1.4 Absolute Values

Solving Absolute Value Equations

By putting into one of 3 categories

What is the definition of “Absolute Value”?

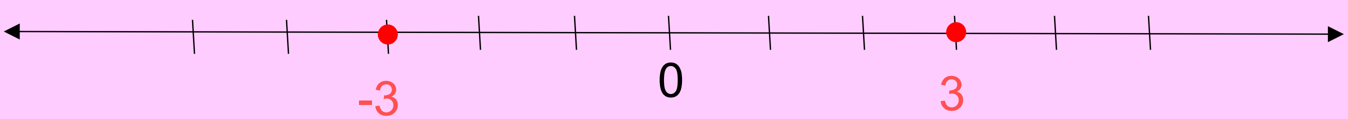
____

____

Mathematically, $|a| = \begin{cases} a \text{ if } a \geq 0 \\ -a \text{ if } a < 0 \end{cases}$

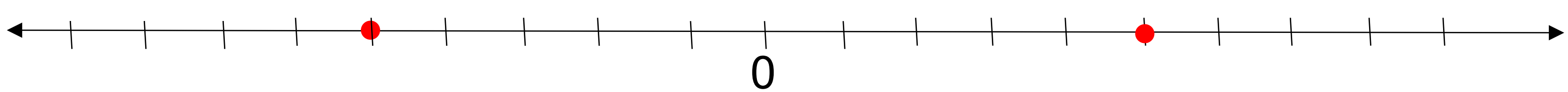
____

For example, in $|x| = 3$, where could x be?



To solve these situations,

Consider $|3x - 2| = 10$



That was Category 1

Category 1 : ______________________________

____

____

$$|x| = a \quad \Rightarrow \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

ExampleCase 1Case 2

$$|x+5| = 3$$

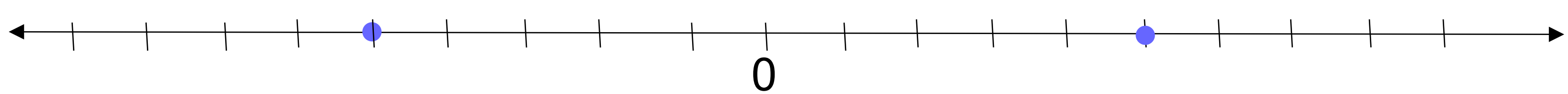
Absolute Value Inequalities

Think logically about another situation.

What does $|x| \leq a$ mean?

For instance, in the equation $|x+6| \leq 5$,

____



How does that translate into a sentence?

____

Now solve for x.

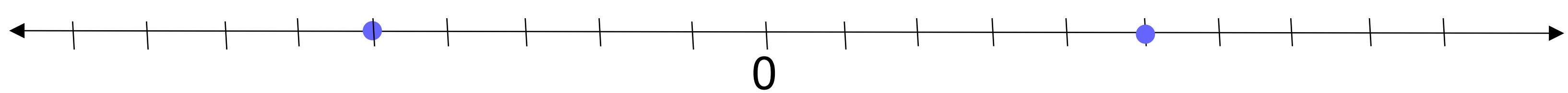
This is Category 2: when x is less than a number

$$|x| \leq a \quad \Rightarrow \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Absolute Value Inequalities

What does $|x| \geq a$ mean?

In the equation $|2x+1| \geq 11$,



$$|x| = a \Rightarrow$$ ______________________________

$$|x| \leq a \Rightarrow$$ ______________________________

Less than = And statement

$$|x| \geq a \Rightarrow$$ ______________________________

Greater than = Or statement

Note: is the same as ; is the same as [illegible]; just have the sign in the rewritten equation match the original.

Isolate Absolute Value

- ____

$$3|x+5| = 12$$

- ____

$$|x+5| = 4$$

- ____

When x is on both sides

$$|3x+4| = x-3$$

- ____
____
- ____

Case 1Case 2

Example inequality with x on other side

$$|2x-1| \le 5-x$$

Case 1Case 2

Examples

1. $|2x + 4| = 24$

Case 1 **Case 2**

2. $|x + 6| \leq 9$

Case 1 **Case 2**

3. $|2x + 1| > 11$

Case 1 **Case 2**

4. $2-5|x-5| \geq 27$

Whats wrong with this?

5. $|3x - 9| < 10$

6. $-4\left|-\frac{1}{2}x + 2\right| \leq -8$

7. $|3x + 4| \geq 2x + 6$

8. $|x + 2| < x = 7$

1-4 Compound Absolute Values Equalities and Inequalities

More than one absolute value in the equation

Some Vocab

- Domain- ______________________________
- Range- _______________________________
- Restriction- ___________________________

$$\frac{x+2}{x-4}=5$$

- ____
  ____

Find a number that works.

$$\left|x+2\right|+\left|x-1\right|>6$$

We will find a more methodical approach to find all the solutions.

In your approach, think about the values of this particular mathematical statement in the 3 different areas on a number line.

$$|2x+2|+|x-5|>9$$

- ____

- ____

- ____

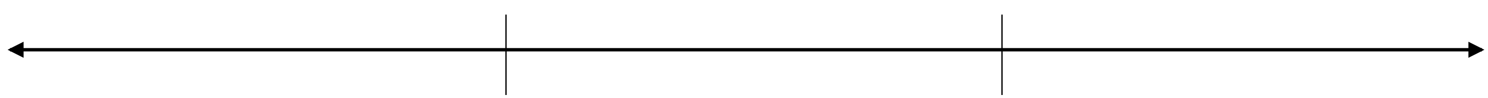
It now forms 3 different areas or cases on the number line.

$$\left|2x+2\right|+\left|x-5\right|>9$$

- ______________________________

____

- ______________________________



Case 1 Domain $x \geq 5$ ____________________

____

____

____

____

Case 2 Domain $-1 \leq x \leq 5$

Case 3 Domain $x \leq -1$

Final Answers

- ______________________________
- ______________________________
- ______________________________

If you get an answer in any of the cases where the variable disappears and the answer is TRUE

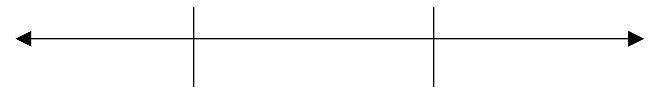
____

____

____

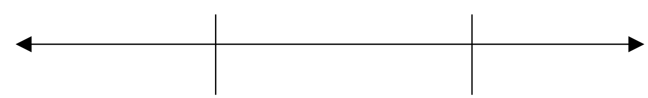
____

$$|3x-6|+|x+2|>12$$



$$\frac{|x-2|}{|x+4|}\geq 5$$

$$|2x+1|+|x+5|>1$$



1-5 Exponential Rules

You know these already

Review of the Basics

1. $a^m a^n =$ ______

2. $a^m \div a^n =$ ______

3. $(a^m)^n =$ ______

4. $(ab)^n =$ ______

5. $\left(\frac{a}{b}\right)^m =$ ______

6. $a^0 =$ ______

7. $a^1 =$ ______

8. $a^{-n} =$ ______

Practice problems

Simplify each expression

1. $x^9 \cdot x^3 =$; $\dfrac{x^9}{x^3} =$; $\dfrac{x^3}{x^9} =$

2. $4^n \cdot 4^{n-7} =$

3. $\left(\dfrac{2}{5}\right)^{-3} =$

4. $\left(\left(\dfrac{1}{3}\right)^3\right)^2 =$

5. $\dfrac{3x^{-3}y^{2}}{2^{-1}x^{2}y^{-5}}$

6. $(-3)^{2}(-2)^{3}$

7. $(-2x^{3}y)^{2}(-3x^{2}y^{2})^{3}$

8. $\dfrac{(x^{4}y^{3})^{2}}{(x^{-5}y^{3})^{3}}$

9. $-7(x-3y)^{-8}(12y-4x)$

10. $\dfrac{2^{5x-9}}{2^{5x-12}}$

11. $4 \cdot 2^{x} + 4 \cdot 2^{x}$

12. $3^{x} \cdot 3^{x+4} = 81$

Do NOT use calculators on the homework, please!! ☺

1-6 Radicals (**Day 1**) and Rational Exponents (Day 2)

What is a Radical?

In simplest term, it is a square root ($\sqrt{\ }$)

$\sqrt{\ }$ = ______________________

The Principle n^{th} root of a:

1. $\sqrt[n]{0} = 0$

If a < 0 and n is odd then $\sqrt[n]{a}$ is a negative number ___________________________

If a < 0 and n is even, __________________

____

Vocab

$$\sqrt[n]{a}$$

Lets Recall

$\sqrt{x^2} =$ ___ $\sqrt{x^4} =$ ___ $\sqrt{x^{20}} =$ ___

$\sqrt[3]{x^3} =$ ___ $\sqrt[3]{x^6} =$ ___ $\sqrt[3]{x^{33}} =$ ___

$\sqrt[5]{x^5} =$ ___ $\sqrt[5]{x^{10}} =$ ___ $\sqrt[5]{x^{60}} =$ ___

Rules of Radicals

1. $\sqrt[n]{a} \cdot \sqrt[n]{b} \Leftrightarrow$ ____________

2. $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} =$ _____

3. $\sqrt[m]{\sqrt[n]{a}} =$ _______

Practice problems

Simplify each expression

1. $\sqrt{20}$

2. $\sqrt[3]{81}$

3. $\sqrt[3]{3^{17}}$

4. $\sqrt[3]{4} \cdot \sqrt[3]{4}$

5. $\sqrt[3]{\sqrt{64}}$

6. $\sqrt{50} + \sqrt{72}$

7. $3\sqrt{27x^2} + 7\sqrt{12x^2}$

8. $\sqrt[3]{\frac{1}{24}} \cdot \sqrt[3]{-81x^6}$

1-6 Radicals (Day 1) and Rational Exponents (**Day 2**)

What is a Rational Exponent?

Don't be overwhelmed by fractions!

These problems are not hard, as long as you remember what each letter means. Notice I used "p" as the numerator and "r" as the denominator.

p= ___________________________________

r= ___________________________________

____

____

____

Practice problems

Simplify each expression

1. $\dfrac{9^{\frac{1}{2}}}{27^{\frac{1}{3}}}$

2. $\left(\dfrac{1}{36}\right)^{-\frac{1}{2}}$

3. $4^{-\frac{3}{2}}$

4. $(-8)^{\frac{2}{3}}$

5. $\left(\dfrac{16x^4}{y^{-8}}\right)^{\frac{3}{4}}$

6. $(243)^{\frac{2}{5}}$

The last part of this topic

What is wrong with this number?

$$\frac{1}{3^{\frac{1}{2}}}$$

Rationalizing

If you see a single radical in the denominator

7. $\dfrac{6}{\sqrt{8}}$

8. $\dfrac{6}{\sqrt[3]{2}}$

Rationalizing

If you see 1 or more radicals in a binomial, what can we do?

10. $\dfrac{7}{\sqrt{6}-\sqrt{5}}$

11. $\dfrac{5}{5+\sqrt{18}}$

What is a Conjugate?

The conjugate of a + b ________. Why?

The conjugate ______________________________

____

____

____

Rationalizing

Multiply top and bottom by the conjugate! Its MAGIC!!

10. $$\frac{7}{\sqrt{6}-\sqrt{5}}$$

11. $$\frac{5}{5+\sqrt{18}}$$

12. $\dfrac{6}{\sqrt{3}} + \sqrt{75} - 3\sqrt{3}$

13. $\left(\dfrac{1}{8} + \dfrac{1}{27}\right)^{\frac{1}{3}}$

1.7 Fundamental Operations

Terms

- Monomial ______________________________
- Binomial _______________________________
- Trinomial ______________________________

Standard form:

$$x^3 + 4x^2 + x + 7$$

Collecting Like terms

$$5x^3 + 7x^2 - x - 3 - 4x^3 - x^2 - x + 6$$

F____________________

O____________________

I ____________________

L____________________

$$(3x+2)(2x-5)$$

Examples

$$(x^2 - 3x + 5)(2x^3 + x^2 - 3x)$$

1-8 Factoring Patterns

What is the first step??

____

Why?

____

____

____

____

Perfect Square Trinomial

Factors as

Difference of squares

Sum/Difference of cubes

3 terms but not a pattern?

This is where you use combinations of the first term with combinations of the third term that collect to be the middle term.

$$6x^2 - 10x - 4$$

4 or more terms?

$$x^4 + 3x^3 - 8x - 24$$

Examples

$$9x^2 - 4y^2 \longrightarrow$$

$$(x + y)^3 + 1 \longrightarrow$$

$$x^6 - y^6 \longrightarrow$$

Solve using difference of Squares

$5x - 7 \longrightarrow$

$16y - 5x^2 \longrightarrow$

Solve using sum of cubes

$27x + 1 \longrightarrow$

Examples of Grouping

$$a^2 - 2b + 2a - ab$$

$$a^2 - 4a + 4 - y^2$$

$$a^3 + 6a^2 + 12a + 8$$

$$4x^5 - x^3 + 108x^2 - 27$$

1.9 Fundamental Operations

What are the Fundamental Operations?

Addition, Subtraction, Multiplication and Division

We will be applying these fundamental operations to rational expressions.

This will all be review. We are working on the little things here.

Simplify the following

Hint: ______________________________

____

____

1. $$\frac{x^2 - 9x + 20}{x - 4}$$

2. What is the domain of problem 1?

5. $$\frac{3x^2-10x+3}{3x^2-13x+12}\cdot\frac{3x^2+17x-28}{x^2-49}$$

6. $$\frac{x^2 - 1}{x^3 - 1} \div \frac{x + 1}{x^2 + x + 1}$$

7. $$\frac{4}{x^3-1}+\frac{7}{x^2-1}$$

9. $$\frac{x^4+3x^2-2x-2}{x-1}$$

Lets Watch.

Polynomial Long Division

$$x-1\overline{)x^4+0x^3+3x^2-2x-2}$$

Polynomial Long Division

$$2x+1\overline{\big)4x^3+0x^2+0x-1}$$

Classwork

1. $$\frac{\frac{x+2}{x+3}}{\frac{x+5}{(x+3)^2}}$$

2. $$\frac{x^{-2}-y^{-2}}{\frac{1}{x}-\frac{1}{y}}$$

1-10 Introduction to Complex Numbers

What is a complex number?

To see a complex number we have to first see where it shows up

Solve both of these

$$x^2 - 81 = 0 \qquad\qquad x^2 + 81 = 0$$

Um, no solution????

$x = \pm\sqrt{-81}$ does not have a real answer.

It has an “imaginary” answer.

To define a complex number ________________

____

This new variable is “ i “

Definition: $\mathrm{i} = \sqrt{-1}$

Note: ______________________________ $\sqrt{-1}$

So, following this definition:

____

____

____

____

And it cycles….

$$i = \sqrt{-1}$$

$$i^2 = -1$$

$$i^3 = -i$$

$$i^4 = 1$$

Do you see a pattern yet?

What is that pattern?

We are looking at the remainder when the power is divided by 4.

Why?

Try it with

Hints to deal with i

1. ________________________________

2. ________________________________

____

- ________________________________

____

Examples

1. $\sqrt{-36} \cdot \sqrt{-81}$

2. $\sqrt{-36} + \sqrt{-81}$

OK, so what is a complex number?

A complex number comes in the form

$$a + bi$$

Lets try these 4 problems.

1. $(8+3i)+(6-2i)$

2. $(8+3i)-(6-2i)$

3. $(8+3i)\times(6-2i)$

5. $6i^{-5}$

6. $\frac{6 - i}{4} + \frac{5}{2 + i}$

Try on your own

7. $i^{23} + 2i^{13} - 3i^{16}$