Little Black Book of Algebra II Properties
Unit 6 - Exponential and Logarithmic Functions

6.1 Laws of Exponents – write rules for adding, subtracting, multiplying and dividing values with exponents, raising an exponent to a power, and using negative and fractional exponents.

6.2 Solving Exponential Equations – write the rules for solving two types of exponential equations: same base and different bases (e.g., solve $2^x = 8^{x-1}$ without calculator; solve $2^x = 3^{x-1}$ with and without calculator).

6.3 Exponential Function with Base *a* – write the definition, give examples of graphs with $a > 1$ and $0 < a < 1$, and locate three ordered pairs, give the domains, ranges, intercepts, and asymptotes for each.

6.4 Exponential Regression Equation - give a set of data and explain how to use the method of finite differences to determine if it is best modeled with an exponential equation, and explain how to find the regression equation.

6.5 Exponential Function Base *e* – define *e*, graph $y = e^x$ and then locate 3 ordered pairs, and give the domain, range, asymptote, intercepts.

6.6 Compound Interest Formula – define continuous and finite, explain and give an example of each symbol.

6.7 Inverse Functions – write the definition, explain one-to-one correspondence, give an example to show the test to determine when two functions are inverses, graph the inverse of a function, find the line of symmetry and the domain and range, explain how to find inverse analytically and how to draw an inverse on calculator.

6.8 Logarithm – write the definition and explain the symbols used, define common logs, characteristic, and mantissa, and list the properties of logarithms.

6.9 Laws of Logs and Change of Base Formula – list the laws and the change of base formula and give examples of each.

6.10 Solving Logarithmic Equations – explain rules for solving equations, identify the domain for an equation, find $\log_2 8$ and $\log_{25} 125$, and solve each of these equations for *x*: $\log_x 9 = 2$, $\log_4 x = 2$, $\log_4(x-3)+\log_4 x=1$).

6.11 Logarithmic Function Base *a* – write the definition, graph $y = \log_a x$ with $a < 1$ and $a > 1$ and locate three ordered pairs, identify the domain, range, intercepts, and asymptotes, and find the domain of $y = \log(x^2+7x+ 10)$.

6.12 Natural Logarithm Function – write the definition and give the approximate value of *e*, graph $y = \ln x$ and give the domain, range, and asymptote, and locate three ordered pairs, solve $\ln x = 2$ for *x*.

6.13 Exponential Growth and Decay – define half-life and solve an example problem, give and solve an example of population growth using $A(t) = Pe^{rt}$.

Algebra II – Date

Simplify and explain in words the law of exponents used

1) $a^2a^3 =$

2) $\dfrac{b^7}{b^3} =$

3) $(c^3)^4 =$

4) $2x^5 + 3x^5 =$

5) $(2x)^3 =$

6) $(a + b)^2 =$

7) $x^0 =$

8) $2^{-1} =$

Unit 6, Activity 2, Graphing Exponential Functions Discovery Worksheet

Name_____ Date_____

Exponential Graph Transformations

	Equation	Sketch	Domain	Range	*x*–intercept	*y*-intercept
1	$f(x) = 2^x$					
2	$f(x) = 3^x$					
3	$f(x) = 5^x$					
4	$f(x) = \frac{1}{2}^x$					
5	$f(x) = \frac{1}{5}^x$					
6	$f(x) = \frac{1}{8}^x$					
7	$f(x) = -2^x$					
8	$f(x) = -\frac{1}{2}^x$					

Analysis of Exponential Graphs

Answer the following questions concerning the graphs of exponential functions:

1) What point do most of the graphs have in common? _____

2) Which ones do not have that point in common and what is different about them? _____

_____

3) What is the effect of putting a negative sign in front of b^x? _____

4) What happens to the graph as b increases in problems 1, 2, and 3? _____

5) Describe the graph if $b = 1$. _____

6) Describe the difference in graphs for problems 1 through 6 if $b > 1$ and if $0 < b < 1$.

_____

7) What is the domain of all the graphs? _____

8) What is the range of the graphs? _____

9) Are there any asymptotes? If so, what is the equation of the asymptote? _____

10) Predict what the graph of $y = 2^{(x-3)} + 4$ would look like before you graph on your calculator and explain why.

_____

Graph without a calculator and check yourself on the calculator. What is the new y-intercept and asymptote?

11) Predict what the graph of $y = 2^{-x}$ would look like before you graph on your calculator and explain why.

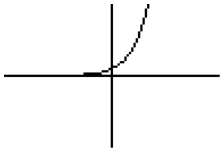
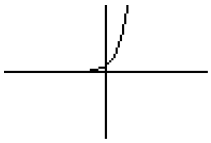
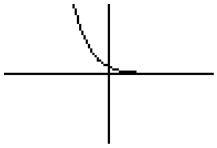
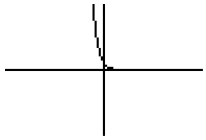
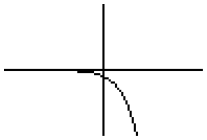
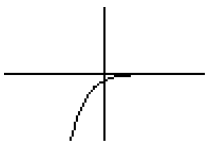
_____

Graph without a calculator and check yourself on the calculator. Is it similar to any of the previous graphs and why?

Unit 6, Activity 2, Graphing Exponential Functions Discovery Worksheet with Answers

Name *Key* Date

Exponential Graph Transformations

	Equation	Sketch	Domain	Range	*x*–intercept	*y*-intercept
1	$f(x) = 2^x$		*all reals*	$y > 0$	*none*	*(0, 1)*
2	$f(x) = 3^x$		*all reals*	$y > 0$	*none*	*(0, 1)*
3	$f(x) = 5^x$		*all reals*	$y > 0$	*none*	*(0, 1)*
4	$f(x) = \frac{1}{2}^x$		*all reals*	$y > 0$	*none*	*(0, 1)*
5	$f(x) = \frac{1}{5}^x$		*all reals*	$y > 0$	*none*	*(0, 1)*
6	$f(x) = \frac{1}{8}^x$		*all reals*	$y > 0$	*none*	*(0, 1)*
7	$f(x) = -2^x$		*all reals*	$y < 0$	*none*	*(0, –1)*
8	$f(x) = -\frac{1}{2}^x$		*all reals*	$y < 0$	*none*	*(0, –1)*

Analysis of Exponential Graphs

Answer the following questions concerning the graphs of exponential functions:

1) What point do most of the graphs have in common? ***y–intercept (0, 1)***

2) Which ones do not have that point in common and what is different about them?

   #7 and 8, negative leading coefficient

3) What is the effect of putting a negative sign in front of b^x? ***rotate on the x–axis***

4) What happens to the graph as b increases in problems 1, 2, and 3? ***it gets steeper***

5) Describe the graph if $b = 1$. ***horizontal line at y = 1***

6) Describe the difference in graphs for # 1 through #6 if $b > 1$ and if $0 < b < 1$.

   If b > 1 then the end-behavior as x approaches ∞ is ∞ and as x approaches – ∞ is 0.

   If b < 1, then it has the opposite end-behavior

7) What is the domain of all the graphs? ***all real numbers***

8) What is the range of the graphs? ***#1–6 y > 0, #7 and 8 y < 0***

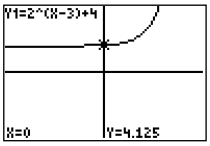
9) Are there any asymptotes? If so, what is the equation of the asymptote? ***y = 0***

10) Predict what the graph of $y = 2^{(x-3)} + 4$ would look like before you graph on your calculator and explain why.

    Shift right 3 and up 4

    Graph without a calculator and check yourself on the calculator. What is the new y–intercept and asymptote?

    y–intercept: (0, 4.125) Asymptote: y = 4

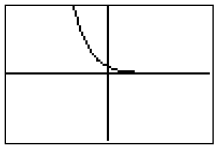


11) Predict what the graph of $y = 2^{-x}$ would look like before you graph on your calculator and explain why.

    rotate $y = 2^x$ on the y–axis

    Graph without a calculator and check yourself on the calculator. Is it similar to any of the previous graphs and why? ***Similar to*** $f(x) = \frac{1}{2}^x$ ***because***

$$2^{-x} = \frac{1}{2^x} = \left(\frac{1}{2}\right)^x$$



Unit 6, Activity 3, Exponential Regression Equations

Name_____ Date_____

Real-World Exponential Data

Enter the following data into your calculator:

To enter data on a TI-84 calculator: STAT, 1:Edit, *enter data into L_1 and L_2 . To set up the plot of the data:* 2ND , [STAT PLOT] (above Y=), 1:PLOT1, ENTER, On, Type:  Xlist: L_1, Ylist: L_2, Mark *(any). To graph the scatter plot:* ZOOM, 9: ZoomStat

Wind tunnel experiments are used to test the wind friction or resistance of an automobile at the following speeds.

Speed (mph)	Resistance (lbs)
10	6.4
21	9.2
34	17.0
40	22.4
45	30.2
55	59.2

Determine a regression equation for the data by entering an equation in your calculator in the form $f(x) = AB^x$ with numbers for the constants *A* and *B*. Change the constants until your graph matches the data or use the Transformation APPS. Do not use the regression feature of the calculator.

(1) Write your equation and discuss why you chose the values *A* and *B*.

(2) When each group is finished, write your equation on the board. Enter all the equations from the other groups into your calculator and vote on which one is the best fit. Discuss why.

(3) Use the best fit equation determined by the class as the best fit to predict the resistance of a car traveling at 50 mph and 75 mph.

(4) At what speed is the car going when the resistance is 25 lbs? Discuss the method you used to find this.

Method of Finite Differences

(1) Evaluate the following table of data using the method of finite differences to determine which data represents a linear, quadratic, or exponential function.

Linear: _____ Quadratic: _____ Exponential:_____

x	***f(x)***	***g(x)***	***h(x)***
0	3	3	3
1	5	4	4
2	7	7	6
3	9	12	10
4	11	19	18

(2) Discuss what happens in the method of finite differences for an exponential function.

(3) Explain the limitations of predictions based on organized sample sets of data. (i.e., Why can't you use Method of Finite Differences on real world data?)

(4) Make a scatter plot on your calculator and find the regression equations for each by either changing constants in Y = or using the Transformation APPS. *(Hint: The linear function is in the form y = mx + b, the quadratic function is in the form* $y = x^2 + b$*, and the exponential function is in the form* $f(x) = b^x + D$*.)*

f(x) = _____ g(x) = _____ h(x) = _____

(5) Use the regression feature of your calculator to find the exponential regression equation (STAT, CALC, 0: ExpReg L1, L2, Y_1) and discuss the differences. Which is better, yours or the calculators?

Name_____ Date_____

Real World Exponential Data

Enter the following data into your calculator:

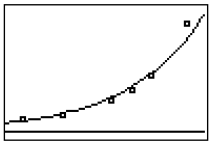
To enter data on a TI 84 calculator: STAT, 1:Edit, *enter data into* L_1 *and* L_2 *. To set up the plot of the data:* 2ND , [STAT PLOT] (above Y=), 1:PLOT1, ENTER, On, Type: Xlist: L_1, Ylist: L_2, Mark *(any). To graph the scatter plot:* ZOOM, 9: ZoomStat

Wind tunnel experiments are used to test the wind friction or resistance of an automobile at the following speeds.

Speed (mph)	Resistance (lbs)
10	6.4
21	9.2
34	17.0
40	22.4
45	30.2
55	59.2

Determine a regression equation for the data by entering an equation in your calculator in the form $f(x) = AB^x$ with numbers for the constants *A* and *B*. Change the constants until your graph matches the data or use the Transformation APPS. Do not use the regression feature of the calculator.

(1) Write your equation and discuss why you chose the values *A* and *B*.



Answers will vary $f(x) = 3.493(1.050)^x$

(2) When each group is finished, write your equation on the board. Enter all the equations from the other groups into your calculator and vote on which one is the best fit. Discuss why.

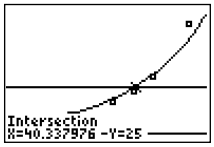
Discussions will vary.

(3) Use the best fit equation determined by the class as the best fit to predict the resistance of a car traveling at 50 mph and 75 mph.

f(50) = 40.058 lbs, f(75) = 135.660 lbs. Actual student answers will vary based on the equation chosen in #1 by the class.

(4) At what speed is the car going when the resistance is 25 lbs? Discuss the method you used to find this.

The car will be going approximately 40. 338 mph. Some students will trace to a y = 25, but the most accurate way is to graph the line y = 25 and find the point of intersection.



Unit 6, Activity 3, Exponential Regression Equations with Answers

Method of Finite Differences

(1) Evaluate the following table of data using the method of finite differences to determine which data represents a linear, quadratic, or exponential function.

Linear: ***f(x)*** Quadratic: ***g(x)*** Exponential: ***h(x)***

x	***f(x)***	***g(x)***	***h(x)***
0	3	3	3
1	5	4	4
2	7	7	6
3	9	12	10
4	11	19	18

(2) Discuss what happens in the method of finite differences for an exponential function.

In an exponential function, the set differences are always the same each time you subtract.

(3) Explain the limitations of predictions based on organized sample sets of data. (i.e., Why can't you use Method of Finite Differences on real-world data?)

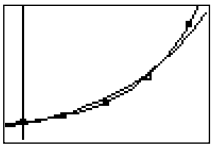
Real world data is not exactly exponential therefore the differences will vary.

(4) Make a scatter plot on your calculator and find the regression equations for each by either changing constants in Y= or using the Transformation APPS. *(Hint: The linear function is in the form $y = mx + b$, the quadratic function is in the form $y = x^2 + b$, and the exponential function is in the form $f(x) = b^x + D$.)*

$f(x) =$ ***$2x + 3$*** $g(x) =$ ***$x^2 + 3$*** $h(x) =$ ***$2^x + 2$***

(5) Use the regression feature of your calculator to find the exponential regression equation (STAT, CALC, 0: ExpReg L1, L2, Y_1) and discuss the differences. Which is better, yours or the calculators?

$y = 2.702(1.568)^x$. This equation did not have a vertical shift but changes the leading coefficient. My equation $y = 2^x + 2$ matched the data better.



Name_____ Due Date_____

When It Grows, It Grows Fast

This is an individual project, and each student must have different data, so be the first to print out your data and claim the topic. Make sure the data creates an exponential scatter plot.

Possible topics include: US Bureau of Statistics, Census, Stocks, Disease, Bacteria Growth, Investments, Land Value, Animal Population, number of stamps produced each year.

Directions:

(1) Search the Internet or newspaper and find data that is exponential in nature. You must have at least ten data points. Print out the data making sure to include the source and date of your data and bring to class to claim your topic.
(2) Plot the data using either your calculator or an Excel spreadsheet.
(3) Print out the table of data and the graph from your calculator or spreadsheet.
(4) Find the mathematical model (regression equation) for the data and state a reasonable domain and range for the topic.
(5) Compose a relevant question that can be answered using your model to extrapolate (make a future prediction) and answer the question.
(6) Type a paragraph (minimum five sentences) about the subject of your study. Discuss any limitations on using the data for predictions.
(7) Include all the above information on a ½ sheet of poster paper with an appropriate title, your name, date, and period.
(8) Present findings to the class.

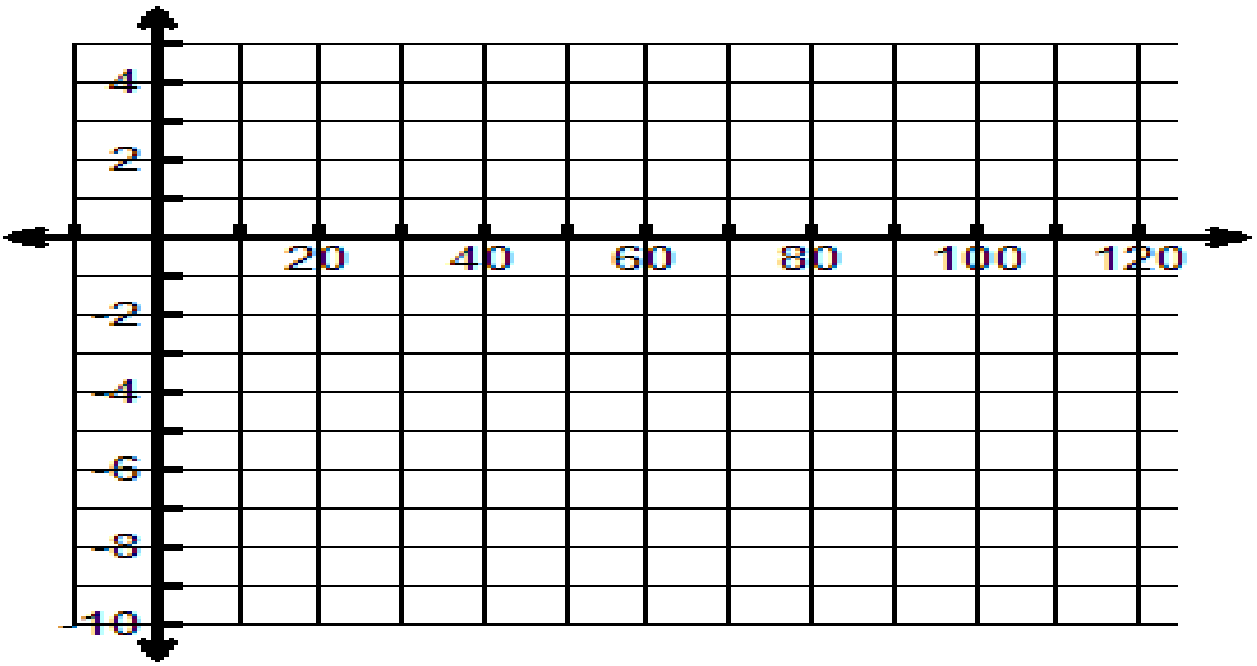
Grading Rubric for Exponential Data Research Project

10 pts. – table of data with proper documentation (source and date of data)
10 pts. – scatterplot with model equation from the calculator or spreadsheet (not by hand)
10 pts. – equations, domain, range
10 pts. – real world problem using extrapolation with correct answer
10 pts. – discussion of subject and limitations of the prediction
10 pts. – poster - neatness, completeness, readability
10 pts. – class presentation

Name_____ Date_____

Characteristics of Parent Logarithm Graph

(1) Graph $f(x) = \log x$ by plotting points by hand on the graph below for x = 1, 10, 100, and 0.1, and connecting the dots.



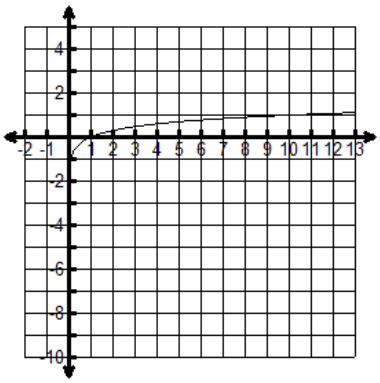
(2) Discuss the shape of the graph, its speed of increasing, its domain, range, asymptotes, end-behavior, and intercepts.

(3) Graph $g(x) = \log_5 x$ by plotting points by hand on the same graph for x = 1, 5, 25, and $\frac{1}{5}$, and connecting the dots. Discuss the similarities and differences in the graphs when changing the base. Why were these x–values chosen to plot?

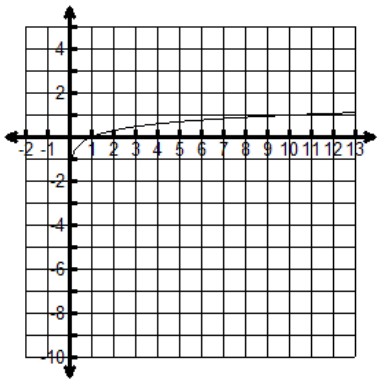
(4) Predict what a graph of log base 20 would look like. What x–values would you choose to plot?

Characteristics of Parent Logarithm Graph

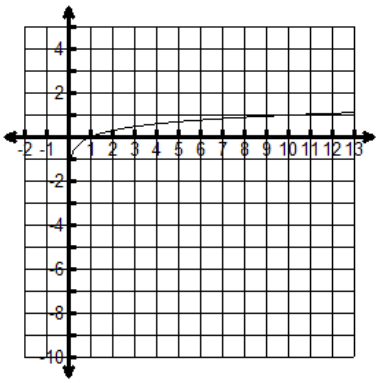
The graphs below contain the graph of $f(x) = \log x$. Graph the following functions by hand on these graphs using your knowledge of shifts and translations in the form $f(x) = A \log B(x - C) + D$. Label the asymptotes and x- and y- intercepts when possible.



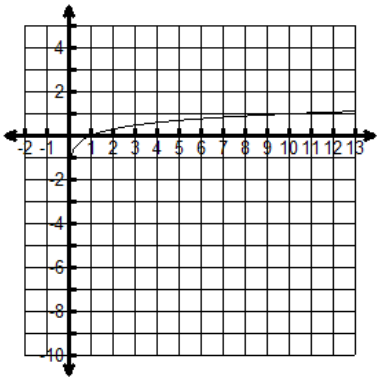
(1) $f(x) = \log (x - 2)$



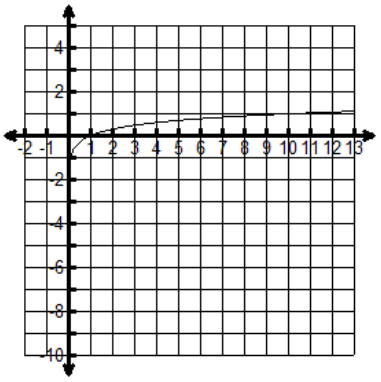
(2) $f(x) = \log x + 2$



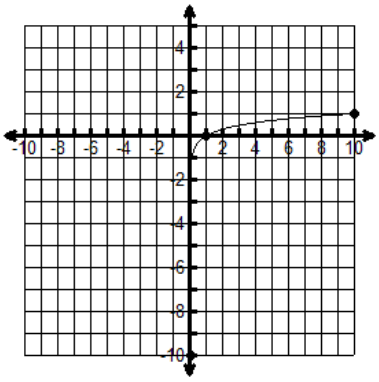
(3) $f(x) = \log 10x$



(4) $f(x) = 3 \log x$



(5) $f(x) = -\log x$



(6) $f(x) = \log(-x)$

(7) Discuss the similarities and differences in #2 and #3.

(8) Analytically find the x–intercepts of all the functions. Show your work.

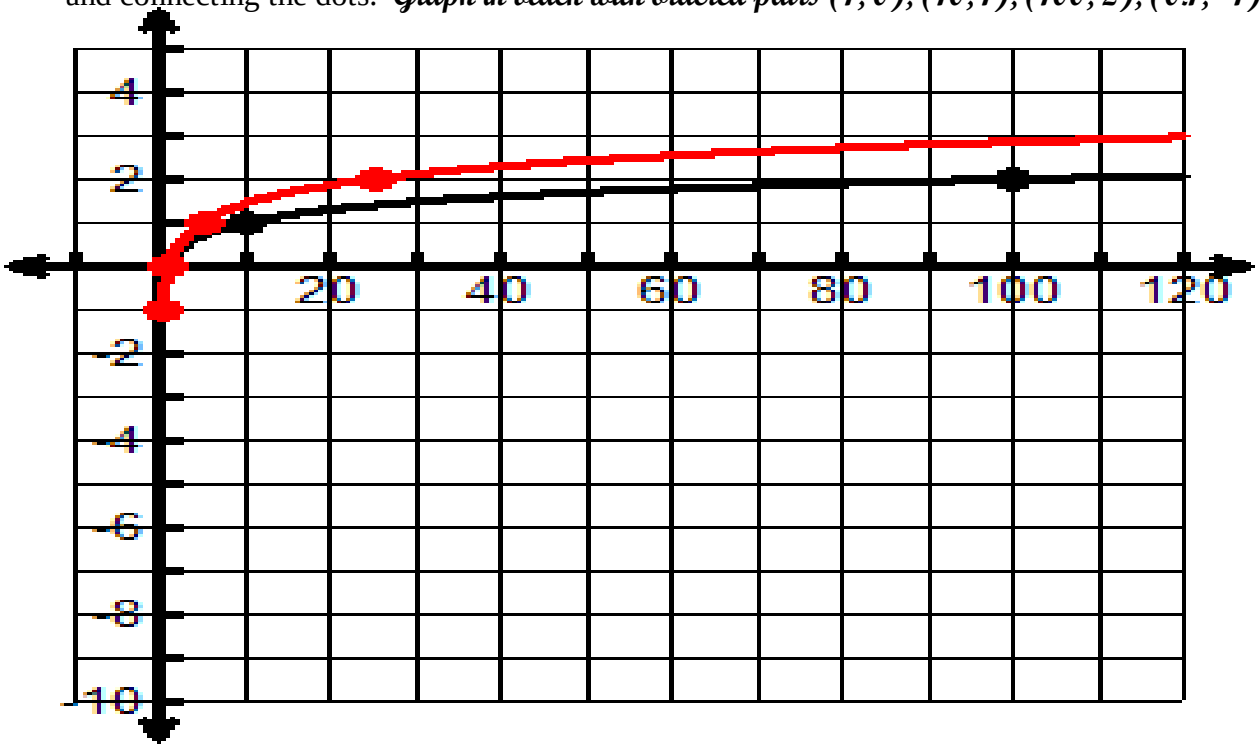
(9) Discuss domain restrictions on #1. Find the domain of $g(x) = \log(x + 1)$ and discuss why you cannot find $g(-2)$, but you can find $g(0)$. Can you find $f(0)$?

Unit 6, Activity 7, Graphing Logarithmic Functions Discovery Worksheet with Answers

Name_____ Date_____

Characteristics of Parent Logarithm Graph

(1) Graph $f(x) = \log x$ by plotting points by hand on the graph below for x = 1, 10, 100, and 0.1, and connecting the dots. ***Graph in black with ordered pairs (1, 0), (10, 1), (100, 2), (0.1, –1)***



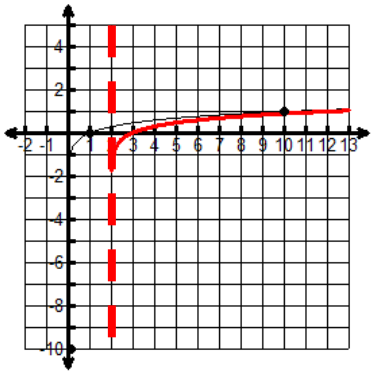
(2) Discuss the shape of the graph, its speed of increasing, its domain, range, asymptotes, end-behavior, and intercepts.
The graph increases very fast from 0 to 1, but then very slowly after 1. The domain is $x > 0$, range = all reals, asymptote $x = 0$. End-behavior: as $x \to 0$, $y \to -\infty$. As $x \to \infty$, $y \to \infty$. There is no y–intercept. The x–intercept is (1, 0).

(3) Graph $g(x) = \log_5 x$ by plotting points by hand on the same graph for x = 1, 5, 25, and $\frac{1}{5}$, and connecting the dots. Discuss the similarities and differences in the graphs when changing the base. Why were these x–values chosen to plot?
Graph in red. Ordered pairs (1, 0), (5, 1), (25, 2), $\left(\frac{1}{5}, -1\right)$. Both graphs have an x-intercept at (1, 0) and the same domain, range, vertical asymptote and end behavior, but $\log_5 x$ has higher y-values for every x value ≠ 1 so there is a vertical stretch. These x-values are powers of 5.

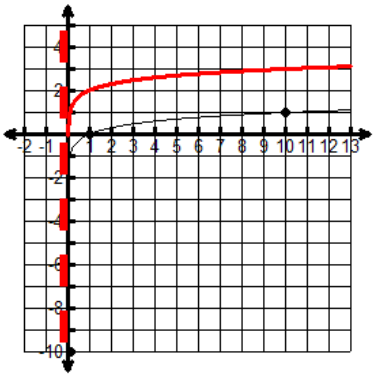
(4) Predict what a graph of log base 20 would look like. What x–values would you choose to plot? ***It would have the same x–intercept, domain, range, vertical asymptote, and end-behavior, but the y–values would be smaller for every x value ≠ 1 so there would be a vertical shrink. $x = \frac{1}{20}$, 1, 20, 400***

Characteristics of Parent Logarithm Graph

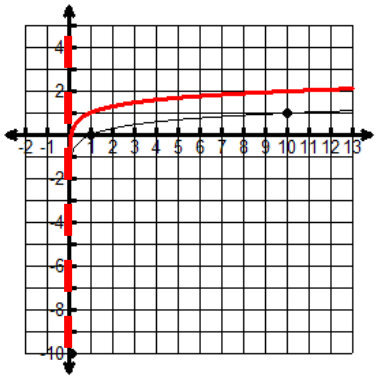
The graphs below contain the graph of $f(x) = \log x$. Graph the following functions by hand on these graphs using your knowledge of shifts and translations in the form $f(x) = A \log B(x - C) + D$. Label the asymptotes and x- and y- intercepts when possible.



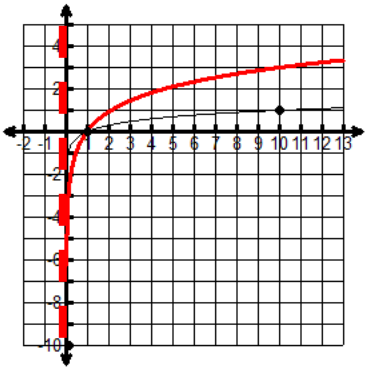
(1) $f(x) = \log (x - 2)$



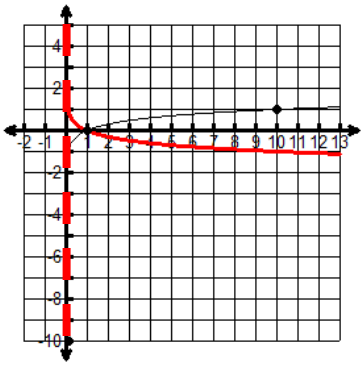
(2) $f(x) = \log x + 2$



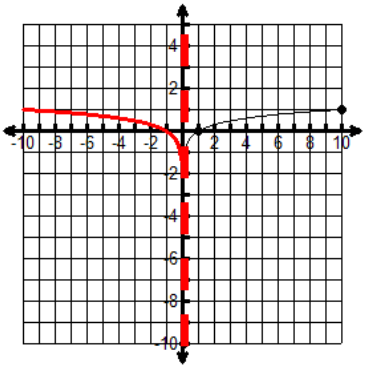
(3) $f(x) = \log 10x$



(4) $f(x) = 3 \log x$



(5) $f(x) = -\log x$



(6) $f(x) = \log(-x)$

(7) Discuss the similarities and differences in #2 and #3.

They both look like they were shifted up but actually #2 was shifted up. In #3 the domain was compressed. The point (10, 1) moved to (1, 1) and (100, 2) moved to (10, 2).

(8) Analytically find the x-intercepts of all the functions. Show your work

(1) $0 = \log (x-2) \Rightarrow 10^0 = x-2 \therefore x = 3$.

(2) $0 = \log x + 2 \Rightarrow -2 = \log x \therefore 10^{-2} = x$

(3) $0 = \log 10x \Rightarrow 10^0 = 10x \therefore x = 0.1$

(4) $0 = 3 \log x \Rightarrow 0 = \log x \Rightarrow 10^0 = x \therefore x = 1$

(5) $0 = -\log x \Rightarrow 0 = \log x \Rightarrow 10^0 = x \therefore x = 1$

(6) $0 = \log (-x) \Rightarrow 10^0 = -x \therefore x = -1$

(9) Discuss domain restrictions on #1. Find the domain of $g(x) = \log(x + 1)$ and discuss why you cannot find $g(-2)$ but can find $g(0)$. Can you find $f(0)$?

The domain of the parent function $f(x) = \log x$ is $x > 0$. Since the graph was shifted to the right 2, the domain was shifted to $x > 2$. The domain of $g(x)$ will be $x > -1$. -2 is not in the domain but 0 is. You cannot find $f(0)$ because 0 is not in the domain of the parent function.

Unit 6, Activity 10, Exponential Growth and Decay Lab

Name_____ Date_____

Exponential Growth Get a cup of about 50 *Skittles*® (or *M & M's*®). Start with 6 *Skittles*®. Pour out the *Skittles*®. Assume that the ones with the S showing have had babies and add that many more *Skittles*® to the cup. Repeat the process until all 50 *Skittles*® have been used.

Year	Population (Number you have after adding babies.)
0	6
1	
2	

(1) Create a scatter plot and find the exponential regression equation on your calculator.

(Round numbers 3 places behind the decimal.) _____

(2) Use your regression equation to predict the population in 20 years. _____

(3) In what year will the population be 100? _____

(4) What is your correlation coefficient? _____ Is this a good correlation? _____

Exponential Decay Pour out the *Skittles*® and remove the ones with the S showing as these represent an organism that has contacted a radioactive substance and has died. Repeat the process until one candy is left.

Year	Population (Number left without S)
0	50
1	
2	

(1) Create a scatter plot and find the exponential regression equation on your calculator.

(Round numbers 3 places behind the decimal.) _____

(2) Use your regression equation to predict the population in 10 years. _____

(3) In what year will the population be 5? _____

(4) What is your correlation coefficient? _____ Is this a good correlation? _____

Unit 6, Activity 13, Money in the Bank Research Project

Name_____ Due Date_____

In this research project, you will choose a financial institution in town or on the Internet. Each student in a class must choose a different bank, so claim your bank early. Contact the bank or go online to find out information about the interest rates available for two different types of accounts, and how they are compounded. Fill in the following information and solve the following problems. When all projects are in, you will report to the class.

Information Sheet: Name of bank, name of person you spoke to, bank address, and phone number or the URL if online, types of accounts, interest rates, and how funds are compounded.

Problem: Create a hypothetical situation in which you invest $500.

(1) Find the equation to model two different accounts for your bank.
(2) Determine how much you will have for each account at the end of high school, at the end of college, and when you retire. (Assume you finish high school in one year and college four years later and retire 50 years after you finish college.)
(3) Determine how many years it will take you to double your money for each account.
(4) Determine in which account you will put your money and discuss why.

Class Presentation: Display all information on a poster board and report to the class.

Grading Rubric for Data Research Project:

10 pts. – Information sheet: Name of bank, name of person you spoke to, bank address, and phone number or the URL if online, types of accounts, interest rates, and how funds are compounded (source and date of data)
10 pts. – Compound interest equation for each situation; account value for both accounts at the end of high school, college, and when you retire in 50 years (show all your work)
10 pts. – Solution showing your work of how long it will take you to double your money in each account
10 pts. – Discussion of where you will put your money and why
10 pts. – Poster - neatness, completeness, readability
10 pts. – Class presentation