a frictionless loop-the-loop. 

For each of the scenarios described above (try arranging all three side-by-side on your board):

a. Draw or re-draw a diagram of the scenario on your whiteboard.

b. Indicate the direction of the radial acceleration $\vec{a}_r$ of the car on your diagram.

c. Draw a force diagram for the car, labeling each force with two subscripts to show what object is exerting a force on the object of interest. Indicate the direction of the radial axis on your diagram.

d. Apply Newton's second law $\vec{a}_r = \frac{\sum \vec{F}}{m}$ to the car, making sure you are consistent with how you chose your radial axis.

Share your ideas with another group and resolve any discrepancies.

5.4.2 Regular problem

PIVOTAL Class: *Equipment per group:* whiteboard and markers

Suppose a loop in a roller coaster track has a 16-m diameter. How fast must the upright roller coaster car move across the top of the loop so that the force that the seats exert on its riders is half the force that Earth exerts on them? Be sure your solution includes all the problem solving steps.

Sketch and translate • Sketch the situation described in the problem statement. Label it with all relevant information. • Choose the system and a specific position to analyze its motion. • Identify the unknown that you need to find and label it with a question mark.	
Simplify and diagram • Decide if the system can be modeled as a point-like object.	

• Determine if the constant speed circular motion approach is appropriate. • Indicate with an arrow the direction of the object's acceleration as it passes the chosen position. • Draw a force diagram for the system at the instant it passes that position. • On the force diagram, draw an axis in the radial direction toward the center of the circle.	
Represent mathematically • Convert the force diagram into the radial *r*-component form of Newton's second law. • For objects moving in a horizontal circle (unlike this example), you may also need to apply a vertical *y*-component form of Newton's second law.	
Solve and evaluate • Solve the equations formulated in the previous two steps. • Evaluate the results to see if they are reasonable (the magnitude of the answer, its units, limiting cases).	

5.4.3 Equation Jeopardy 1

Class: *Equipment per group:* whiteboard and markers

A situation involving circular motion is described mathematically below.

$$(50\text{ kg}) \times v^2/(12\text{ m}) = (900\text{ N}) - (50\text{ kg}) \times (9.8\text{ N/kg})$$

a. Construct a sketch of a situation the equation might describe.

b. Write in words a problem for which the equation could be a solution.

5.4.4 Equation Jeopardy 2

Class: *Equipment per group:* whiteboard and markers

A situation involving circular motion is described mathematically below.

$$(50\text{ kg})\times v^2/(12\text{ m}) = (200\text{ N}) + (50\text{ kg})\times(9.8\text{ N/kg})$$

a. Construct a sketch of a situation the equation might describe.

b. Write in words a problem for which the equation could be a solution.

5.4.5 Regular problem

Class: *Equipment per group:* whiteboard and markers

The third turn at the Alamo Cart Dirt Speedway is tilted at 30° and has a 240-m radius. Determine the speed of a Cart car going around that turn, assuming the car has no help from friction.

5.4.6 Regular problem

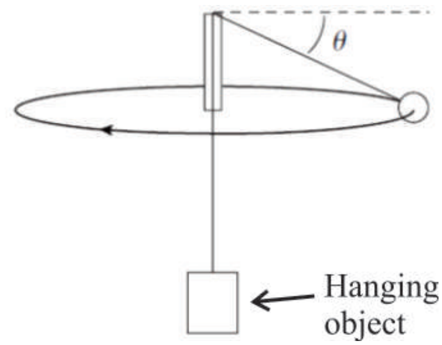
Class: *Equipment per group:* whiteboard and markers

In an old-fashioned amusement park ride, passengers stand inside a 5-m diameter hollow steel cylinder with their backs against the wall. The cylinder begins to rotate about a vertical axis, then the floor on which the passengers are standing suddenly drops away. If all goes well, the passengers will “stick” to the wall and not slide down. If clothing has a coefficient of static friction of 0.6–1 and a coefficient of kinetic friction of 0.4–0.7, what is the minimum angular velocity in rpm for which the ride is safe?

5.4.7 Application experiment

Lab: *Equipment per group:* whiteboard and markers, ruler, string, objects, glass tube, digital scale, stopwatch, tape.

Apply your knowledge of circular motion to estimate the mass of the vertically hanging object using the whirly-gig apparatus shown in the diagram on the right without putting it on a scale. In the whirly-gig apparatus a string connecting two objects (the hanging object and the swinging ball) passes through a smooth glass tube. A person holds and the tube and sets the ball in circular motion by whirling the top of the tube in a small horizontal circle. The hanging object provides the necessary tension in the string.



a. Describe your experimental procedure. What data do you need to collect prior to performing the experiment?

b. Decide what assumptions about the objects, interactions, and processes you need to make to solve the problem. How might these assumptions affect the result? Be specific. Considering one of the relevant assumptions, evaluate how making the assumption will affect the results.

c. Draw force diagrams for the two objects in your system (recall your assumptions). Include appropriate sets of coordinate axes. Use the force diagrams to devise a mathematical procedure to determine the mass of the object.

d. What are the sources of experimental uncertainty? Which measurement is the most uncertain? How did you decide? Explain what steps you will take to minimize experimental uncertainty.

e. Collect necessary data, then perform the experiment and record your results in an appropriate format. What is the outcome of the experiment?

f. Now use the scale to measure the mass of the hanging object.

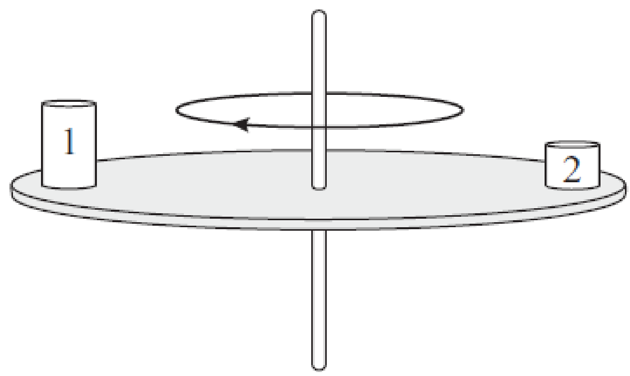
g. Compare the two values you obtained for the mass of the object. Taking into account experimental uncertainties and the assumptions you made, decide if these two values are consistent or not. If they are not consistent, explain possible reasons for how this could have happened.

h. Describe the shortcomings you noticed in the experiments. Suggest specific improvements.

5.4.8 Test the relation

Lab or class: *Equipment per group or one set up for class:* Two objects of different mass with similar surfaces, rotating platform

You have two objects of mass 500 g and 100 g with identical bottom surfaces (the masses of the objects you have might differ from these values). Imagine that you place them on a rotating platform as shown in the figure. The platform can turn with increasing speed. Work with your group members to answer the questions that follow.



a. Draw force diagrams for each of the objects.

b. Apply the vertical *y*-component form of Newton's second law to each object.

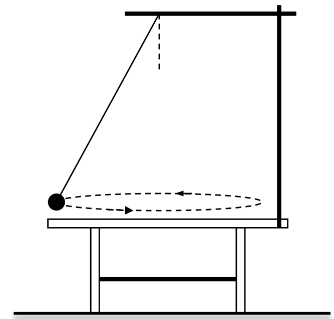
c. Apply the radial *r*-component form of Newton's second law (Equation 5.6).

d. Use the results of your analysis to predict which object flies off the platform first or if they fly off at the same time.

e. Then conduct the experiment and compare the outcome to the prediction. Was your prediction consistent with the outcome of the experiment? Explain. What might have caused any discrepancy? (Hint: Think about the assumptions that you made.)

5.4.9 Application experiment

Lab: *Equipment per group:* whiteboard and markers, conical pendulum set-up (large ring stand with long side arm and clamps, string, bob – see figure), spring scale (alternative: force probe with laptop computer), digital scale.



Design two independent methods to determine the sum of the forces $\sum \vec{F}$ exerted on the bob of a conical pendulum by other objects as the bob moves at constant speed in a horizontal circle of a chosen radius. *Hint:* For more accurate measurements, use a circle with a large radius. Be sure to address the following points for *each* method.

a. Write a description in words of each of the methods you will use to determine the sum of the forces that other objects exert on the bob.

b. Draw a labeled sketch of your set-up.

c. Draw a force diagram (as needed).

d. Write the physical quantities that you will measure and quantities you will calculate.

e. Write the mathematical procedure you will use to determine the net force.

f. List any assumptions that need to be made in your mathematical procedure.

g. List sources of experimental uncertainty and ways to minimize them.

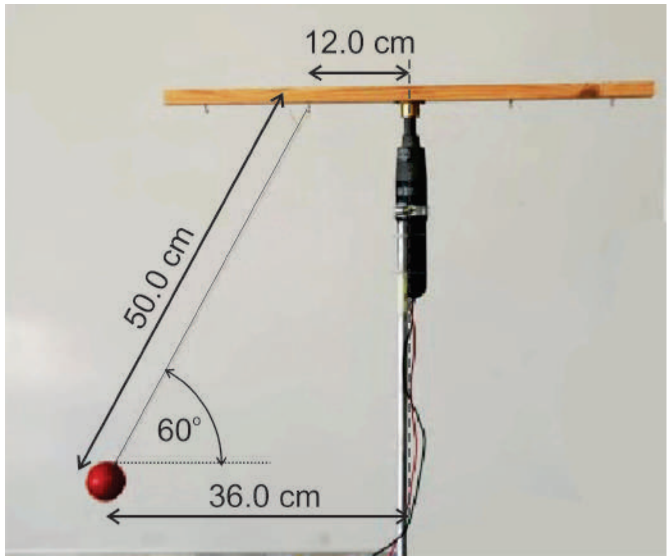
h. Then perform the two experiments, record your results, calculate your uncertainties, and compare the two results. Discuss how assumptions and experimental uncertainties contribute to the discrepancy between the outcomes of the two experiments.

5.4.10 Evaluate the solution

PIVOTAL class: *Equipment per group:* whiteboard and markers

A group of students were solving the following problem:

You attached a sphere to a wooden rod by a light string. The rod and the sphere rotate at constant speed around the vertical axis (see the photo in the figure below). The photo shows the instant when the sphere is in the plane perpendicular to the direction of sight. Determine the period of the revolution of the sphere using the data in the figure.



Here is the solution that the students devised:

The vertical component of the force exerted by the string on the sphere should be equal to the force exerted by Earth on the sphere:

$$F_{\text{S on Sp}\,y} = F_{\text{S on Sp}} \sin 60° = F_{\text{E on Sp}}$$

Therefore the force exerted by the string on the sphere is

$$F_{\text{S on Sp}} = \frac{F_{\text{E on Sp}}}{\sin 60°} = \frac{mg}{\sin 60°}$$

The force exerted by the string on the sphere also makes the sphere rotate at constant speed. Therefore:

$$F_{\text{S on Sp}} = m\frac{v^2}{R} \Rightarrow$$

$$\frac{v^2}{R} = \frac{g}{\sin 60°} \Rightarrow v = \sqrt{\frac{Rg}{\sin 60°}} = \sqrt{\frac{(0.36\text{ m} - 0.12\text{ m}) \times (9.8\text{ m/s}^2)}{0.5}} = 2.17\text{ m/s}$$

$$T = \frac{2\pi R}{v} = \frac{2\pi \times (0.36\text{ m} - 0.12\text{ m})}{2.17\text{ m/s}} = 0.70\text{ s}$$

Evaluate the solution. Identify any mistakes and provide the correct solution.

5.5 The law of universal gravitation

5.5.1 Reason

PIVOTAL Class: *Equipment per group:* whiteboard and markers

a. Newton found that the radial acceleration of the Moon when circling Earth was $2.69\times10^{-3}\,\mathrm{m/s^2}$. This finding was based on a time for one orbit (27.3 days) and its orbital radius (about $3.8\times10^{8}\,\mathrm{m}$ —the distance between the centers of Earth and the Moon). Work together with your group to confirm the value of the Moon's acceleration with your own calculation.

This tiny number is exactly $1/3600$ times the free-fall acceleration on Earth's surface. Interestingly, the radius of the Moon's orbit about Earth is 60 times greater than the radius of Earth $\left(6.4\times10^{6}\,\mathrm{m}\right)$.

b. Newton assumed that the Moon moves in a circular orbit due to the interaction with Earth.

Estimate the gravitational force exerted by Earth on the Moon. Draw a force diagram for the Moon. Apply Newton's second law to the Moon. (What type of motion is the Moon undergoing?)

c. Now Newton is sitting under a tree holding an apple. Estimate the gravitational force exerted by Earth on the apple in Newton's hand. Draw a force diagram for the apple and explain how you know the magnitude of the force exerted by Earth on the apple. How far is the apple located from the center of mass of Earth?

d. Imagine (as Newton must have done) that you could relocate the Moon to the surface of Earth. Imagine holding the Moon in your hand (very compressed) like an apple. Estimate the force exerted by Earth on the Moon.

e. What relation between the force that Earth exerts on an object and the distance between the centers of Earth and the object could Newton propose based on these data? Explain your reasoning so another student can understand.

5.5.2 Explain

PIVOTAL Class: *Equipment per group:* whiteboard and markers

Data on the accelerations of objects of different masses falling from small heights in vacuum tubes on Earth show that all of them fall with the same acceleration, 9.8 m/s^2. Newton assumed that in the absence of air, the only force causing this acceleration is the gravitational force Earth exerts on the falling object. What relation between the force exerted by Earth on an object and the mass of the object could he propose based on these data? Explain your reasoning so another student can understand.

5.5.3 Explain

PIVOTAL Class: *Equipment per group:* whiteboard and markers

According to Newton's third law, the force that a falling object exerts on Earth is equal in magnitude to the force that Earth exerts on the object. What relationship between the gravitational force that Earth exerts on a falling object and the mass of Earth could Newton propose based on his own third law? Explain your reasoning so another student can understand.

5.5.4 Observe and find a pattern

Class: *Equipment per group:* whiteboard and markers

Find a relationship between the periods and radii of the orbits of the planets. Use the Internet. Find the average radius of each of the planetary orbits about the Sun and the time that each planet takes to make one complete revolution around the Sun (the orbital period). Find a mathematical relationship between the periods and the radii from the data you've collected.

Note: From extensive observations of the motions of the planets, medieval astronomers had figured out that all the observable planets were traveling in roughly circular orbits around the Sun and they had precisely measured the periods, and could roughly estimate the radius of each circle. These data are what Kepler used to devise empirical relationships that were consistent for all the observable planets.

5.5.5 Test your explanation

Class: *Equipment per group:* whiteboard and markers

Using reasoning similar to that above, Newton decided that the gravitational force that Earth of mass M exerts on any object of mass m is directly proportional to the product of the two masses and inversely proportional to the square of the distance r between the centers of the objects $\left(F_g = GMm/r^2\right)$ where G is some constant number unknown to Newton—it was found later to be $G = 6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}$.

Kepler, somewhat before Newton's time, devised several empirical relationships describing planetary motion using observational data collected by other astronomers. According to Kepler's third law, the period T squared of a planet about the Sun divided by the cube of the mean distance r of the planet from the Sun equals the same constant for all of the planets—that is, $T^2/r^3 = K$, a constant. Use Newton's law of gravitation and his second law of motion as applied to circular motion to derive Kepler's third law.

5.5.6 Evaluate

Class: *Equipment per group:* whiteboard and markers

James thinks that the weight of a person on the Moon is less than on Earth because the Moon is farther from the center of Earth than is the surface of Earth where we normally weigh ourselves. What physics-based reasons would he have for such an opinion? Do you agree or disagree with him? If you disagree, what is your explanation?

5.5.7 Real-life application

Class: *Equipment per group:* Whiteboard and whiteboard markers

Advise NASA: How fast does the space shuttle have to be traveling to maintain a circular low-Earth orbit, 200 km above Earth's surface? How many minutes does it take to make one complete orbit of Earth at this altitude?

5.5.8 Reason

PIVOTAL ***Class:*** *Equipment per group:* whiteboard and markers

Sally Ride floats in the space shuttle as it moves in a circular orbit about Earth. If she stands on a bathroom scale, it reads zero. Does this mean Earth is not exerting any force on her? If not, explain why the scale reads zero. Use pictures and diagrams to help you.

5.5.9 Application

Class: *Equipment per group:* whiteboard and markers

Calculate the radius of the geostationary orbit. This is the orbit above Earth's equator in which the object, rotating about Earth, appears motionless for an observer on Earth.

5.5.10 Represent and Reason

Class: *Equipment per group:* whiteboard and markers

Use your knowledge of gravitation to understand and explain the force that Earth exerts on an object on its surface. Useful info: Earth's mass is 5.98×10^{24} kg, the radius of Earth at the equator is 6.38×10^{6} m.

a. Find the force exerted by Earth on a 70 kg man standing on its surface.

b. Calculate the gravitational acceleration of freely falling objects at Earth's surface (commonly referred to as "*g*".)

c. If the 70 kg man in part a. was to stand on a scale on the equator, what difference would there be in the scale reading as compared to the same 70 kg man standing on the same scale at the geographic South Pole? (*Hint*: We're trying to factor in the effect of Earth's rotation on the scale reading. Start by drawing a force diagram for the man and indicate the direction of the radial acceleration if you're stuck. Does the man have a radial acceleration if he is standing on the geographic South Pole?)

5.5.11 Evaluate

Class: *Equipment per group:* whiteboard and markers

In the movie *Inception* the main heroes are in a bus that is falling from a bridge. You can see them floating freely inside the bus. One of them says that they are in zero gravity. Do you agree or disagree? If you disagree, how can you explain the floating? Discuss with your group members possible reasons for why he would he say this.

5.5.12 Apply/Estimate

Class: *Equipment per group:* whiteboard and markers

Some people believe that the end of the world is coming soon. One hypothesis is that when all the planets in the solar system align, this event would produce a great gravitational pull on Earth, causing a catastrophe. A quick search on the NASA website can reveal that there will be no alignment of the planets in the next few decades. Therefore, there is no need to panic.

If all the planets *did* align, which arrangement would result in the greatest gravitational pull exerted on Earth? (Think carefully about the relative positions of the planets in their orbits and draw them on a whiteboard.) Estimate how big the sum of the forces exerted on Earth due to this arrangement would be. (There are a *lot* of calculations to do so distribute your resources. You could assign each group to calculate one of the forces and then pool your results. Think about how you'd quickly evaluate each group's calculation to make sure it is correct.) Once you have a complete answer, how does this combined force exerted by the planets compare to the force exerted by the Sun on Earth? (How many times greater is it?)

In addition, estimate how large the pull of the Milky Way (our galaxy) is on Earth? What assumptions do you need to make to make this estimation?

5.5.13 Reading exercise

Read Section 5.5 in the textbook and answer Review Question 5.5.