Chapter 2

Kinematics: Motion in One Dimension

2.1 What is motion?

2.1.1 Describe

Class: *Equipment per group:* none.

Each member of your group plays a role in the story described below. Assign roles and discuss the answers to the questions. Then get up and enact the story for the rest of the class.

Story: A person sits in the passenger seat of a car that is traveling along a street. Describe the person's motion as seen by each of the following observers:

a. another person sitting in the backseat of the car;

The person in the front seat is not moving relative to another person sitting in the backseat of the car.

b. a pedestrian standing on the sidewalk as the car passes; and

The pedestrian standing on the sidewalk sees the person in the car is moving in the direction the car is traveling.

c. the driver of a second car moving in the same direction and passing the first car.

The driver of the second car sees the person in the first car moving backwards relative to them.

2.1.2 Describe

Class: *Equipment per group:* none.

Each member of your group plays a role in the story. Assign roles and discuss the answers to the questions. Then get up and enact the story for the rest of the class.

Story: A person stands near a bus stop. Describe the standing person's motion as seen by the following observers:

a. a person sitting in an approaching bus;

The person at a bus stop is moving toward the person sitting in the approaching bus.

b. a person riding in a car moving away from the bus stop; and

The person at a bus stop is moving backwards away from the person riding in a car moving away from the bus stop.

c. another person standing at the bus stop.

The person at the bus stop is not moving relative to another person standing at the bus stop.

2.1.3 Explain

Class: *Equipment per group:* none.

Discuss with your group members your analyses for Activities 2.1.1 and 2.1.2 and answer the questions that follow. Did you agree on your answers? If not, what were the contentious points?

a. Do any observers say that the person sitting in the passenger seat of the car in Activity 2.1.1 was moving? Explain.

The observers in 2.1.1b and 2.1.1c said that the passenger in the car was moving since the passenger was changing position relative to those observers.

b. Do any observers say that the person sitting in the passenger seat of the car in Activity 2.1.1 was not moving? Explain.

The observer in 2.1.1a said that the passenger in the car was not moving because the passenger was NOT changing position relative to the observer.

c. Do any observers say that the person standing near a bus stop in Activity 2.1.2 was moving?

The observers in 2.1.2a and 2.1.2b said the person at the bus stop was moving because the person at the bus stop was changing position relative to those observers.

d. Do any observers say that the person standing near a bus stop in Activity 2.1.2 was not moving?

The observer in 2.1.2c says that the bus stop person was not moving as the bus person's position was not changing position relative to the observer.

e. Based on your answers in parts a. through d., explain what it means when someone says an object is "moving." List all explanations on the board and discuss which is the most comprehensive.

If an observer says an object is "moving," it means the object's position is changing relative to THAT observer during a certain time interval.

2.1.4 Describe

Class: *Equipment per group:* none.

Work with your group members to devise several situations in which

a. you *can* model Earth as a point-like object, and

Cases where you can model Earth as a point-like object involve scenarios in which the length scales are much larger than the diameter of Earth. For example, studying Earth as an object orbiting the sun, it is reasonable to model Earth as a point-like object.

We also model Earth as a point-like object when considering the gravitational interaction between an object and Earth. We know, because of the spherical symmetry of Earth it is a good approximation to assume all of Earth's mass is concentrated at Earth's center and exerts a single force on, say, an apple on Earth's surface.

b. you *cannot* model Earth as a point-like object.

Tidal forces (forces that cause the ocean tides) cannot be modeled by assuming Earth is a point-like object.

What are the features that the part a. situations have in common and the part b. situations have in common? Can you come to a consensus in your group?

2.1.5 Reading exercise

Read Section 2.1 in the textbook and answer Review Question 2.1.

2.2 A conceptual description of motion

2.2.1 Observe

PIVOTAL ***Lab or class:*** *Equipment per group:* metronome (download a metronome app.) or any device to keep track of time in seconds, low-speed battery-operated car, sugar packets (or any other marking device), meter stick (or any other length-measuring device), whiteboard and markers.

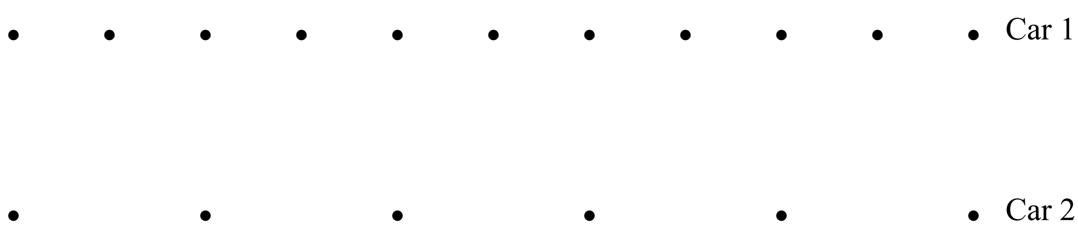
This activity requires collaboration and coordination of all group members. Set a metronome to about one beat per second. Person 1 turns on a battery-operated toy car and releases it to roll across the floor. Person 2 places sugar packets on the floor at the points where the car is located at every blip of the metronome (instead of sugar packets you can use anything else that will allow you to mark the floor where the car was every second, be creative!). Do not try to put the sugar packets where the car was just released. After about 5 to 7 blips, stop the car and draw a sketch showing the locations of the sugar packets as dots. Discuss with the group members how you can use the dots to describe the motion of the car. After you come to a consensus, draw your representations on the board and share it with the class. [If you do not have a toy car, you can use a hard ball such as a billiard ball or bowling ball that you roll on a smooth floor.]

Students should conclude that the object (car or bowling ball) is moving at a constant rate. Key points for additional discussion: a. Ask them how do they know that the rate is constant? Answer: the spacing between the sugar packets is equal. b. Not all students will conclude that the rate is constant. They will point to the different distances between the sugar packets and argue it is speeding up/slowing down. This should initiate a discussion: should they (the students) be worried about the variation in the spaces between the sugar packets? Is this an indication of change of rate or is it a reflection of your (the student's) ability to drop sugar packets perfectly precisely? This can be leveraged into a discussion about experimental uncertainties.

2.2.2 Represent and reason

Class: *Equipment per group**:*** none.

You have two battery-operated toy cars that you can release simultaneously on a smooth floor and a metronome set to 1-second intervals. You and a friend each walk next to one of the cars, and at every blip of the metronome, you place a sugar packet at your car's location. The dots in the figure below represent the locations of the packets for the two cars. The cars start simultaneously at the dot on the left and move to the right.



Discuss with your group members how to answer the questions below. Make sure that you can defend your point of view using the evidence presented in the diagrams.

a. Were the cars ever next to each other? If so, where?

Only at time zero—the first positions on the very left. It appears that they are at the same positions at other times—but are NOT. For example, the third dot for car 1 and second dot for car 2 were at different times.

b. If there were a passenger in car 1, how would the passenger describe the motion of car 2?

Car 2 was moving ahead of you at a constant rate relative to you, the observer in car 1.

c. If there were a passenger in car 2, how would the passenger describe the motion of car 1?

Car 2 was moving backwards behind you at a constant rate relative to you, the observer in car 1.

2.2.3 Observe

PIVOTAL ***Lab or class:*** *Equipment per group:* metronome or any device to keep track of time in seconds, any light/soft rubber ball or under-inflated basket ball, sugar packets (or any other marking device), meter stick (or any other length-measuring device), whiteboard and markers.

This activity requires collaboration and coordination of all group members. One group member sets a metronome to 1-second intervals. Another person places a flexible ball such as a hollow rubber ball at rest on the floor (The ball should be flexible enough to change shape a little when on a surface.) This person abruptly pushes on the ball once with a ruler so that the ball rolls away from the ruler, moving with considerable speed. The third group member moves beside the ball and places sugar packets on the floor to indicate positions of the ball every second after the ruler no longer touches the ball. Discuss together: describe the motion of the ball in simple words, draw a sketch representing the sugar packets with dots, and describe the relative distance between the packets. How does the distance between the packets correspond to the observed motion of the ball?

Students should conclude the ball is slowing down. Ask similar questions as 2.1.1: How do you know it is slowing down? Answer: The gaps between sugar packets get smaller and smaller.

2.2.4 Explain

Class: *Equipment per group:* none.

Examine Figure 2.2 in the textbook. Explain the changes in the light traces of the LED in each experiment. In particular:

a. What can the length of the light trace tell you about the motion of the cart?

Since the light is on for a fixed/known and repeatable amount of time, the length of the light trace can tell you how fast the cart is moving.

b. If each subsequent light trace gets shorter, what does that tell you about the motion of the cart?

The cart is slowing down.

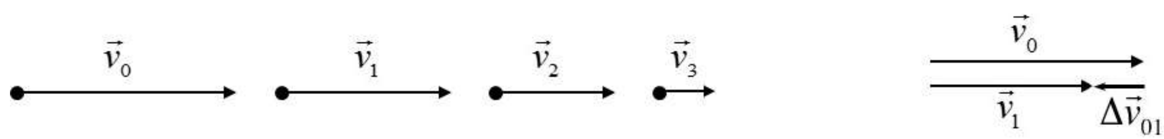
c. If each subsequent light trace gets longer, what does that tell you about the motion of the cart?

The cart is speeding up.

2.2.5 Represent and reason

PIVOTAL ***Class:*** *Equipment per group:* none.

The illustration below relates to the experiment you performed with the flexible ball in Activity 2.2.3. The dots represent the locations of the ball measured each second. The arrows represent the direction of motion and how fast the ball was moving (we call them *velocity arrows*). Consider velocity arrows 0 and 1. Move them side by side with their tails at the same horizontal position. Decide what change arrow $\Delta\vec{v}_{01}$ you would have to add to arrow 0 to make it the same length as arrow 1. Repeat for arrow 1—what change arrow is needed to change it into arrow 2, and what change arrow is needed to change arrow 2 into arrow 3? We call these *velocity change arrows*.

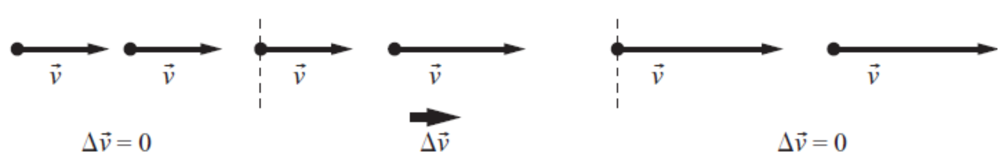


Use the Physics Tool Box 2.1 to learn how to represent motion using qualitative motion diagrams.

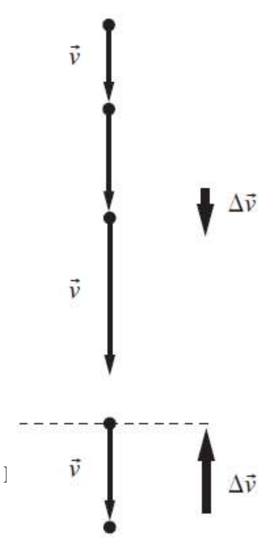
2.2.6 Represent and reason

***PIVOTAL* Class:** *Equipment per group:* none.

The illustration below is a motion diagram for an object. Remember that the dots represent the object's position after equal time intervals. Work with your group members to describe the object's motion in words by devising a story that is consistent with this diagram. Note that the process has three distinct parts: vertical dashed lines separate the parts. Share your story with another group.



For the horizontal motion diagram: the object could be a car first moving at constant speed, then speeding up and finally moving at a higher constant speed.



2.2.7 Represent and reason

Class: *Equipment per group:* none.

The illustration to the right is a motion diagram for an object. Work with your group members to describe the object's motion in words by devising a story that is consistent with this diagram. Note that the process has two distinct parts: the horizontal dashed line separates the parts. Share your story with another group.

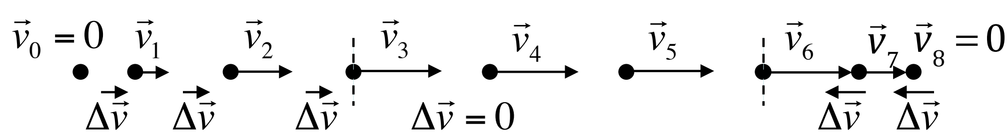
A ball is in free fall when it arrives at a pile of leaves which opposes its downward motion and causes its downward motion to stop.

2.2.8 Represent and reason

Class: *Equipment per group:* whiteboard and markers.

A car stops for a red light. When the light turns green, the car moves forward for 3 s at a steadily increasing speed. The car then travels at constant speed for another 3 s. Finally, when approaching another red light, the car steadily slows to a stop during the next 3 s. Using the whiteboard and markers, work with your group members to draw a motion diagram that describes this process. What difficulties did you encounter? What parts of the activity led to discussions? Share your diagram with another group.

Solution:



2.2.9 Reading exercise

Read Section 2.2 in the textbook and answer Review Question 2.2.

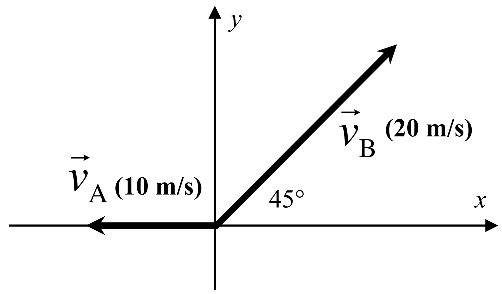
2.3 Operations with vectors

2.3.1 Read and analyze

Work through Section 2.3 in the textbook and discuss with your group members what a vector is and how operations with vectors are different from operations with numbers.

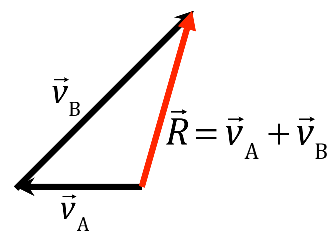
2.3.2 Practice

Class: *Equipment per group:* whiteboard and markers, meter stick, protractor.

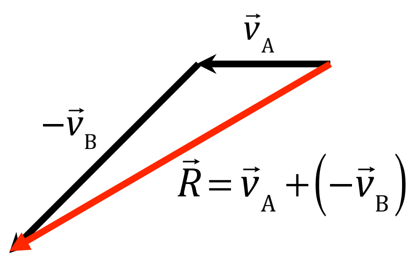


Do this together with your group on a whiteboard. For the velocity vectors $\vec{v}_\mathrm{A}$ and $\vec{v}_\mathrm{B}$ shown in the figure to the right, use a meter stick and markers to draw the vectors head-to-tail to find the magnitude and direction of

a. the sum of the two vectors $\vec{v}_\mathrm{A} + \vec{v}_\mathrm{B}$, and

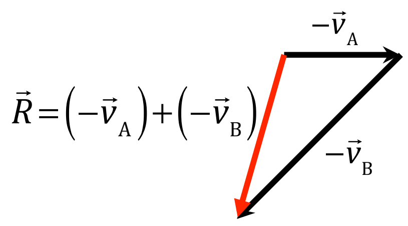
Joining the vectors head to tail, the red vector represents the resultant: $\vec{R} = \vec{v}_\mathrm{A} + \vec{v}_\mathrm{B}$. If students draw the two arrows to scale and at approximately the right angle, they should find the length of R to be about 14.7 m/s	

b. the difference of the two vectors: $\vec{v}_\mathrm{A} - \vec{v}_\mathrm{B}$.

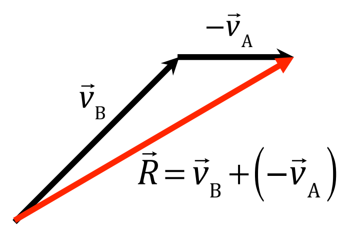
One way to think about this is: Subtracting $\vec{v}_\mathrm{B}$ from $\vec{v}_\mathrm{A}$, is equivalent to adding a $-\vec{v}_\mathrm{B}$ to $\vec{v}_\mathrm{A}$. In the diagram, the red vector represents the resultant: $\vec{R} = \vec{v}_\mathrm{A} + \left(-\vec{v}_\mathrm{B}\right)$. The length of R should be about 28 m/s	

Next, use your answers from parts a. and b. to find the magnitude and direction of

c. $-\vec{v}_\mathrm{A} - \vec{v}_\mathrm{B}$, and

This is exactly the opposite of ***a.***: $\vec{R} = \vec{v}_A + \vec{v}_B$.	

d. $\vec{v}_B - \vec{v}_A$.

This is the reverse of ***b.*** $\vec{R} = \vec{v}_A + (-\vec{v}_B)$.	

(Note: to get accurate results, you need to draw your vectors to scale.)

2.3.3 Practice

PIVOTAL Class: *Equipment per group:* whiteboard and markers, meter stick, protractor.

On your group's whiteboard (or in your notebook) draw arbitrary vectors $\vec{A}$, $\vec{B}$, $\vec{C}$, $\vec{D}$, and $\vec{E}$ that have different magnitudes and directions.

a. Brainstorm with your group about what operations with vectors you know. Consult textbook Section 2.3. Make a list.

b. For each operation, draw at least two examples on the board using your vectors A to E. Do all group members agree with the examples?

c. Invite another group to evaluate your examples while you are evaluating theirs.

d. Based on your evaluations, decide if you need to revise the examples.

2.3.4 Evaluate

Class: *Equipment per group:* none.

Discuss with your group how finding the direction and magnitude of the $\Delta\vec{v}$ arrow on the motion diagram relates to an operation with vectors. Can you use vector addition to find $\Delta\vec{v}$? Can you use vector subtraction?

Students should realize that vector operations in one dimension are the same: vectors are added head to tail. $\Delta\vec{v}$ *can be found either by subtraction:* $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$, *or by addition: rearranging* *the equation:* $\vec{v}_2 = \vec{v}_1 + \Delta\vec{v}$ *so then the operation becomes "what must I add to* $\vec{v}_1$ *to achieve the resultant* $\vec{v}_2$*?"*

2.3.5 Reading exercise

Read Section 2.3 in the textbook and answer Review Question 2.3.

2.4 Quantities for describing motion

2.4.1 Explain

Class: *Equipment per group:* none.

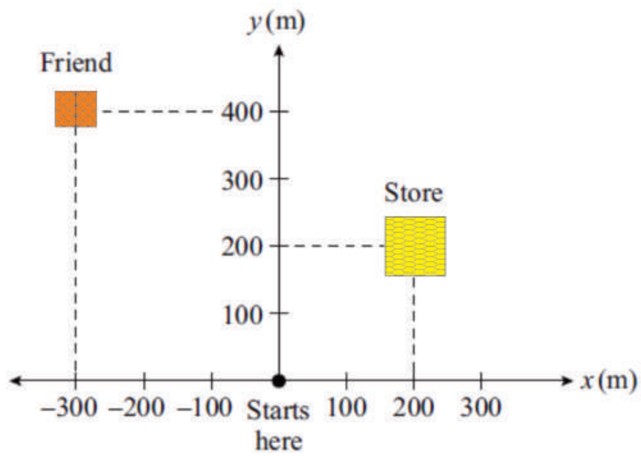
Work with Chapter 1 in the textbook to learn what a physical quantity is. Discuss with your group: What is the difference between a physical quantity and unit? Give examples of physical quantities that have different units in the SI system and the British system. What are the quantities that have the same units in both systems?

A physical quantity is the NAME assigned to a quantity used to describe something about a physical system. For example the physical quantity of SPEED describes the rate at which something is moving. We use a letter of the Greek or Roman alphabets to label that quantity. Speed is represented by letter v. The units of a physical quantity describe what the quantity is measured in. For example, the rate at which something is moving (speed) can be measured in meters per second (m/s) or kilometers per hour (km/h) or feet per second (ft/s) or miles per hour (mi/h). The physical quantity of speed is just one concept: a rate of change of distance with time, but can be measured in many different units for both distance and time. Length measurements have different units in SI and British systems (meters versus feet or yards). Time is measured in the same units in both systems (seconds, minutes, hours)

2.4.2 Represent and reason

PIVOTAL Class: *Equipment per group:* whiteboard and markers, meter stick.

Henry is traveling to a store and then to his friend's house. His trip is represented on a sketch below (with a bird's eye view).



a. What is Henry's initial position? Final position? What is the position of the store?

Henry's initial position: (0m,0m). Final position: (-300m,+400m). Position of the store: (+200m,+200m).

b. Draw a vector to represent his total displacement, determine the scalar x-component of the displacement, the total distance traveled, and the path length.

To draw a vector representing his total displacement, draw an arrow starting at (0,0) and ending at (-300m,+400m). The scalar x-component of the displacement is -300 m. The total distance traveled is 500 m. Assuming Henry walks in a straight line from his starting point to the store and then in a straight line to his friend's house, the path length is 283m+539m=822m.

c. Choose a different origin for the coordinate system and repeat parts a. and b. What quantities changed? What quantities remained the same?

Students should find that all position quantities (part a.) have changed, but total distance, displacement, and path length (all the quantities in part b.) are the same as before.

2.4.3 Reason and represent

Class: *Equipment per group:* whiteboard and markers, meter stick.

Work together with your group on a whiteboard to tell a story and draw pictures representing Heather's trips A and B, as described below. On the pictures, show the coordinate axis and the displacement vector. Think of where you will choose the origin. d_x is the x-component of the displacement and l is the path length.

a. Trip A: $d_x = 0.7\text{mi}$; $l = 2.4\text{mi}$.

b. Trip B: $d_x = -3.7\,\text{mi};\ l = 4.4\,\text{mi}.$

c. What are Heather's initial and final positions for each trip? How do they depend on the choice of the direction of the coordinate axis and the location of the origin?

d. Who is the observer for the trips described above? Find an observer (per trip) for whom during the same trips Heather's displacements and path lengths traveled are all zero.

To me, the observer is a person standing still (relative to Heather) at the origin of the chosen coordinate system. If this same observe decided to follow Heather around exactly matching her motion, displacement, distance and path length would all be zero relative to this observer.

2.4.4 Pose your own problem

Class: *Equipment per group:* none.

Imagine any motion you participate in every day, such as going to classes or to a movie and dinner afterward. Pose a problem to solve about this motion in which one needs to understand the difference between the physical quantities position, displacement, distance, and path length, and the difference between a vector and the scalar component of a vector.

2.4.5 Reason

Class: *Equipment per group:* none.

Discuss with your group: In each of the following measurements, how many significant figures does the measurement have and what is the absolute uncertainty in each measurement?

a. Ulani says that she used a meter stick and measured her pencil to be 0.153 m long.

Ulani's measurement has three significant figures. Absolute uncertainty is ±0.0005 m.

b. Hermes says that the college swimming pool is 50 m long.

Hermes' estimate has two significant figures. Absolute uncertainty is ±0.5 m

c. Ulani used a chemical balance to weigh her pencil and says that her pencil weighs 0.00478 kg.

Ulani's measurement has three significant figures. Absolutely uncertainty is ±0.000005 kg.

d. Hermes says that he estimates that there are about 2 million liters of water in the college swimming pool.

Hermes' estimate has one significant figure. Absolute uncertainty is ±0.5 million liters.

2.4.6 Explain

PIVOTAL ***Class:*** *Equipment per group:* none.

Discuss with your group. Suppose your friend tells you that she measured her bed to be 2 m long, then she tells you that she looked up on Wikipedia that the circumference of Earth is 40,075,017 m. How many significant figures does each of these measurements have? Which of these two measurements is more precise? Discuss why. Evaluate both values and their number of significant figures.

Bed measurement: 1 significant figure, absolute uncertainty ±0.5m.

Earth's circumference: 8 significant figures, absolute uncertainty ±0.5m.

Although their absolute uncertainties are the same, Earth's circumference measurement is far more precise. The precision of a measurement is given by the ratio of the uncertainty to the size of the measured quantity: The bed measurement is (0.5m/2m) = 0.25 or 25% uncertain – not very precise! Earth's circumference measurement is (0.5m/40,075,017m) = 1.2×10^{-8} *or* 1.2×10^{-6} *% uncertain – far more precise than the bed measurement.*

2.4.7 Reading exercise

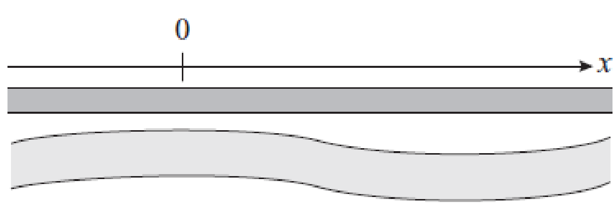
Read Section 2.4 in the textbook and answer Review Question 2.4.

2.5 Representing motion with data tables and graphs

2.5.1 Observe and describe

PIVOTAL ***Class:*** *Equipment per group:* whiteboard and markers, meter stick.

Imagine that you and your friend ride bicycles along a straight path beside a river. A coordinate axis is shown alongside the path.



The table indicates your position along the path at different clock readings.

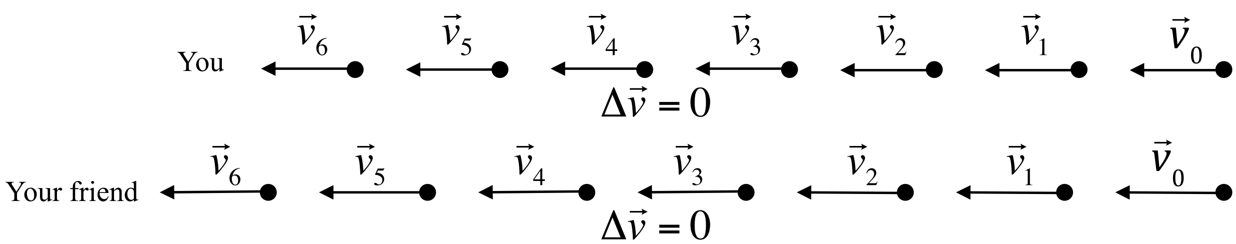
Clock reading t (s)	Your position x (m)	Your friend's position
$t_0 = 0$	$x_0 = 640$	$x_0 = 640$
$t_1 = 20$	$x_1 = 500$	$x_1 = 490$
$t_2 = 40$	$x_2 = 360$	$x_2 = 340$
$t_3 = 60$	$x_3 = 220$	$x_3 = 190$
$t_4 = 80$	$x_4 = 80$	$x_4 = 40$
$t_5 = 100$	$x_5 = -60$	$x_5 = -110$
$t_6 = 120$	$x_6 = -200$	$x_6 = -260$

a. Work with your group members to write everything you can about the bike rides and indicate any pattern in the data. What was happening at the clock reading of zero?

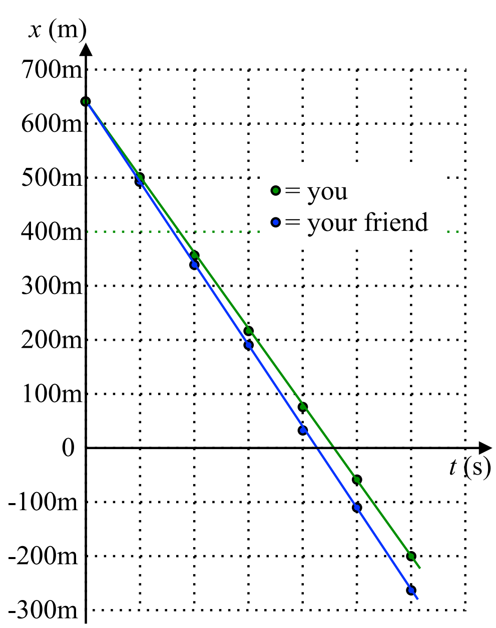
Your motion is constant rate motion because you cover the same distance (140m) every 20 second time interval. Your friend's motion is also constant rate motion because she cover's the same distance (150m) every 20 second time interval. Because your friend covers more distance every 20 seconds, she's going faster than you. Because the position numbers are decreasing, both you and your friend are traveling to the left using the coordinate system on the diagram. Both you and your friend are already moving at t=0 and you both are at the same point, 640m to the right of the origin.

b. Draw motion diagrams for both bikes.

Solution:



c. Construct position-versus-clock-reading graphs for both bike trips using the same coordinate axes in which *x* is a dependent variable and *t* is an independent variable. Compare and contrast the graphs – how do the graph lines represent the differences in the bikes' motions? Check with other groups – are their graphs the same or different from the graph of your group?



Both lines have negative slopes, indicating that both you and your friend are moving in the –x direction. Both lines are straight, showing that the motions are constant rate motions. Your friend's line has a steeper negative slope than your line because your friend is going faster than you are.

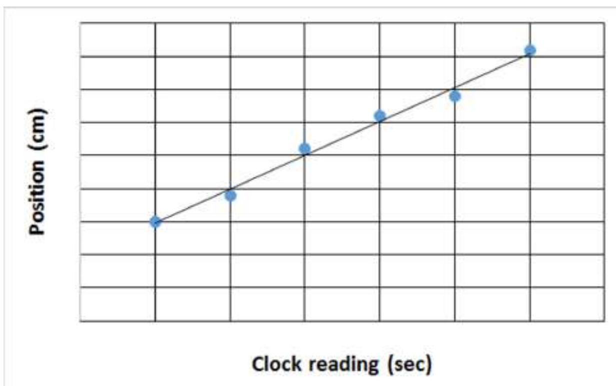
d. Discuss with your group members how the motion diagrams in part b. correspond to the graphs. How do you need to position the motion diagrams with respect to the graph axes so that it helps you visualize the motions?

You can think of each motion diagram as residing on the vertical axis of the position versus clock reading graph. The points on each motion diagram are horizontally aligned with the points on the graph.

2.5.2 Represent and reason

Class: *Equipment per group:* whiteboard and markers, meter stick.

With your group members, examine the graph below. On a whiteboard, describe the motion in words, construct a motion diagram for the motion, and a table of data.



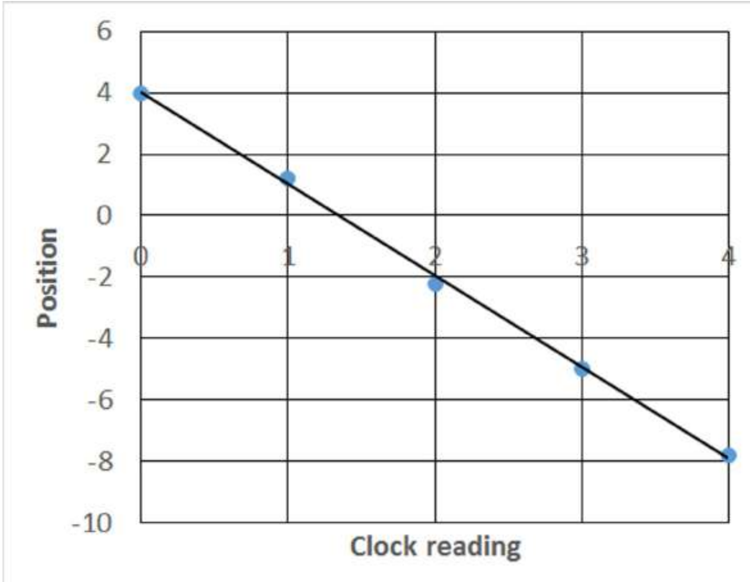
Solution:

Words: This is a constant rate motion (constant velocity) because the graph is a straight line. An object is moving at a constant rate in the +x direction. To construct the table of data we need to assign the unit on the position and time axes, depending on the units the data table will be different.

2.5.3 Represent and reason

Class: *Equipment per group:* whiteboard and markers, meter stick.

Examine the graph below. Is the graph complete? If not, complete it and represent the same information with a table and the motion diagram.



There is no scale on the time axis. Assign the scale and proceed to do the table and the motion diagram. The time scale needs to be reasonable.

2.5.4 Reading exercise

Read Section 2.5 in the textbook and answer Review Question 2.5.

2.6 Constant velocity linear motion

2.6.1 Represent mathematically

PIVOTAL ***Class:*** *Equipment per group:* whiteboard and markers.

Work together with your group members to answer the following questions:

a. Examine the graphs you drew in Activity 2.5.1c. Write two function expressions $x(t)$ for the graphs. Consider your labeling system: how can you distinguish the function for your bike from the function for your friend's bike?

$x = 640\text{ m} + (-7.0\text{ m/s})t$ *you*

$x = 640\text{ m} + (-7.5\text{ m/s})t$ *friend*

b. What are the physical meanings of the slope of each function and the intercepts? What common name can you use for the slope? Explain the meanings of positive or negative values for these quantities.

The slope equals the velocity of the object and the x-intercept equals the initial position of the object (the location of the object at time zero). A positive slope indicates velocity in the positive x direction and a negative slope indicates velocity in the negative x-direction.

c. Compare and contrast how we write linear functions in mathematics to how you just wrote the position-versus-time functions for motion. What is the same between them? What is different?

We wrote the position-versus-time functions for motion the same way linear functions are written in mathematics. The function has a slope and an intercept just like a mathematical function. What is different: in physics each term (slope and intercept, as well as the position and time variables) has a physical meaning, describing or telling us something about the motion of an object.

2.6.2 Test your idea

***PIVOTAL* Lab:** *Equipment per group:* whiteboard and markers, metronome or any device to keep track of seconds, 2 battery-operated toy cars that move with *different* speeds, sugar packets (or any other marking device), meter stick or ideally a longer tape measure.

Work with your group members on the following assignments. Make sure that your group keeps detailed records of the experiments so that another group from your class can repeat the experiments and get the same results.

a. For car A, design an experiment to decide if the car moves with constant velocity. If it does, determine the magnitude of the velocity (the car's speed).

b. For car B, use the same equipment and method to decide if this car moves with constant velocity. If it does, determine the magnitude of the velocity (the car's speed).

c. Predict where the cars will meet if you simultaneously release them from 2.0 m apart moving straight toward each other. List all assumptions that you made about how the cars move. If the assumptions were not valid, how would your prediction change?

d. Decide how you will record the data. How will you represent the data? In your representation, mark the predicted value for the meeting location. Perform the experiment and collect data.

e. Did the outcome match your prediction? How many times do you need to conduct the experiment to be able to say for sure whether the outcome of the experiment matches the prediction or not? Write the result of the experiment (meeting location) accounting for the discrepancies in the meeting location in different repetitions of the experiment.

Some pointers for instructors:

There should be two cars, a fast car and a slow car. The slow car is achieved by removing one battery and replacing it with a "shunt" which is basically a piece of conducting material that fills up the space previously taken up by the battery. If you have two cars going at the same speed the entire task of predicting where they collide becomes kind of stupid.

Students tend to use the sugar packets and drop them counting out the time each second. Ask them to look at their data and characterize how much uncertainty there is in this method. For example, if each interval is assumed to be 1 second: you might see variations like 35cm, 40cm 45cm. In other words 40cm±5cm (easily, it is likely even bigger than this), which is a 12.5% uncertainty at the very least. This is a large uncertainty! Ask them to consider another method. For example, measure out 1 meter and time how long it takes for the car to do that meter, say, 5 times. Then ask them to estimate the uncertainty in their measurements by looking at the variations in the time measurements and compare against the previous method. Clearly the second method will be much more precise. So, not just telling them to do it a different way from what they might initially try, but also embed into it a discussion of uncertainties as a way of seeing why it is a better method (it reduces the uncertainty in your measurements).

Using their knowledge of motion to make a prediction about where the cars will meet: BEFORE this lab, presumably in lecture/discussion (but also could be a homework reading exercise) the students need to learn how to set up equations of motion in different coordinate systems. This is covered in textbook conceptual exercises. IF you didn't do this with them before lab, you need to do it in the lab in some way shape or form.

There are various ways that students like to solve this problem. One way is to skip setting up the coordinate system and just say $v_A t + v_B t = 2m$*. Now, this is correct. BUT the concern is: do they really understand what they're doing when they do it this way? Instead of telling them "no" I ask them to do it both ways: Do it this intuitive way, but also please try to do it by setting up two positions equations in one coordinate system. Why? Because part of the reason to do this lab is to get practice at doing that. I specifically tell them if they skip doing it that way, they won't get any practice which is necessary for harder problems. Ask them also to compare the intuitive way with the proper coordinate system method and see that the equation is basically the same.*

2.6.3 Represent

Class: *Equipment per group:* none.

The motion of two objects is represented by the expressions below. Study the motions and act them out with your classmates. Note that it is important to focus on what was happening at $t = 0$. What are your assumptions about the observer?

$$x_A = (-7.5\text{ m}) + (1.7\text{ m/s})t$$
$$x_B = (5.2\text{ m}) + (-0.8\text{ m/s})t$$

Here an important thing about zero time is that the object is already moving. Thus to act out the first scenario the students should first agree on what their zero of the coordinate system is what the positive and negative directions are. Let's imagine that the positive direction is to the right. The actor should start about 10 m to the left of the zero point and jog in the positive direction getting to the speed of 1.7 m/s before she passes the mark that is 7.5 to the left of the zero point. While she gets up to speed the rest of the class should have their eyes closed. When she passes this ark, her speed should be constant and this is when the rest of the class should open their eyes (she can clap for example) and observe her passing this mark jogging in the positive direction at constant speed.

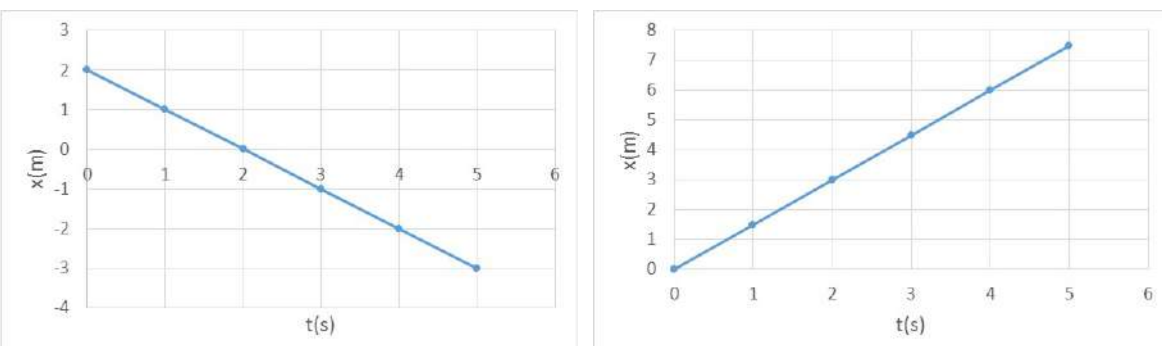
2.6.4 Apply

***PIVOTAL* Lab:** *Equipment per group:* whiteboard and markers, motion detector, meter stick or any other length-measuring device.

Practice using the motion detector until you are comfortable with it, and you know the range of distances it detects and what direction of motion is positive.

a. Discuss with your group members what the position-versus-time graphs of the motions described by the expressions below will look like, and sketch the graphs.

$$x = 2\text{ m} + \left(-1.0\text{ m/s}\right)t$$
$$x = \left(1.5\text{ m/s}\right)t$$



b. Choose one group member to perform the motions in front of the motion detector and compare the graphs on the motion detector's screen with those predicted in part a.

When the observer starts observing the represented motion, the moving person is passing the 2 m position moving in the negative direction at a constant speed 1.0 m/s. The second equation describes a person moving in the positive direction at speed 1.5 m/s (the person was already *moving when the observer started observing). Note that the default direction for motion detectors is: moving away from the motion detector is positive, moving towards the motion detector is negative.*

2.6.5 Reason

Class: *Equipment per group:* whiteboard and markers.

Work together with your group members. You are sitting on a park bench. A jogger is passing the bench, jogging at a relatively constant speed of 6 mph when you start observing her. Write a position-versus-time function for the jogger that will allow you to predict her position at any clock reading. What assumptions do you need to make to write this function? How many "correct" functions can you write?

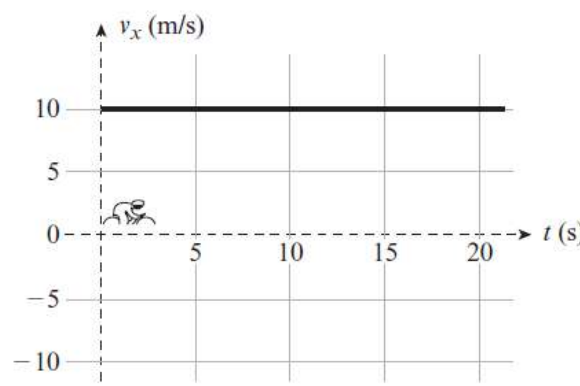
x = [6 miles/hour (1 hour/3600 s)(1600 m/mile)]t = (2.68 m/s)t

Could write other functions using different units. Also the above assumes that she is at position zero in front of you at time zero. It also assumes the jogger is moving in the +x direction. There are an infinite number of correct functions because you are free to pick the origin of your coordinate system. You can also choose which direction is + and -

2.6.6 Analyze

PIVOTAL Class: *Equipment per group:* whiteboard and markers.

Share ideas with your group members and figure this out together. The figure at the right shows a *velocity*-versus-time graph that represents the motion of a bicycle moving along a straight bike path. The positive direction of the velocity coordinate axis is toward the east.



a. Use the graph to estimate the bike's displacement during the time interval from clock reading 10 s to clock reading 15 s.

The bike's displacement during the time interval from clock reading 10 s to clock reading 15 s is: (10 m/s)(15 s –10 s) = 50 m

b. Use the graph to estimate its displacement during the time interval from 0 s to 20 s.

(10 m/s)(20 s – 0) = 200 m.

c. Formulate a general rule for using a velocity-versus-time graph to determine an object's displacement during some time interval if the object is moving at constant velocity.

A general rule for using a velocity-versus-time graph to determine an object's displacement during some time interval if the object is moving at constant velocity is to determine the area under the velocity-versus-time graph or for the special case of constant velocity to multiply the speed and time interval.

2.6.7 Test an idea

Lab: *Equipment per group:* whiteboard and markers, motion detector (if available), a small object that can be dropped and will not bounce, meter stick or any other length-measuring device, video camera. [https://mediaplayer.pearsoncmg.com/assets/_frames.true/sci-phys-egv2e-alg-2-6-7]

Use available equipment to design an experiment to test if the motion of the object that you drop from certain height can be described mathematically as $y = v_y t$. Consider the location from which the object is dropped to be the origin.

a. Discuss with your group members what it means to test an idea. In physics, experimental testing consists of the following steps:

1) you accept the idea being tested as true;

2) you design an experiment whose outcome you can predict using this idea;

3) you make the prediction of the outcome (here you also need to think of what you assume to be true in addition to the idea you are testing—these are called *assumptions*);

4) you perform the experiment and compare the outcome to the prediction, and based on the comparison you make your judgment concerning the idea being tested.

Students need to devise a way to measure the y position of a falling object at various time intervals. This could be done with a motion detector or with a video that can be stepped frame by frame. Students take position and time data and could plot a graph of position versus time.

b. Design an experiment and make a prediction of the expected results based on the idea being tested. Be sure to write your prediction down.

Prediction: Expect to get a linear graph of position versus time

c. Then perform the experiment as many times as necessary for you to be convinced that the data you collected are sufficient to support or reject the idea. Record the data and justify your judgment.

2.6.8 Apply

A total solar eclipse is a rare phenomenon that happens at the same location once in about 200 years. During this phenomenon, the Moon passes directly in front of the Sun as seen from Earth.

Given the visible diameter of the Moon is very close to the visible diameter of the Sun, the Moon covers the Sun completely and the part of Earth in the Moon's shadow plunges into darkness during the daytime. The average shadow of the Moon on Earth is about 200 km wide and it slowly travels across Earth during the eclipse day. On August 21st 2017, this rare phenomenon occurred in the US. Below are the data about the eclipse. Work with your group members to answer the questions below. (The photo above shows the Sun in Franklin, NC, about 5 minutes before the total solar eclipse in 2017).

a. The 2017 total solar eclipse started on Monday August 21 in Madras, Oregon at about 10:20 am (Pacific daylight time) and ended in Columbia, South Carolina at 2:44 pm (Eastern daylight time). Estimate the average speed of the Moon's shadow moving across the United States and compare it to the speed of sound in air (340 m/s). Indicate any assumptions that you made.

In order to estimate the average speed we need to estimate the distance traveled by the shadow and divide it by the corresponding time interval. We will assume that the shadow is traveling along the straight line and we will neglect curvature of Earth surface. On the Internet you can find that the straight line distance between the Madras, OR and Columbia, SC is about 4000 km. If you take car travel route (for example using Google maps), you get about 4300 km, which still useful approximation for our purpose, since the route is quite straight.

To determine the time interval you first need to know that the difference between PDT and EDT is 3 hours, which means you have to subtract 3 hours from the difference between local times. The time interval between start in Madras, Oregon (t_i = 10:20 am, PDT) and end in Columbia, South Carolina (t_f = 2:44 pm, EDT) is therefore

$$\Delta t = t_\mathrm{f} - t_\mathrm{i} = (12\text{ h} + 2\text{ h} + 44\text{ min}) - (10\text{ h} + 20\text{ min}) - 3\text{ h} = 1\text{ h} + 24\text{ min} = 1.4\text{ h}$$

Average speed of the shadow is therefore

$$\bar{v}_\text{Shadow} = \frac{4000\text{ km}}{1.4\text{ h}} = 2850\text{ km/h} = 1770\text{ mi/h} = 790\text{ m/s}$$

This is more than two times faster than the speed of sound in air (340 m/s, 767 mi/h). If we assume that the uncertainty of the distance between Madras and Columbia is about 100 km then the uncertainty of the average speed is about

$$\frac{100\text{ km}}{1.4\text{ h}} = 70\text{ km/h} \approx 20\text{m/s}.$$

b. During the same total solar eclipse in Franklin, North Carolina, the Moon cast on Earth a circular shadow with a diameter of about 109 km. The total solar eclipse in Franklin lasted for 2 minutes and 30 seconds. Estimate the speed of the Moon's shadow moving across Franklin. Compare this answer with the answer in part a. and try to explain any discrepancies.

The duration of the total solar eclipse is time interval needed for the circular shadow (umbra) to pass certain point on Earth. Because the time interval for total solar eclipse is given but the location of the Franklin with respect to the trajectory of the circular shadow is not given, we can only estimate the maximum average speed of the shadow. The shadow moves the longest distance during the total eclipse if the place of observation is along the line that passes through the center of the shadow in the direction of motion (point A in the figure on the right). For any other point the path length moved by the shadow during the total solar eclipse in that point is shorter than diameter of the shadow. Therefore, the maximum speed of the shadow can be determined as

$$\overline{v}_{\text{Shadow\_max}} = \frac{D}{\Delta t} = \frac{109\ \text{km}}{150\ \text{s}} = 730\ \text{m/s}.$$

The value is close to the value that we obtained in the previous calculation, although we see that even if we take into account the uncertainty of the previous calculation the results are still different. The discrepancy between the results in a. and b. suggests that we should verify if the assumptions that we made are valid. In reality, Earth is not flat but spherical. As a result, the speed of the Moon shadow changes as the shadow is moving over the Earth's surface.

2.6.9 Reading exercise

Read Section 2.6 in the textbook and answer Review Question 2.6.

2.7 Motion at constant acceleration

2.7.1 Observe and analyze

PIVOTAL ***Lab or class:*** *Equipment per group:* whiteboard and markers, motion detector (if available), a small object that can be dropped and will not bounce, meter stick or any other length-measuring device, video camera.[https://mediaplayer.pearsoncmg.com/assets/_frames.true/sci-phys-egv2e-alg-2-7-1]

a. Use the available equipment to design an experiment to record position-versus-time data for a ball falling from the height of about 2 meters. It helps to position the motion detector above the falling ball, not below.

Time (s)	*position (m)*	*displacement*	*avg time*	*avg velocity*
0.000	*2.010*			
0.033	*2.000*	*-0.010*	*0.017*	*-0.300*

0.067	*1.980*	*-0.020*	*0.050*	*-0.600*
0.100	*1.950*	*-0.030*	*0.083*	*-0.900*
0.133	*1.900*	*-0.050*	*0.117*	*-1.500*
0.167	*1.850*	*-0.050*	*0.150*	*-1.500*
0.200	*1.790*	*-0.060*	*0.183*	*-1.800*
0.233	*1.720*	*-0.070*	*0.217*	*-2.100*
0.267	*1.640*	*-0.080*	*0.250*	*-2.400*
0.300	*1.540*	*-0.100*	*0.283*	*-3.000*
0.333	*1.430*	*-0.110*	*0.317*	*-3.300*
0.367	*1.300*	*-0.130*	*0.350*	*-3.900*
0.400	*1.170*	*-0.130*	*0.383*	*-3.900*
0.433	*1.030*	*-0.140*	*0.417*	*-4.200*
0.467	*0.880*	*-0.150*	*0.450*	*-4.500*
0.500	*0.720*	*-0.160*	*0.483*	*-4.800*
0.533	*0.550*	*-0.170*	*0.517*	*-5.100*
0.567	*0.360*	*-0.190*	*0.550*	*-5.700*

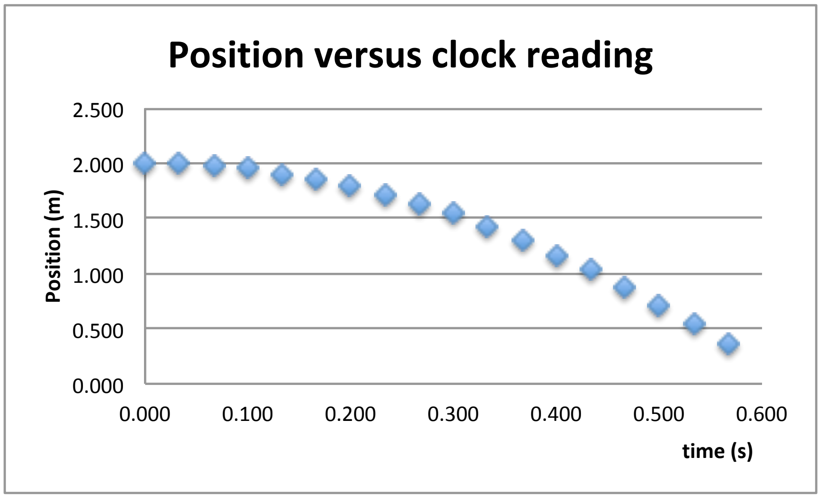
b. Perform the experiment and collect data. If you are using a motion detector, the data will be represented as a graph right away. If you are analyzing a video, you will need to figure out how to collect position and time data from it. Repeat the experiment a few times. What can you say about the motion of the ball based on the data you collected?

See table in a. above. The ball is speeding up as it falls.

c. Draw a motion diagram for the ball.

d. Draw a position-versus-time graph for the ball. Discuss whether the graph resembles a position-versus-time graph for an object moving at constant velocity.

The graph is not linear, it is not the same as the graph for an object moving at a constant velocity

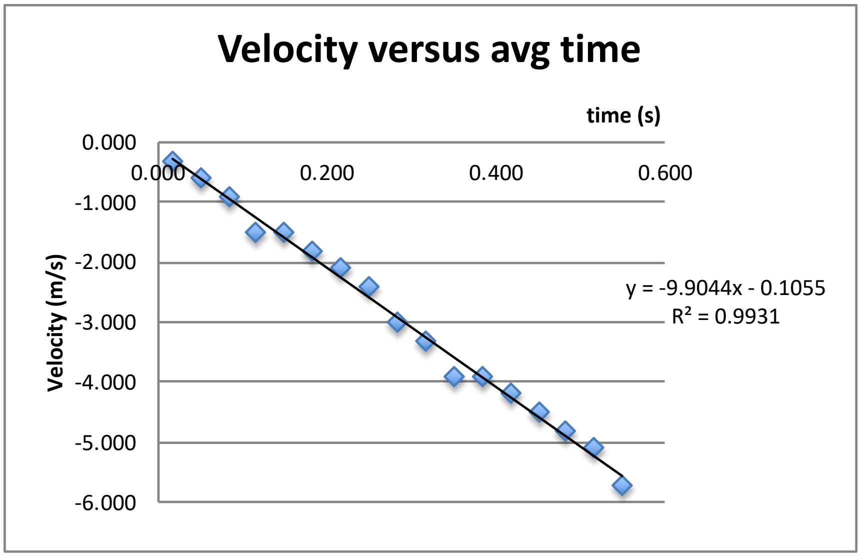


e. Determine the scalar component of the average velocity for the ball for each time interval by completing the following table.

Time interval $\Delta t = t_n - t_{n-1}$	Displacement $\Delta x = x_n - x_{n-1}$	Average time $(t_n + t_{n-1})/2$	Average velocity $\frac{\Delta x}{\Delta t}$

See table in part a.

f. Plot this average velocity v_x on a velocity-versus-time graph. The time coordinate for each average velocity coordinate should be in the middle of the corresponding time interval (the average time for that time interval). Draw a best-fit line for your graph.



g. Discuss with your group the shape of the graph: How does the speed change as time elapses? Suggest a name for the slope of the graph.
This graph is linear. The speed increases at a constant rate as time elapses. The slope is called acceleration. Notice that the slope is -9.9 m/s² – close to the "accepted" 9.8m/s² for free fall acceleration.

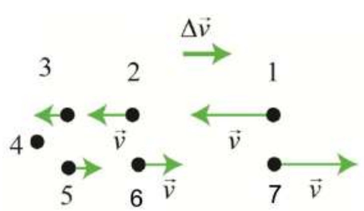
2.7.2 Analyze

Class: *Equipment per group:* whiteboard and markers.

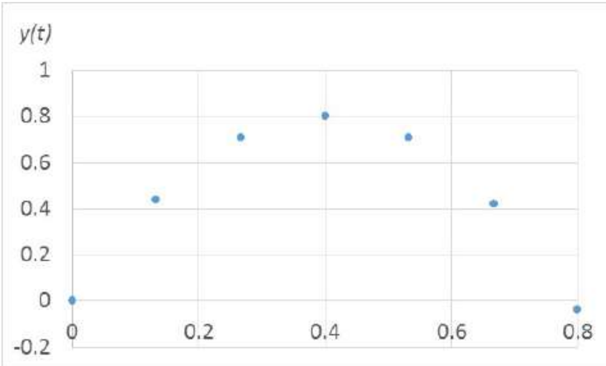
If you did not perform the experiment in Activity 2.7.1, work with the data recorded in the table at the right for the up and down motion of the center of a ball thrown upward (the y-axis points up).

Clock reading t (s)	**Position y (m)**
0.000	0.00
0.133	0.44
0.267	0.71
0.400	0.80
0.533	0.71
0.667	0.42
0.800	- 0.04

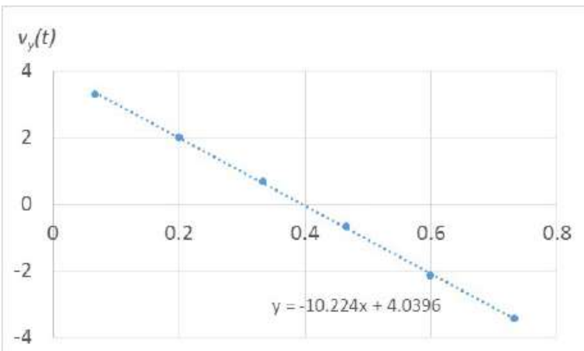
a. Sketch a motion diagram for the ball modeled as a point-like object.



b. Draw a position-versus-time graph.



c. Draw a velocity-versus-time graph. Find its slope. What do you call this slope?



Slope tells the acceleration of the ball ($-10.2\ \text{m/s}^2$*).*

d. Use the velocity-versus-time graph to determine the ball's acceleration at the very top of its trajectory?

The ball's acceleration at the top of its trajectory is 9.8 m/s^2 *downwards.*

e. What is the ball's velocity at the top?

The ball's velocity at the top is zero.

f. Can you reconcile these two answers? Explain.

g. Use the velocity-versus-time graph to determine the distance that the ball traveled during the trip from clock reading 0.000 s to 0.800 s.

The distance traveled is zero.

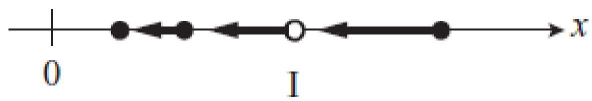
2.7.3 Represent and reason

Class: *Equipment per group:* whiteboard and markers.

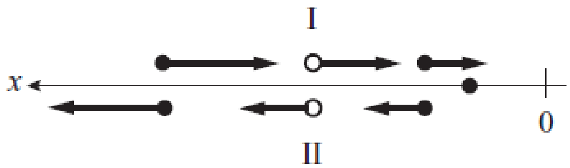
The motion diagrams in the illustrations below represent the motion of different objects. The arrows are velocity arrows.

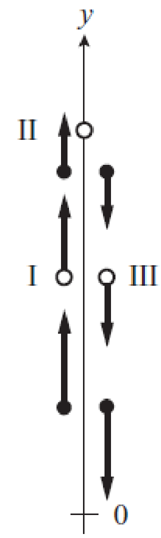
c.

a.



b.





A different coordinate axis is provided for each of the three motion diagrams. An open circle indicates a location of interest (there are three locations: I, II, and III). Add a single velocity change arrow for each diagram. Then, determine the signs of the position, velocity component, velocity change component, and acceleration component at the position(s) of the open dots for each diagram. Note: what assumptions about motion do you need to make to use only *one* velocity change arrow for part b. and one for part c.

a. position +, velocity -, velocity change +, acceleration +

b I. position +, velocity -, velocity change +, acceleration +

b II. position +, velocity +, velocity change +, acceleration +

c I. position +, velocity +, velocity change -, acceleration -

c II. position +, velocity 0, velocity change -, acceleration -

c III. position +, velocity -, velocity change -, acceleration -

2.7.4 Reading exercise

Read Section 2.7 in the textbook and answer Review Question 2.7.

2.8 Displacement of an object moving at constant acceleration

2.8.1 Derive

PIVOTAL Class: *Equipment per group:* whiteboard and markers.

Discuss with your group members how you can construct a function $x(t)$ for a cart that moves at constant acceleration. Choose the simple case first: when the cart starts at the origin of the coordinate system and has zero initial speed. There are many ways of doing this. Think of average velocity or a velocity-versus-time graph. Once you agree on the method, follow through and derive the expression. Evaluate the expression using limiting case analysis – for example, does your equation work for constant-velocity motion? Share the expression with the class – are you in agreement concerning the function $x(t)$?

See two different ways to approach this task on pages 34-35 and 36-37 in the textbook.

2.8.2 Design an experiment

Lab: *Equipment per group:* Low-friction track and a cart, motion detector or a stopwatch, meter stick.

Design an experiment to investigate the motion of a cart moving up and down an inclined plane. You push it forcefully at the bottom of the plane so that it moves up the slope, and then it stops and moves back down. You can use a motion detector, a stopwatch, a meter stick, and other equipment available in the lab.

a. Describe your experiment in words and draw the setup.

Students can to use sugar packets to track the motion of the cart or use motion detector. The motion of the cart will depend on the slope of the plane, the initial speed of the cart, and if the cart is not frictionless, on the cart itself. The students need to decide what they are going to investigate.

b. What data will you collect?

They need to collect position vs time data, if they use the motion detector they can use velocity vs time data (acceleration vs time data are usually too messy to use), they need to collect the data pertaining to the inclined plane too.

c. How will you organize and report your data so that others can understand it?

The answer depends on the set up and what aspect of the motion students decided to investigate

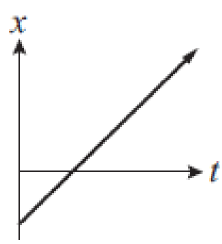
d. After you have made all the decisions, conduct the experiment and write down your findings. What can you say about the motion of the cart?

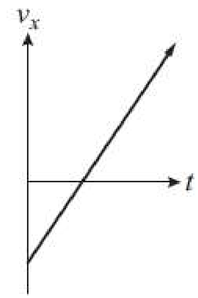
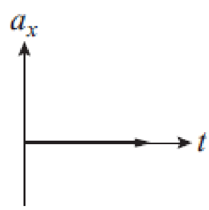
The answer depends on the set up and materials.

2.8.3 Summarize

***PIVOTAL* Class:** *Equipment per group:* whiteboard and markers.

This is a really helpful activity to do with your group: Use different representations of the two types of motion we have studied to fill in the empty cells in the table. Some cells are completed to give you an idea of the motions and the direction of the coordinate axis for each case. Your responses should relate to the motion already described. Completing the table will help you summarize everything you have learned about the description of motion. Resolve any confusion you may have by talking with your group members.

Motion with constant velocity	**Motion with constant acceleration**
Describe the motion in words, providing an example.	Describe the motion in words, providing an example. The object's velocity is decreasing by the same amount every second—for example, a cart going up a smooth track tilted at an angle.
Provide a motion diagram that describes this type of motion.	Provide a motion diagram that describes this type of motion.
Provide a position-versus-time graph that describes this type of motion. 	Provide a position-versus-time graph that describes this type of motion.
Describe the motion mathematically as $x(t)$.	Describe the motion mathematically as $x(t)$. $x = -v_0 t + \frac{1}{2} a_{x0} t^2$
Provide a velocity-versus-time graph that describes this type of motion.	Provide a velocity-versus-time graph that describes this type of motion.

	
Describe the motion mathematically as $v_x(t)$. $v_x = v_{x0}$	Describe the motion mathematically as $v_x(t)$.
Provide an acceleration-versus-time graph that describes this type of motion. 	Provide an acceleration-versus-time graph that describes this type of motion.
Describe motion mathematically as $a_x(t)$.	Describe motion mathematically as $a_x(t)$. $a_x = a_{x0}$

2.8.4 Reading exercise

Read Section 2.8 in the textbook and answer Review Question 2.8.

2.9 Skills for analyzing situations involving motion

2.9.1. Represent and reason

Class: *Equipment per group:* whiteboard and markers.

You have two identical billiard balls at the top of an inclined track. Assume that the balls move along the incline in the same way a small cart does (i.e., with constant acceleration). Now imagine you release one ball and it moves down the track. When it is about 10 cm down the track, you release the second ball. Draw a picture of the situation and describe it using motion diagrams, graphs, and mathematical equations. Use those representations to predict what will happen to the distance between the two balls – will it increase, decrease, or stay the same (about 10 cm)?

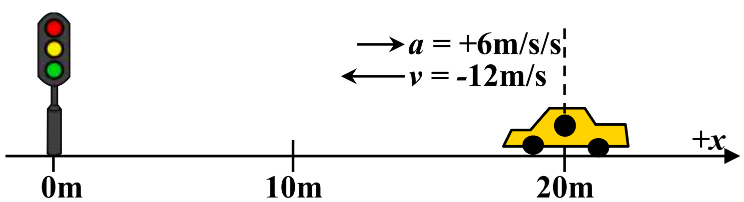
They have the same acceleration. But the first ball is moving when you release the second ball. Thus, its speed is more than that of the second ball at every instant as it continues to gain speed on top of an already moving speed. Thus, the first ball gets farther and farther ahead of the second ball. Students need to use motion diagrams, graphs or mathematical representations to reason to the same answer.

2.9.2 Represent and reason

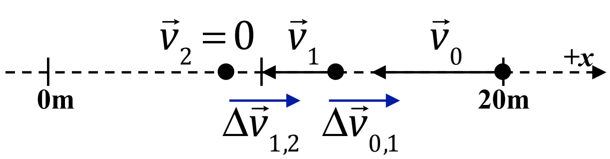
***PIVOTAL* Class:** *Equipment per group:* whiteboard and markers.

Practice representing a process involving motion on a whiteboard together with your group members. A stoplight turns yellow when you are 20 m from the edge of an intersection. Your car is traveling at 12 m/s. After you hit the brakes, your car's speed decreases at a rate of 6.0 m/s each second until the car stops. Ignore the reaction time needed to move your foot from the floor to the brake pedal.

a. Sketch the process. Indicate the origin of the coordinate system and the direction of the x-axis.

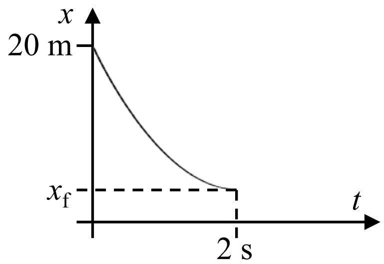


b. Draw a motion diagram representing the process. What are the signs of the v_x, Δv_x and a_x?

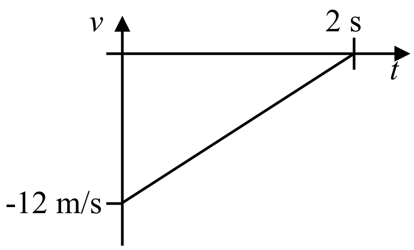


v_x -, Δv_x +, a_x +.

c. Construct an $x(t)$ graph for the process.



d. Construct a $v_x(t)$ graph for the process.



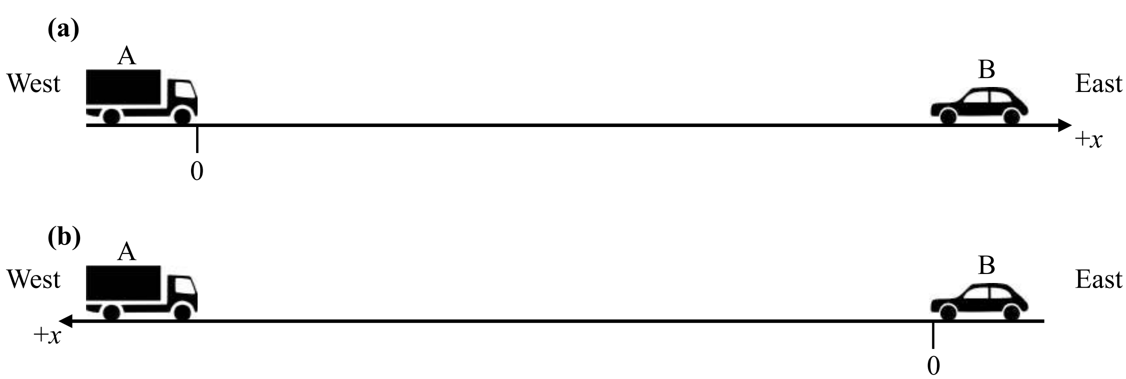
e. Write $x(t)$ and $v_x(t)$ expressions representing the process.

$x(t) = 20\text{ m} - (12\text{ m/s})t + \frac{1}{2}(6\text{ m/s}^2)t^2$, $v_x(t) = -12\text{ m/s} + (6\text{ m/s}^2)t$

2.9.3 Represent and reason

PIVOTAL ***Class:*** *Equipment per group:* whiteboard and markers.

Truck A starts at rest and moves faster and faster toward the east so that its speed increases by 3.0 m/s each second. At the same time that truck A begins to move, car B is 200 m east of truck A. Car B is initially moving at speed 16 m/s toward the west and begins to slow down by 1.0 m/s each second.



Fill in the table that follows using a different description for each coordinate axis shown above.

	Truck A: coordinate system (a)	*Car B: coordinate system (a)*	*Truck A: coordinate system (b)*	*Car B: coordinate system (b)*
Indicate the initial clock reading.	0 s	0 s	0 s	0 s
Indicate the initial position.	0 m	+ 200 m	+ 200 m	0 m
Indicate the initial velocity.	0 m/s	- 16 m/s	0 m/s	+ 16 m/s
Indicate the acceleration.	+ 3 m/s^2	+ 1 m/s^2	- 3 m/s^2	- 1 m/s^2
Write equations that can be used to determine the position and velocity of the vehicle at any given clock reading in the future (before the car stops).	$x_A(t) = +\frac{1}{2}(3\text{m/s}^2)t^2$ $x_B(t) = 200\text{m} - (16\text{m/s})t + \frac{1}{2}(1\text{m/s}^2)t^2$ $v_A(t) = (3\text{m/s}^2)t$ $v_B(t) = -16\text{m/s} + (1\text{m/s}^2)t$		$x_A(t) = 200\text{m} - \frac{1}{2}(3\text{m/s}^2)t^2$ $x_B(t) = (16\text{m/s})t - \frac{1}{2}(1\text{m/s}^2)t^2$ $v_A(t) = -(3\text{m/s}^2)t$ $v_B(t) = 16\text{m/s} - (1\text{m/s}^2)t$	

2.9.4 Regular problem

PIVOTAL Class: *Equipment per group:* whiteboard and markers.

Work together with your group to figure this out: You ride your bike west at speed 8.0 m/s. Your friend, 400 m east of you, is riding her bike west at speed 12 m/s. Complete the following steps to determine when your friend passes you.

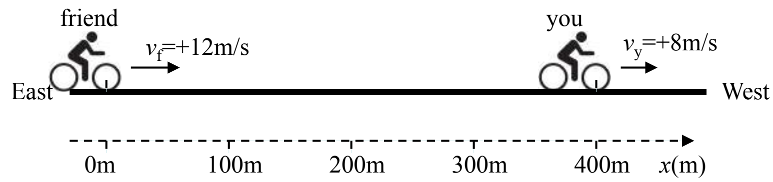
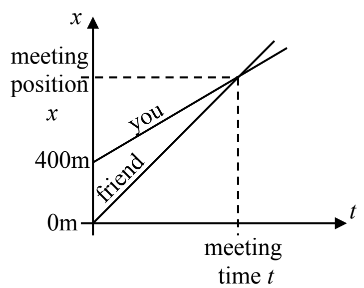
a. *Sketch and Translate* Draw a sketch of the initial situation and choose a coordinate system to describe the motion of both bikes. Put all given information on the sketch; identify the unknown.

b. *Simplify and Diagram* Draw a motion diagram for each bike. Sketch a position-versus-time graph for each bike using the same coordinate axes.

c. *Represent Mathematically* Construct equations that describe the positions of each bicycle as a function of time relative to the chosen coordinate system.

d. *Solve and Evaluate* Use the equations to determine when the bicycles are at the same position. Does your result make intuitive sense? How do you know?

Sketch and Translate:	*The diagram shows the initial*

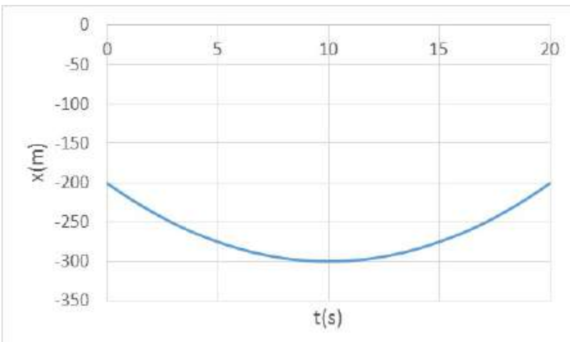
 	*situation at time t = 0. Both cyclists are traveling at constant velocity in the +x direction. The goal is to find the position x at some later time t where they meet.*
Simplify and Diagram:  	*The position versus time graph illustrates how, when both you and your friend are placed on the same coordinate system you can see that they will share some position x and time t – this is their meeting point.*
Represent Mathematically: $x_f(t) = +(12\text{m/s})t$ $x_y(t) = 400\text{m} + (8\text{m/s})t$	*My numbers and signs are consistent with my chosen coordinate system: friend's initial position is zero while your initial position is at 400m. Both are moving in the +x direction.*
Solve: $(12\text{m/s})t = 400\text{m} + (8\text{m/s})t$ $(4\text{m/s})t = 400\text{m}$ $t = 100\text{s}$ $x_\text{f} = (12\text{m/s})(100\text{s}) = 1200\text{m}$ *Evaluate*: $x_\text{y} = 400\text{m} + (8\text{m/s})(100\text{s}) = 1200\text{m}$	*We put these two equations equal to each other and solve for time. Then we substitute that time back into either equation and solve for the final position. A quick evaluation is to check that both equations return the same final position of 1200m. This is indeed the case. The answer of 1200m also makes sense because it is positive and substantially larger than 400m. It is consistent with our chosen coordinate system.*

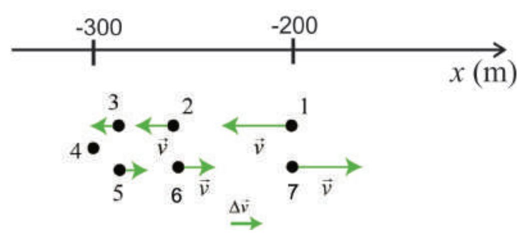
2.9.5 Represent and reason

Class: *Equipment per group:* whiteboard and markers.

An imaginary object moves horizontally. The position-versus-time function represents the object's motion mathematically. Describe in different ways a process that the equation below might represent. (The equation could represent many different processes.) Work together with your group, and when you're done or if you get stuck, compare your ideas with another group.

$$x(t) = (-200.0 \text{ m}) + (-20.0 \text{ m/s})t + (1.0 \text{ m/s}^2)t^2$$





a. Describe the motion in words. Note that it is important to focus on what was happening at $t = 0$. Use physical quantities to write down all of the information that you can "extract" from the function. If you can write other functional dependencies – do it!

The object starts at position –200 m at time 0. It's velocity at time zero is –20 m/s. It has an acceleration of +2.0 m/s². This means that the object is moving in the negative direction for 10 s slowing down each second until it stops and starts moving in the positive direction. When it stops, it is at position -300 m.

b. Draw a motion diagram that represents the process.

c. Draw a position-versus-time graph that represents the process.

d. Draw a velocity-versus-time graph that represents the process.

e. Determine when and where the object for your chosen process stops.

f. If you decided to let the process continue beyond the point where it stops in e. above, what would the object be doing?

The object would reverse direction and start speeding up in the +x direction.

2.9.6 Evaluate

Class: *Equipment per group:* whiteboard and markers.

Discuss with your group: You learned that the equation describing position-versus-time of an object moving at constant acceleration is $x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$. Use both algebraic and graphical approaches to show that, in a limiting case of $a_x = 0$, this equation describes the motion of an object that is traveling at constant velocity.

If $a_x = 0$, then the constant acceleration equation becomes $x = xo + v_x\, t$, which is the equation for constant velocity motion.

2.9.7 Evaluate

Class: *Equipment per group:* whiteboard and markers.

Discuss with your group: You learned that in the equation describing an object moving at constant acceleration, the position as a function of time is $x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$. Use algebraic and graphical approaches to show that, in the case where $x_0 = 0$ and $v_{x0} = 0$, the successive displacements of the object change in proportion as the integers squared: 1, 4, 9, 25, etc.

If $x_0 = 0$ and $v_{x0} = 0$, then $2x/a = t^2$. Thus displacement x increases with time as the integers squared: 1, 4, 9, 25, ….

2.9.8 Evaluate the solution

Class: *Equipment per group:* whiteboard and markers.

Discuss with your group: Identify any errors in the proposed solution to the following problem and provide a corrected solution if there are errors.

The problem You are driving at 20 m/s and slam on the brakes to avoid a goose walking across the road. You stop in 1.2 s. How far did you travel after hitting the brakes?

Proposed solution

$$(x - x_0) = vt = (20\text{ m/s})(1.2\text{ s}) = 24\text{ m}$$

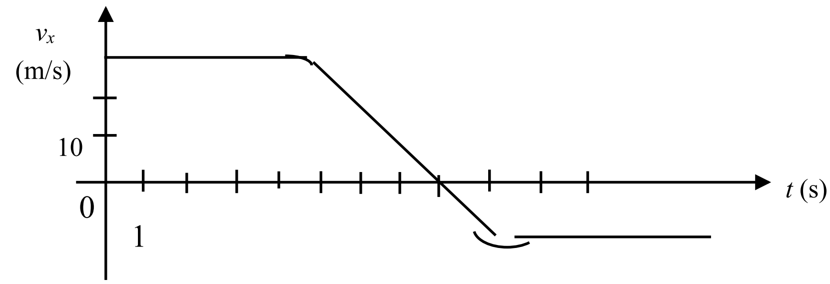
The proposed solution only works if the velocity of the car is constant. The velocity is not constant. The more appropriate approach is to assume constant acceleration as the car comes to a stop. If the acceleration is constant, $\left(x - x_0\right) = \frac{1}{2}\left(v_\text{f} + v_0\right)\Delta t = \frac{1}{2}\left(0 + 20\text{m/s}\right)\left(1.2\text{s}\right) = 12\text{m}$. *In words: the average velocity while stopping is 10 m/s and the car should stop in 12 m.*

2.9.9 Evaluate the solution

Class: *Equipment per group:* whiteboard and markers.

Discuss with your group: Identify any errors in the proposed solution to the following problem and provide a corrected solution if there are errors.

Problem: Use the following graphical representation of motion to determine how far the object traveled before it stopped.



Proposed solution The object was at rest for about 5 seconds, then started moving in the negative direction and stopped after about 9 seconds. During this time, its position changed from 30 m to – 10 m, so the total distance that it traveled was 40 m.

Answer

The interpretation of the graph is incorrect for every segment. The solver probably confused the velocity vs time graph with the position vs time graph, thus the answer to the second question is also incorrect.

Corrected solution: A car was moving at a speed of 30 m/s for 4.5 seconds in the positive direction, then it slowed down for 3.5 seconds at constant acceleration, reversed direction and started speeding up with the same magnitude of acceleration for 1 second and continued at constant speed of 10 m/s in the negative direction for about 5 seconds. The total path length is 135 m +52.5 m +5 m +50 m = 242.5 m

2.9.10 Observe and analyze

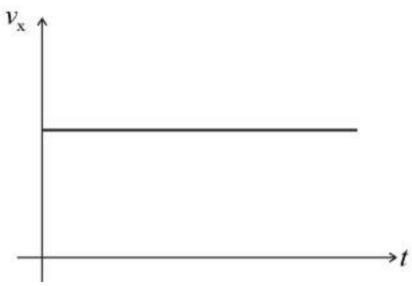
Class: *Equipment per group:* whiteboard and markers.

Collaborate together with your group to figure this out: The figure below shows long-exposure photos of two experiments with a blinking LED that was fixed on a moving cart. In both cases the cart was moving from right to left. The duration of the ON and OFF times for the LED is 154 ms, and the length of the cart is 17 cm.

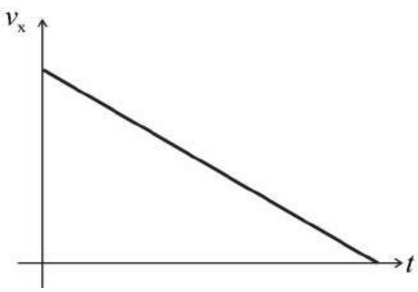
a. Specify a coordinate system and draw a qualitative velocity-versus-time graph for the motion of the cart in both experiments.

The x axis of coordinate system points to the right. The first photo shows the cart that moves with constant speed and the second the cart that is slowing down until it stops. These are the velocity-versus-time graphs for these two cases:

Constant speed



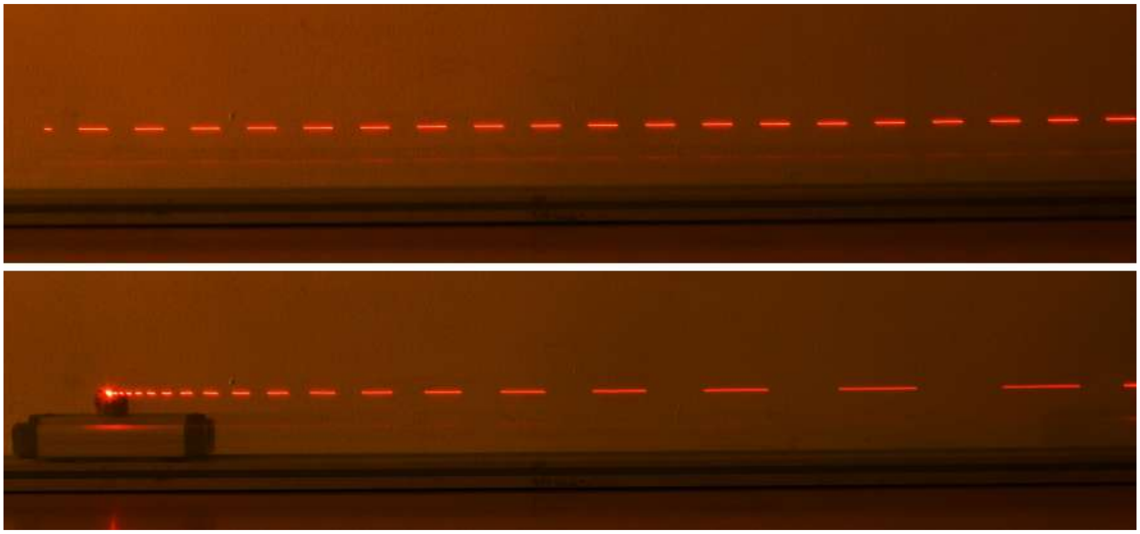
Slowing down (we assume constant acceleration)



b. Estimate the speed of the cart in the first experiment. Both photos were taken from the same spot and with the same settings. Indicate any assumptions that you made.

First, measure the length of the cart on the lower photo using a ruler. Then measure the length of the traces when the LED is on the length of the intervals when it is off (they are about equally long). You will find out that you can fit about 7 on or off LED intervals into the length of the cart.

Because the real length of the cart is 17 cm, this means that the cart on the first photo moved $d = \frac{17\text{ cm}}{7} = 2.4\text{ cm}$ *every 0.154 s. Therefore, the speed of the car is* $v = \frac{2.4\text{ cm}}{0.154\text{ s}} = 0.16\text{ m/s}$. *This means the cart needs about 6 seconds to move 1 m, which is very reasonable speed for the equipment that was used in this experiment.*



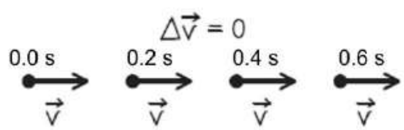
2.9.11 Observe and analyze

Class: *Equipment per group:* whiteboard and markers.

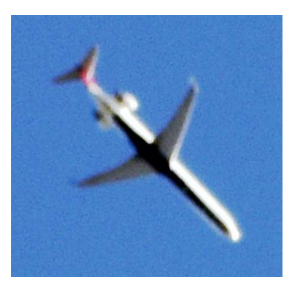
Collaborate together with your group to figure this out: Daniel fixed a camera on a tripod and took four successive photos of an airplane that was flying above him. The time interval between the photos was 0.2 s (see the first figure below; a straight line was added later to help you compare the position of the airplane in the different photos).

a. Draw a motion diagram for the airplane.

Using ruler you can find out that the airplane moves about the same distance in every time interval, therefore it moves with constant speed. The corresponding motion diagram looks like this



b. Estimate the length of the airplane using the magnified photo (shown in the second figure below) of the airplane and data that you can find on the Internet. Indicate any assumptions that you made.

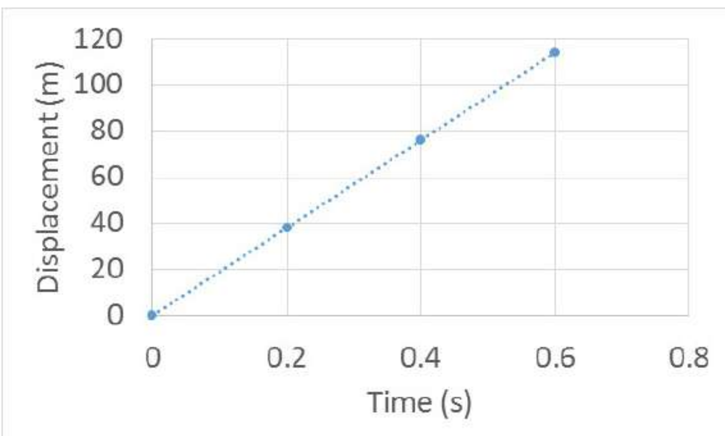


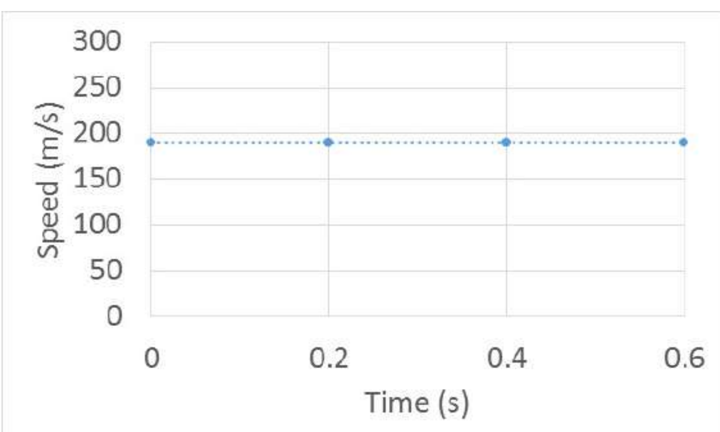
Notice that the airplane has the engine on the tail. On the Internet you can find that the length of the airplanes that have engines on a tail varies from 26 m to 47 m. The numerical values in the answers to this activity depend on what length of the airplane you use. We will assume this was

Boeing 717, which has engines on the tail and is 38 m long (data from Internet).

c. Draw labeled position-versus-time and velocity-versus-time graphs for the airplane's motion. Indicate your assumptions (they will relate to the airplane you choose and the direction of motion) and the choice of a coordinate system. Make sure the axes of your graphs contain units. Note that you will need a ruler to solve this problem.

We will assume that the x axis points in the direction of airplane motion and that at t = 0 the airplane was at x = 0. Analyzing the photos, we find out that the airplane moves for about the distance equal to its own length in the time interval between the successive frames. This allows us to determine the speed of the airplane ($v = \dfrac{38\ \text{m}}{0.02\ \text{s}} = 190\ \text{m/s} = 680\ \text{km/h}$ *) and to draw the graphs:*





2.9.12 Linearize

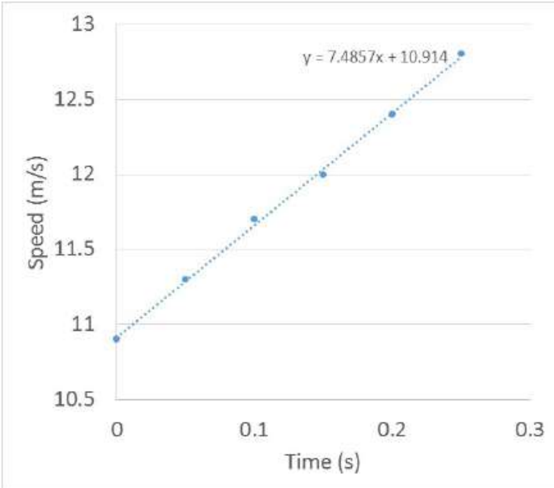
Class: *Equipment per group:* none.

You are testing your new motion detector at an open window when suddenly a stuffed Piglet passes by. While Piglet is moving downward, you manage to record the data shown in the table at right about its motion with your motion detector.

t **(s)**	***v*** **(m/s)**
0.00	10.9
0.05	11.3
0.10	11.7
0.15	12.0
0.20	12.4
0.25	12.8

Estimate how far above your window lives the child who dropped Piglet. What else can you estimate from these data? Indicate any assumptions that you made (Note: the acceleration of falling Piglet may be significantly less than g).

First, we plot the velocity-versus-time graph using the data in the table:



The linear shape of the graph suggests that the Piglet was moving at constant acceleration during the time of measurement.

The best-fit linear equation describes time dependence of the speed of the Piglet (we rounded the values to two significant digits, as this is the lowest precision of the original data):

$$v_y = 11 \text{ m/s} + 7.5 \text{ m/s}^2 \times t$$

The slope of the line is equal to the acceleration of the Piglet $a_y = 7.5 \text{ m/s}^2$ *(y axis points vertically down). As expected, this acceleration is smaller than g.*

Assuming the Piglet was released from rest, the previous equation allows you to estimate the time when the Piglet was released, knowing that $t = 0$ *is the moment when you started the measurement.*

At the moment when the Piglet is released, $v_y = 0 \text{ m/s}$*, therefore*

$$t_{\text{rel}} = -\frac{10.9 \text{ m/s}}{7.49 \text{ m/s}^2} = -1.5 \text{ s}.$$

Again, we rounded the values to two significant digits. You can obtain the same result graphically, by extending the best-fit line to the right until it crosses the horizontal axis.

The Piglet was released about 1.5 seconds before you started the measurement. After the Piglet was released and before it reached you, it traveled a distance:

$$y_1 = \frac{1}{2} a_y t_{\text{rel}}^{\;2} = 0.5 \times 7.49 \text{ m/s}^2 \times (1.455 \text{ s})^2 = 7.9 \text{ m}$$

Therefore, the person who released the Piglet lives about 8 m above you, which is a reasonable height for a typical residential building with several floors.

You can also estimate the distance traveled by the Piglet during the time of measurement. In order to do so, you need to choose the time origin at the moment when the Piglet was released (you do this by adding 1.46 s *to every time reading).*

$$\Delta y = y_{\mathrm{f}} - y_1 = \frac{1}{2} a_y t_{\mathrm{f}}^2 - \frac{1}{2} a_y t_{\mathrm{rel}}^2 =$$
$$= 0.5 \times 7.49\ \mathrm{m/s^2} \times (0.25\ \mathrm{s} + 1.46\ \mathrm{s})^2 - 0.5 \times 7.49\ \mathrm{m/s^2} \times (0 + 1.46\ \mathrm{s})^2 = 3.0\ \mathrm{m}$$

The Piglet moved about 3 m during the time of measurement.

2.9.12 Design an experiment

Lab: *Equipment per group:* a cotton ball, a stopwatch, and a meter stick.

Work together with your group.

a. Describe in detail (including a sketch) an experiment that you can perform to determine whether a cotton ball falls with constant speed, constant acceleration, or changing acceleration.

b. Write the physical quantities that you will measure and the quantities that you will calculate.

c. List experimental uncertainties and how you will minimize them.

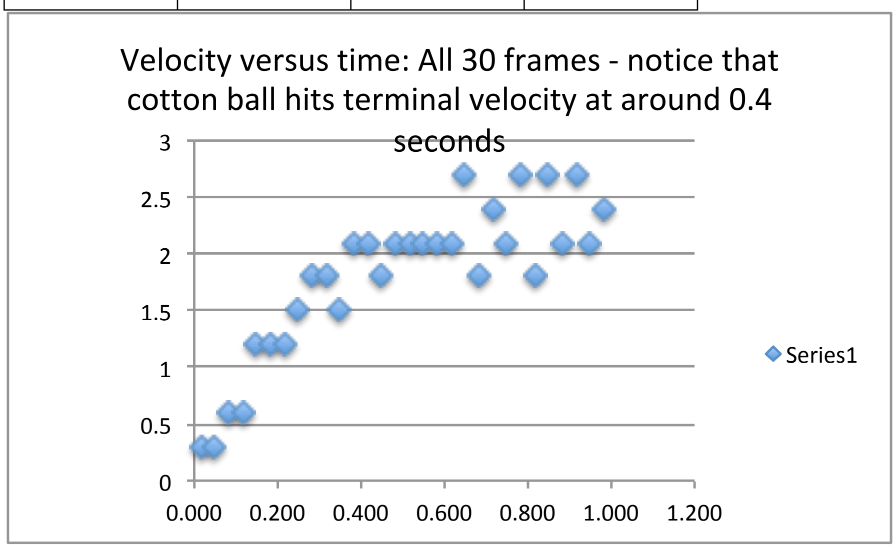
d. Perform the experiment; record the data in a table, and use a best-fit curve for the data to make a judgment.

e. Write your analysis and conclusion about the cotton ball's motion.

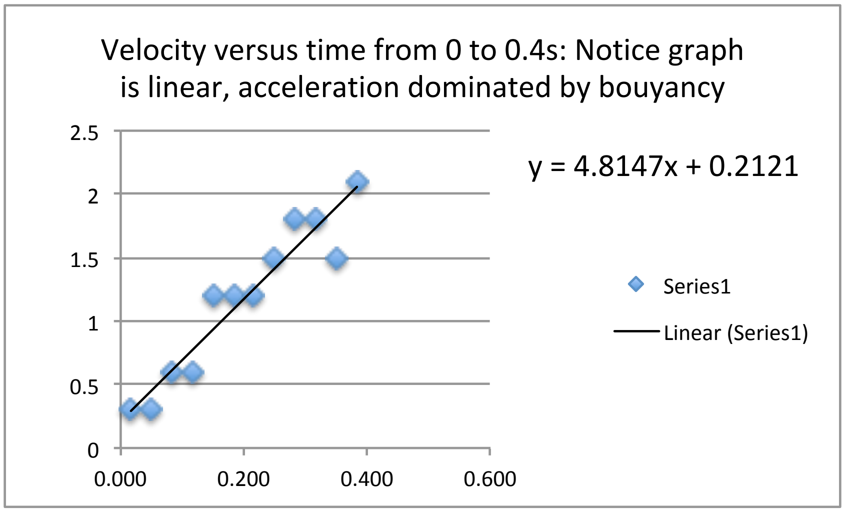
Some notes for the instructor: We are including some real data from a falling cotton ball experiment. This is important so that you (the instructor) understand the factors at play.

Time (s)	Position	v(avg)	t avg
0.000	0		
0.033	0.01	0.3	0.017
0.067	0.02	0.3	0.050
0.100	0.04	0.6	0.083
0.133	0.06	0.6	0.117
0.167	0.1	1.2	0.150
0.200	0.14	1.2	0.183
0.233	0.18	1.2	0.217
0.267	0.23	1.5	0.250
0.300	0.29	1.8	0.283

0.333	0.35	1.8	0.317
0.367	0.4	1.5	0.350
0.400	0.47	2.1	0.383
0.433	0.54	2.1	0.417
0.467	0.6	1.8	0.450
0.500	0.67	2.1	0.483
0.533	0.74	2.1	0.517
0.567	0.81	2.1	0.550
0.600	0.88	2.1	0.583
0.633	0.95	2.1	0.617
0.667	1.04	2.7	0.650
0.700	1.1	1.8	0.683
0.733	1.18	2.4	0.717
0.767	1.25	2.1	0.750
0.800	1.34	2.7	0.783
0.833	1.4	1.8	0.817
0.867	1.49	2.7	0.850
0.900	1.56	2.1	0.883
0.933	1.65	2.7	0.917
0.967	1.72	2.1	0.950
1.000	1.8	2.4	0.983



I suspect that the "noise" in the v versus t graph happened because the (irregularly shaped) ball started to tumble and rotate as it fell.



If you want to guarantee that your students observe deviations from constant acceleration, you need to encourage them to sample drops 1 meter or higher. This is especially necessary if they are conducting an experiment where they drop from different heights and measure the time with a stopwatch.

While cotton balls fall at a constant acceleration for the first 0.5m – 1m, it looks like the acceleration is significantly less than 9.8m/s/s. This is likely because of buoyancy.

2.9.13 Reading exercise

Read Section 2.9 in the textbook and answer Review Question 2.9.

2.9.14 Observe and analyze

Class: *Equipment per group:* whiteboard and markers.
[https://mediaplayer.pearsoncmg.com/assets/_frames.true/sci-phys-egv2e-alg-2-9-14]

Together with your group, watch the video at link above. Take relevant data from the video, draw motion diagram(s), and plot appropriate graphs to fully analyze the motion of the cotton ball. Write up a short report of your results and analysis. In conclusion, what can you say about the motion of the cotton ball from your analysis?

See the data analysis in 2.9.12. Please note this is NOT the data taken from this video, but it should give you a sense of how to analyze the motion of the cotton ball.

2.9.15 Evaluate the solution

Lab: *Equipment per group:* whiteboard and markers, stopwatch, cotton ball, meter stick.

Allison proposes the following experiment to investigate the motion of cotton balls: Drop one ball from 1 m above a surface and record the time during which it falls. If it falls at constant speed, then when it is dropped from a height of 2 m, the time of fall should double. If it falls at constant acceleration, the time should be equal to the square root of twice the time it took the ball to fall from 1 m. Now that the predictions for the two models are made, perform the experiment with the aid of your group members, dropping the ball from first 1 m and then 2 m and record the times of fall. Depending on the outcome, discard one or both models of the motion. What do you think of Allison's design, her description of the experiment, and her mathematical analysis? Compare your ideas with another group.