Unit 5, Ongoing Activity, Little Black Book of Algebra II Properties

Little Black Book of Algebra II Properties
Unit 5 - Quadratic & Higher Order Polynomial Functions

5.1 *Quadratic Function* – give examples in standard form and demonstrate how to find the vertex and axis of symmetry.
5.2 *Translations and Shifts of Quadratic Functions* – discuss the effects of the symbol $\pm$ before the leading coefficient, the effect of the magnitude of the leading coefficient, the vertical shift of equation $y = x^2 \pm c$, the horizontal shift of $y = (x - c)^2$.
5.3 *Three ways to Solve a Quadratic Equation* – write one quadratic equation and show how to solve it by factoring, completing the square, and using the quadratic formula.
5.4 *Discriminant* – give the definition and indicate how it is used to determine the nature of the roots and the information that it provides about the graph of a quadratic equation.
5.5 *Factors, x-intercept, y-intercept, Roots, Zeroes* – write definitions and explain the difference between a root and a zero.
5.6 *Comparing Linear functions to Quadratic Functions* – give examples to compare and contrast $y = mx + b$, $y = x(mx + b)$, and $y = x^2 + mx + b$, explain how to determine if data generates a linear or quadratic graph.
5.7 *How Varying the Coefficients in $y = ax^2 + bx + c$ Affects the Graph* - discuss and give examples.
5.8 *Quadratic Form* – define, explain, and give several examples.
5.9 *Solving Quadratic Inequalities* – show an example using a graph and a sign chart.
5.10 *Polynomial Function* – define polynomial function, degree of a polynomial, leading coefficient, and descending order.
5.11 *Synthetic Division* – identify the steps for using synthetic division to divide a polynomial by a binomial.
5.12 *Remainder Theorem, Factor Theorem* – state each theorem and give an explanation and example of each, explain how and why each is used, state their relationships to synthetic division and depressed equations.
5.13 *Fundamental Theorem of Algebra, Number of Roots Theorem* – give an example of each theorem.
5.14 *Intermediate Value Theorem* – state theorem and explain with a picture.
5.15 *Rational Root Theorem* – state the theorem and give an example.
5.16 *General Observations of Graphing a Polynomial* – explain the effects of even/odd degrees on graphs, explain the effect of the use of $\pm$ leading coefficient on even and odd degree polynomials, identify the number of zeros, explain and show an example of double root.
5.17 *Steps for Solving a Polynomial of 4th degree* – work all parts of a problem to find all roots and graph.

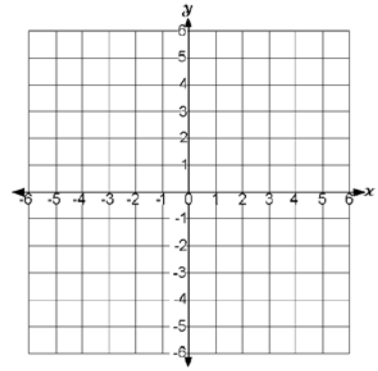
Algebra II – Date

One side, *s*, of a rectangle is four inches less than the other side. Draw a rectangle with these sides and find an equation for the area *A*(*s*) of the rectangle.

Unit 5, Activity 1, Zeroes of a Quadratic Function

Name_______________________________________ Date________________________

Zeroes



Graph the function from the Bellringer $y = x^2 - 4x$ on your calculators. This graph is called a parabola. Sketch the graph making sure to accurately find the *x*- and *y*-intercepts and the minimum value of the function.

(1) In the context of the Bellringer, what do the *x*–values represent?________ ______________________ the *y*–values?___________________________

(2) From the graph, list the zeros of the equation. ______________________

(3) What is the real-world meaning of the zeros for the Bellringer?

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(4) Solve for the zeros analytically showing your work. What property of equations did you use to find the zeros?

Local and Global Characteristics of a Parabola

(1) In your own words, define axis of symmetry: ____

____

(2) Write the equation of the axis of symmetry in the graph above. _________________________

(3) In your own words, define vertex: ____

____

(4) What are the coordinates of the vertex of this parabola? ________________________________

(5) What is the domain of the graph above? ________________ range? ____________________

(6) What domain has meaning for the Bellringer and why? _______________________________

(7) What range has meaning for the Bellringer and why? _________________________________

Unit 5, Activity 1, Zeroes of a Quadratic Function

Reviewing 2nd Degree Polynomial Graphs

Graph the following equations and answer the questions in your notebook.
(1) $y = x^2$ and $y = -x^2$. How does the sign of the leading coefficient affect the graph of the parabola?

(2) $y = x^2$, $y = 4x^2$, $y = 0.5x^2$. How does the magnitude of the leading coefficient affect the zeros and the shape of the parabola as compared to $y = x^2$?

(3) $y = (x - 3)(x + 4)$, $y = (x - 1)(x + 6)$. Make conjectures about the zeros.

(4) $y = 2(x - 5)(x + 4)$, $y = -2(x - 5)(x + 4)$. Make conjectures about the zeros and end-behavior.

Application

A tunnel in the shape of a parabola over a two-lane highway has the following features. It is 30 feet wide at the base and 23 feet high in the center.

(1) Make a sketch of the tunnel on a coordinate plane with the ground as the *x*-axis and the left side of the base of the tunnel at (2, 0). Find two more ordered pairs and graph as a scatter plot in your calculator.________________________________

(2) Enter the quadratic equation $y = a(x - b)(x - c)$ in your calculator substituting your *x*-intercepts from your sketch into b and c. Experiment with various numbers for "a" to find the parabola that best fits this data. Write your equation.

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(3) An 8-foot wide 12-foot high truck wants to go through the tunnel. Determine whether the truck will fit and the allowable location of the truck. Explain your answer.

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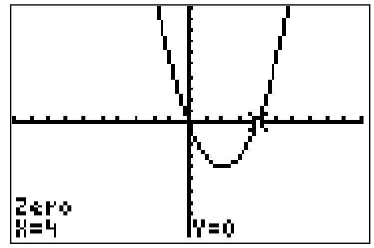
Unit 5, Activity 1, Zeroes of a Quadratic Function with Answers

Name ___*Key*___________________ Date________________

Zeroes

Graph the function from the Bellringer $y = x^2 - 4x$ on your calculators. This graph is called a parabola. Sketch the graph making sure to accurately find the *x*- and *y*-intercepts and the minimum value of the function.



(1) In the context of the Bellringer, what do the *x*–values represent? *the length of the sides* the *y*–values? *the area*

(2) From the graph, list the zeros of the equation. *0 and 4*

(3) What is the real-world meaning of the zeros for the Bellringer?

The length of the side for which the area is zero.

(4) Solve for the zeroes analytically showing your work. What property of equations did you use to find the zeros?
$0 = x^2 - 4x \Rightarrow 0 = x(x-4) \Rightarrow x = 0$ *or* $x - 4 = 0$ *by the Zero Property of Equations* $\Rightarrow \{0, 4\}$

Local and Global Characteristics of a Parabola

(1) In your own words, define axis of symmetry: *a line about which pairs of points on the parabola are equidistant*

(2) Write the equation of the axis of symmetry in the graph above. $x = 2$

(3) In your own words, define vertex: *The point where the parabola intersects the axis of symmetry*

(4) What are the coordinates of the vertex of this parabola? $(2, -4)$

(5) What is the domain of the graph above? *all real numbers* range? $y \geq -4$

(6) What domain has meaning for the Bellringer and why? $x > 4$ *because those sides create positive area.*

(7) What range has meaning for the Bellringer and why? $y > 0$ *because you want an area* > 0

Unit 5, Activity 1, Zeroes of a Quadratic Function with Answers

Reviewing 2nd Degree Polynomial Graphs

Graph the following equations and answer the questions in your notebook.

(1) $y = x^2$ and $y = -x^2$. How does the sign of the leading coefficient affect the graph of the parabola?
Even exponent polynomial has similar end-behavior (either both ends go up or both ends go down). Positive leading coefficient starts up and ends up, negative leading coefficient starts down and ends down.

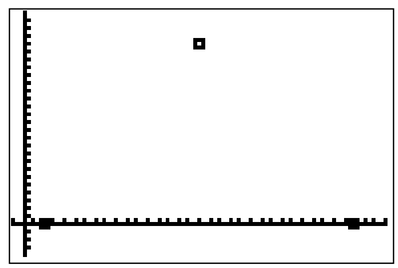
(2) $y = x^2$, $y = 4x^2$, $y = 0.5x^2$. How does the magnitude of the leading coefficient affect the zeros and the shape of the parabola as compared to $y = x^2$?
It does not affect the zeros. If constant is > 1, the graph is steeper than $y=x^2$, and if the coefficient is less than 1, the graph is wider than $y = x^2$.

(3) $y = (x - 3)(x + 4)$, $y = (x - 1)(x + 6)$. Make conjectures about the zeros. ***When the function is factored, the zeros of the parabola are at the solutions to the factors set = 0.***

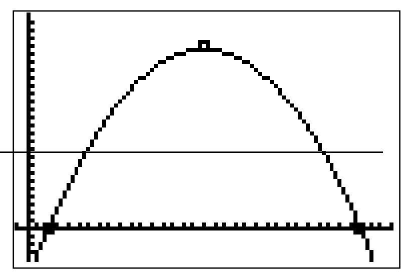
(4) $y = 2(x - 5)(x + 4)$, $y = -2(x - 5)(x + 4)$. Make conjectures about the zeros and end-behavior.
Same zeros opposite end–behaviors.

Application

A tunnel in the shape of a parabola over a two-lane highway has the following features. It is 30 feet wide at the base and 23 feet high in the center.

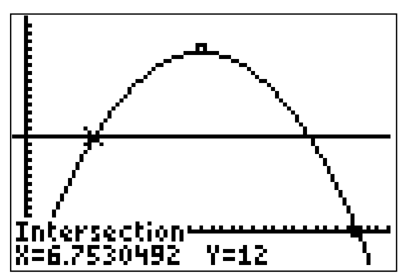


(1) Make a sketch of the tunnel on a coordinate plane with the ground as the x-axis and the left side of the base of the tunnel at (2, 0). Find two more ordered pairs and graph as a scatter plot in your calculator. ***(32, 0) and (17, 23)***



(2) Enter the quadratic equation $y = a(x - b)(x - c)$ in your calculator substituting your x-intercepts from your sketch into b and c. Experiment with various numbers for "a" to find the parabola that best fits this data. Write your equation.

$y = -0.1(x - 2)(x - 32)$



(3) An 8-foot wide 12-foot high truck wants to go through the tunnel. Determine whether the truck will fit and the allowable location of the truck. Explain your answer.

The truck must travel 4.75 feet from the base of the tunnel. It is 8 feet wide and the center of the tunnel is 15 feet from the base so the truck can stay in its lane

Unit 5, Activity 7, Graphing Parabolas Anticipation Guide

Name______________________________________ Date________________________

Give your opinion of what will happen to the graphs in the following situations based upon your prior knowledge of translations and transformations of graphs.

(1) Predict what will happen to the graphs of form $y = x^2 + 5x + \mathrm{c}$ for the following values of c: {8, 4, 0, –4, –8}.

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(2) Predict what will happen to the graphs of form $y = x^2 + \mathrm{b}x + 4$ for the following values of b: {6, 3, 0, –3, –6}

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(3) Predict what will happen to the graphs of form $y = ax^2 + 5x + 4$ for the following values of a: {–2, –1, – ½ , 0, ½ , 1, 2 }

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Unit 5, Activity 7, The Changing Parabola Discovery Worksheet

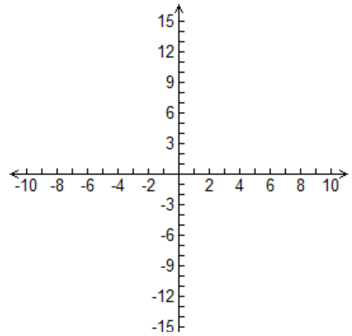
Name_______________________________________ Date________________________

(1) Graph $y = x^2 + 5x + 4$ which is in the form $y = ax^2 + bx + c$ (without a calculator). Determine the following global characteristics:

Vertex: _______________ x–intercept: ______, y–intercept: ______

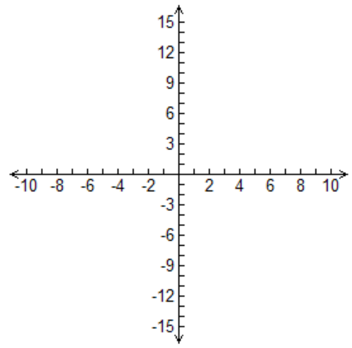
Domain: ____________________ Range: ______________________

End–behavior: ____



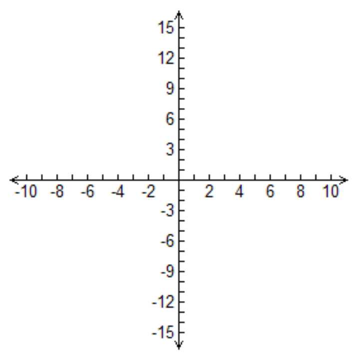
(2) Graph $y = x^2 + 5x + \text{c}$ on your calculator for the following values of c: {8, 4, 0, –4, –8} and sketch. (WINDOW: x: [–10, 10], y: [–15, 15])

- What special case occurs at c = 0? ________________________
- Check your predictions on your *anticipation guide*. Were you correct? If you were incorrect, draw a line through your answer on the *anticipation guide* and write the correct answer. Explain why the patterns occur.



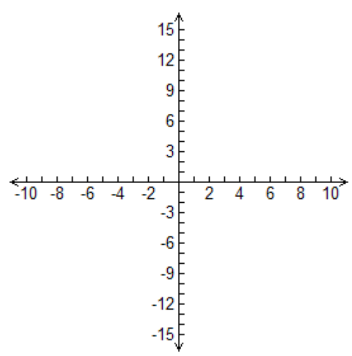
(3) Graph $y = x^2 + \text{b}x + 4$ on your calculator for the following values of b: {6, 3, 0, –3, –6} and sketch.

- What special case occurs at b = 0? ________________________
- Check your predictions on your *anticipation guide*. Were you correct? If you were incorrect, draw a line through your answer on the *anticipation guide* and write the correct answer. Explain why the patterns occur.



(4) Graph $y = \text{a}x^2 + 5x + 4$ on your calculator for the following values of a: {2, 1, 0.5, 0, –0.5, –1, –2} and sketch.

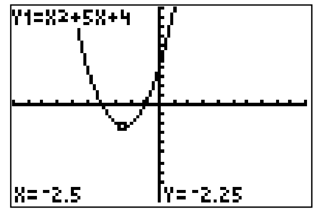
- What special case occurs at $a = 0$? ________________________
- Check your predictions on your *anticipation guide*. Were you correct? If you were incorrect, draw a line through your answer on the *anticipation guide* and write the correct answer. Explain why the patterns occur.



Unit 5, Activity 7, The Changing Parabola Discovery Worksheet with Answers

Name *Key* ______________________________ Date ________________________

(1) Graph $y = x^2 + 5x + 4$ which is in the form $y = ax^2 + bx + c$ (without a calculator). Determine the following global characteristics:

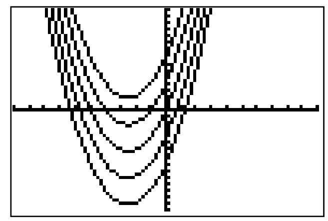


Vertex: $\left(-\frac{5}{2}, -\frac{9}{4}\right)$ x–intercept: ***{–4, –1}***, y–intercept: {4}

Domain: ***All Reals*** __________ Range: $y \geq -\frac{9}{4}$

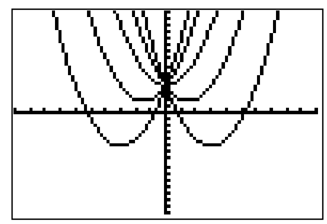
End–behavior: ***as*** $x \to \pm\infty, y \to \infty$

(2) Graph $y = x^2 + 5x + c$ on your calculator for the following values of c: {8, 4, 0, –4, –8} and sketch. (WINDOW: x: [–10, 10], y: [–15, 15])



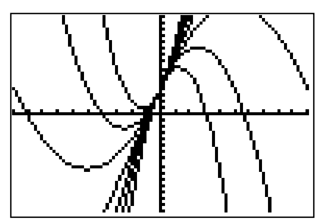
- What special case occurs at c = 0? ***The parabola passes through the origin.***
- Check your predictions on your *anticipation guide*. Were you correct? Explain why the patterns occur. ***There are vertical shifts because you are just adding or subtracting a constant to the graph of $y = x^2 + 5x$, so the y-values of the vertices and y-intercepts change.***

(3) Graph $y = x^2 + bx + 4$ on your calculator for the following values of b: {6, 3, 0, –3, –6} and sketch.



- What special case occurs at $b = 0$? ***the y–axis is the axis of symmetry and the vertex is at (0, 4)***
- Check your predictions on your *anticipation guide*. Were you correct? Explain why the patterns occur. ***There are oblique shifts with the y–intercept remaining the same, but the vertex is moving down because the vertex is affected by b found using $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$ and a is 1. The axis of symmetry is $x = -\frac{b}{2a}$, so when b > 0, it moves left, and when b < 0, the axis of symmetry moves right. Since real zeroes are determined by the discriminant $b^2 - 4ac$ which in this case is $b^2 - 16$, when $|b| \geq 4$, there will be real zeroes.***

(4) Graph $y = ax^2 + 5x + 4$ on your calculator for the following values of a: {2, 1, 0.5, 0, –0.5, –1, –2} and sketch.



- What special case occurs at a = 0? ***the graph is the line y=5x+4***
- Check your predictions on your *anticipation guide*. Were you correct? Explain why the patterns occur. ***The y–intercept remains the same. When $|a| > 1$, the parabola is skinny and when $|a| < 1$ the parabola is wide. When a is positive, the parabola opens up; and when a is negative, the parabola opens down. The axis of symmetry is affected by a, so as $|a|$ gets bigger, the axis of symmetry approaches x = 0. Since real zeroes are determined by the discriminant, which in this case is 25–14a, when $|a| \geq \frac{25}{14}$ there will be real zeroes.***

Activity
10

What Goes Up: Position and Time for a Cart on a Ramp

When a cart is given a brief push up a ramp, it will roll back down again after reaching its highest point. Algebraically, the relationship between the position and elapsed time for the cart is quadratic in the general form

$$y = ax^2 + bx + c$$

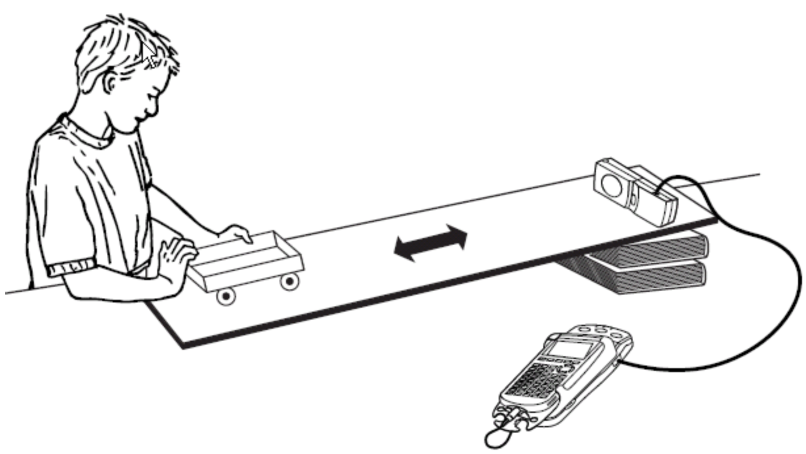
where y represents the position of the cart on the ramp and x represents the elapsed time. The quantities a, b, and c are parameters which depend on such things as the inclination angle of the ramp and the cart's initial speed. Although the cart moves back and forth in a straight-line path, a plot of its position along the ramp graphed as a function of time is parabolic.

Parabolas have several important points including the vertex (the maximum or minimum point), the y-intercept (where the function crosses the y-axis), and the x-intercepts (where the function crosses the x-axis). The x- and y-intercepts are related to the parameters a, b, and c given in the equation above according to the following properties:

1. The y-intercept is equal to the parameter c.
2. The product of the x-intercepts is equal to the ratio $\frac{c}{a}$
3. The sum of the x-intercepts is equal to $-\frac{b}{a}$.

These properties mean that if you know the x- and y-intercepts of a parabola, you can find its general equation.

In this activity, you will use a Motion Detector to measure how the position of a cart on a ramp changes with time. When the cart is freely rolling, the position versus time graph will be parabolic, so you can analyze this data in terms of the key locations on the parabolic curve.



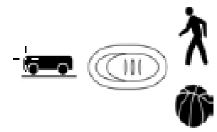
OBJECTIVES

- Record position versus time data for a cart rolling up and down a ramp.
- Determine an appropriate parabolic model for the position data using the *x*- and *y*–intercept information.

MATERIALS

TI-83 Plus or TI-84 Plus graphing calculator
EasyData application
CBR 2 or Go! Motion and direct calculator cable
or Motion Detector and data-collection interface
4-wheeled cart
board or track at least 1.2 m
books to support ramp

PROCEDURE



1. Set up the Motion Detector and calculator.
   a. Open the pivoting head of the Motion Detector. If your Motion Detector has a sensitivity switch, set it to Normal as shown.
   b. Turn on the calculator and connect it to the Motion Detector. (This may require the use of a data-collection interface.)

2. Place one or two books beneath one end of the board to make an inclined ramp. The inclination angle should only be a few degrees. Place the Motion Detector at the top of the ramp. Remember that the cart must never get closer than 0.4 m to the detector, so if you have a short ramp, you may want to use another object to support the detector.

3. Set up EasyData for data collection.
   a. Start the EasyData application, if it is not already running.
   b. Select **File** from the Main screen, and then select **New** to reset the application.

4. So that the zero reference position of the Motion Detector will be about a quarter of the way up the ramp, you will zero the detector while the cart is in this position.
   a. Select **Setup** from the Main screen, and then select **Zero…**
   b. Hold the cart still, about a quarter of the way up the ramp. The exact position is not critical, but the cart must be freely rolling through this point in Step 6.
   c. Select **Zero** to zero the Motion Detector.

5. Practice rolling the cart up the ramp so that you release the cart below the point where you zeroed the detector, and so that the cart never gets closer than 0.4 m to the detector. Be sure to pull your hands away from the cart after it starts moving so the Motion Detector does not detect your hands.

6. Select **Start** to begin data collection. Wait for about a second, and then roll the cart as you practiced earlier.

7. When data collection is complete, a graph of distance versus time will be displayed. Examine the distance versus time graph. The graph should contain an area of smoothly changing distance. The smoothly changing portion must include two $y = 0$ crossings.

   Check with your teacher if you are not sure whether you need to repeat the data collection. To repeat data collection, select **Main** to return to the Main screen and repeat Step 6.

ANALYSIS

1. Since the cart may not have been rolling freely on the ramp the whole time data was collected, you need to remove the data that does not correspond to the free-rolling times. In other words, you only want the portion of the graph that appears parabolic. EasyData allows you to select the region you want using the following steps.

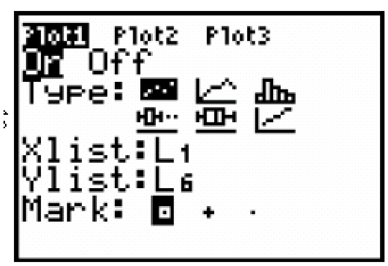
   a. From the distance graph, select **Anlyz** and then select **Select Region…** from the menu.
   b. If a warning is displayed on the screen; select to begin the region selection process.
   c. Use the % and & keys to move the cursor to the left edge of the parabolic region and select **OK** to mark the left bound.
   d. Use the % and & keys to move the cursor to the right edge of the parabolic region and select **OK** to select the region.
   e. Once the calculator finishes performing the selection, you will see the selected portion of the graph filling the width of the screen.

2. Since the cart was not rolling freely when data collection started, adjust the time origin for the graph so that it starts with zero. To do this, you will need to leave EasyData.

   a. Select **Main** to return to the Main screen.
   b. Exit EasyData by selecting **Quit** from the Main screen and then selecting **OK**.

3. To adjust the time origin, subtract the minimum time in the time series from all the values in the series. That will start the time series from zero.
   a. Press **2nd** [L1].
   b. Press –.
   c. To enter the min() function press **Math** , use % to highlight the NUM menu, and press the number adjacent min(to paste the command to the home screen.
   d. Press **2nd** [L1] again and press) to close the minimum function.
   e. Press **STO** , and press **2nd** [L1] a third time to complete the expression L1 – min(L1) _ L1. Press to perform the calculation.

4. You can find the two *x*-intercepts and the *y*-intercept by tracing across the parabola. Redisplay the graph with the individual points highlighted.



   a. Press **2nd** [STAT PLOT] and press **ENTER** to select Plot 1.
   b. Change the Plot1 settings to match the screen shown here. Press **ENTER** to select any of the settings you change.
   c. Press **ZOOM** and then select ZoomStat (use cursor keys to scroll to ZoomStat) to draw a graph with the *x* and *y* ranges set to fill the screen with data.
   d. Press **TRACE** to determine the coordinates of a point on the graph using the cursor keys.

   Trace across the graph to determine the *y*-intercept along with the first and second *x*-intercepts. You will not be able to get to exact *x*-intercepts because of the discrete points, but choose the points closest to the zero crossing. Round these values to 0.01, and record them in the first Data Table on the *Data Collection and Analysis* sheet.

Activity 10

5. Determine the product and sum of the *x*-intercepts. Record these values in the second DataTable on the *Data Collection and Analysis* sheet.

6. Use the intercept values, along with the three intercept properties discussed in the introduction, to determine the values of *a*, *b*, and *c* for the general form parabolic expression $y = ax^2 + bx + c$. Record these values in the third Data Table.

   Hint: Write an equation for each of the three properties; then solve this system of equations for *a*, *b*, and *c*.

⇒ Answer Question 1 on the *Data Collection and Analysis* sheet.

7. Now that you have determined the equation for the parabola, plot it along with your data.

   a. Press **y =** .
   b. Press **CLEAR** to remove any existing equation.
   c. Enter the equation for the parabola you determined in the Y_1 field. For example, if your equation is $y = 5x^2 + 4x + 3$, enter 5*x2+4*x+3 on the Y_1 line.
   d. Press & until the icon to the left of Y_1 is blinking. Press **ENTER** until a bold diagonal line is shown which will display your model with a thick line
   e. Press **GRAPH** to see the data with the model graph superimposed.

⇒ Answer Question 2 on the *Data Collection and Analysis* sheet.

8. You can also determine the parameters of the parabola using the calculator's quadratic regression function.

   a. Press **STAT** and use the cursor keys to highlight CALC.
   b. Press the number adjacent to QuadReg to copy the command to the home screen.
   c. Press **2nd** [L1] **,** **2nd** [L6] **,** to enter the lists containing the data.
   d. Press **VARS** and use the cursor keys to highlight Y-VARS.
   e. Select Function by pressing **ENTER**.
   f. Press **ENTER** to copy Y_1 to the expression.

   On the home screen, you will now see the entry QuadReg L_1, L_6, Y_1. This command will perform a quadratic regression using the *x*-values in L_1 and the *y*-values in L_6. The resulting regression line will be stored in equation variable Y_1.

   g. Press **ENTER** to perform the regression.

   ⇒ Record the regression equation with its parameters in Question 3 on the *Data Collection and Analysis* sheet.

   a. Press **GRAPH** to see the graph.

   ⇒ Answer Questions 4-6 on the *Data Collection and Analysis* sheet.

Activity

10

DATA COLLECTION AND ANALYSIS

Name ___________________________
Date ___________________________

DATA TABLES

y intercept	First *x* intercept	Second *x* intercept	Product of *x* intercepts	Sum of *x* intercepts

a	
b	
c	

QUESTIONS

1. Substitute the values of *a*, *b*, and *c* you just found into the equation $y = ax^2 + bx + c$. Record the completed modeling equation here.

2. Is your parabola a good fit for the data?

3. Record the regression equation from Step 8 with its parameters.

4. Are the values of *a*, *b*, and *c* in the quadratic regression equation above consistent with your results from your earlier calculation?

5. In the experiment you just conducted, the vertex on the parabolic distance versus time plot corresponds to a minimum on the graph even though this is the position at which the cart reaches its maximum distance from the starting point along the ramp. Explain why this is so.

6. Suppose that the experiment is repeated, but this time the Motion Detector is placed at the bottom of the ramp instead of at the top. Make a rough sketch of your predicted distance versus time plot for this situation. Discuss how the coefficient *a* would be affected, if at all.

TEACHER INFORMATION 10

What Goes Up: Position and Time for a Cart on a Ramp

1. There are currently four Motion Detectors that can be used for this lab activity. Listed below is the best method for connecting your type of Motion Detector. Optional methods are also included:

   Vernier Motion Detector: Connect the Vernier Motion Detector to a CBL 2 or LabPro using the Motion Detector Cable included with this sensor. The CBL 2 or LabPro connects to the calculator using the black unit-to-unit link cable that was included with the CBL 2 or LabPro.

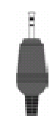
MDC cable

   CBR: Connect the CBR directly to the graphing calculator's I/O port using the extended length I/O cable that comes with the CBR.

I/O cable

   Optionally, the CBR can connect to a CBL 2 or LabPro using a Motion Detector Cable. This cable is not included with the CBR, but can be purchased from Vernier Software & Technology (order code: MDC-BTD).

   CBR 2: The CBR 2 includes two cables: an extended length I/O cable and a Calculator USB cable. The I/O cable connects the CBR 2 to the I/O port on any TI graphing calculator. The Calculator USB cable is used to connect the CBR 2 to the USB port located at the top right corner of any TI-84 Plus calculator.



I/O cable

USB cable

   Optionally, the CBR 2 can connect to a CBL 2 or LabPro using the Motion Detector Cable. This cable is not included with the CBR 2, but can be purchased from Vernier Software & Technology (order code: MDC-BTD).

   Go! Motion: This sensor does not include any cables to connect to a graphing calculator. The cable that is included with it is intended for connecting to a computer's USB port. To connect a Go! Motion to a TI graphing calculator, select one of the options listed below:

   Option I–the Go! Motion connects to a CBL 2 or LabPro using the Motion Detector Cable (order code: MDC-BTD) sold separately by Vernier Software & Technology.

   Option II–the Go! Motion connects to the graphing calculator's I/O port using an extended length I/O cable (order code: GM-CALC) sold separately by Vernier Software & Technology.

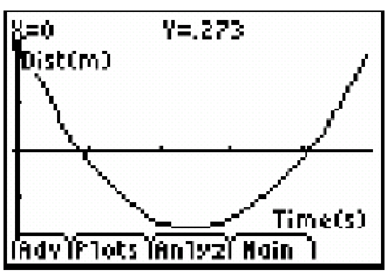
   Option III–the Go! Motion connects to the TI-84 Plus graphing calculator's USB port using a Calculator USB cable (order code: GM-MINI) sold separately by Vernier Software & Technology.

2. When connecting a CBR 2 or Go! Motion to a TI-84 calculator using USB, the EasyData application automatically launches when the calculator is turned on and at the home screen.

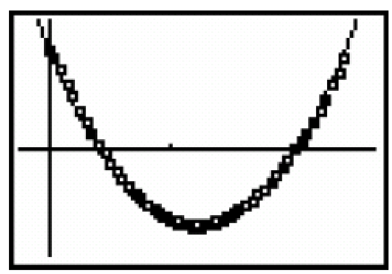
Activity 10

3. A four-wheeled dynamics cart is the best choice for this activity. (Your physics teacher probably has a collection of dynamics carts.) A toy car such as a Hot Wheels or Matchbox car is too small, but a larger, freely-rolling car can be used. A ball can be used, but it is very difficult to have the ball roll directly up and down the ramp. As a result the data quality is strongly dependent on the skill of the experimenter when a ball is used.
4. If a channeled track which forces a ball to roll along a line is used as the ramp, a ball will yield satisfactory data.
5. Note that the ramp angle should only be a few degrees above horizontal. We suggest an angle of five degrees. Most students will create ramps with angles much larger than this, so you might want to have them calculate the angles of their tracks. That will serve both as a trigonometry review and ensure that the ramps are not too steep.
6. It is critical that the student zeroes the Motion Detector in a location that will be crossed by the cart during its roll. If the cart does cross the zero location (both on the way up and the way down), there will be two *x*-axis crossings as required by the analysis. If the student does not zero the Motion Detector, or zeroes it in a location that is not crossed by the cart during data collection, then the analysis as presented is not possible.
7. If the experimenter uses care, it is possible to have the cart freely rolling throughout data collection. In this case (as in the sample data below) there is no need to select a region or adjust the time origin, saving several steps.

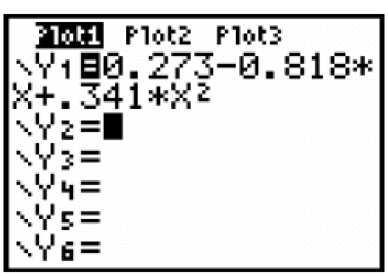
SAMPLE RESULTS



Raw Data in EasyData



Data with parabolic model



Model equation

DATA TABLES

y intercept	First *x* intercept	Second *x* intercept	Product of *x* intercepts	Sum of *x* intercepts
0.273	0.40	2.0	.8	2.4

a	***0.341***
b	***–0.818***
c	***0.273***

ANSWERS TO QUESTIONS

1. Model equation is $y = 0.341x^2 - 0.818\, x + 0.273$ (depends on data collected).
2. Model parabola is an excellent fit, as expected since the vertices were taken from the experimental data.
3. Regression quadratic equation is $y = 0.285 - 0.797\, x + 0.326\, x^2$, or nearly the same as that obtained using the vertex form.
4. The parameters in the calculator's regression are nearly the same as those determined from the vertex form of the equation.
5. The Motion Detector records distance *away* from itself. Since the detector was at the top of the ramp, the cart was at its closest (minimum distance) to the detector when the cart was at its highest point.
6. If the experiment were repeated with the Motion Detector at the bottom of the ramp, the distance data would still be parabolic. However, the parabola would open downward, and the coefficient *a* would change sign.

Unit 5, Activity 10, Solving Quadratic Inequalities by Graphing

Name_______________________________________ Date________________________

SPAWN In your Bellringer, you found the zeros and end-behavior of the related graph to help you solve the inequality. What if your equation had only imaginary roots and no real zeros, how could you use the related graph?

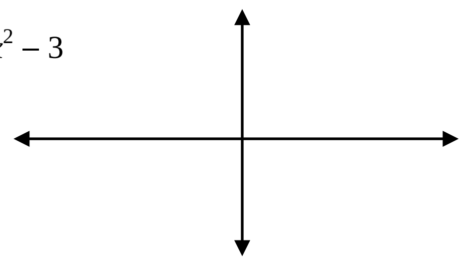
____

____

____

Quadratic Inequalities Find the roots and zeros of the following quadratic equations and fast graph, paying attention only to the x-intercepts and the end-behavior. Use the graphs to help you solve the one-variable inequalities by looking at the positive and negative values of *y*.

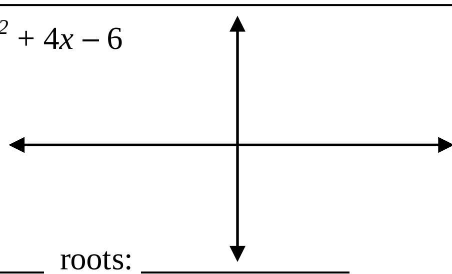
(1) Graph $y = x^2 - 3$



zeros: ________ roots: _______________

Solve for x: $x^2 - 3 > 0$

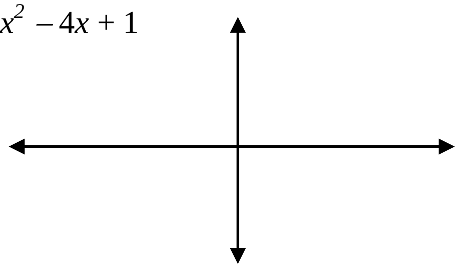
(2) Graph $y = x^2 + 4x - 6$



zeros: ________ roots: _______________

Solve for x: $x^2 + 4x \leq 6$

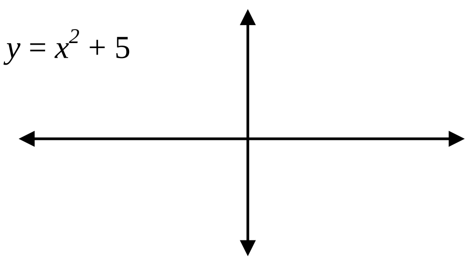
(3) Graph $y = 4x^2 - 4x + 1$



zeros: ________ roots: _______________

Solve for x: $4x^2 - 4x + 1 < 0$

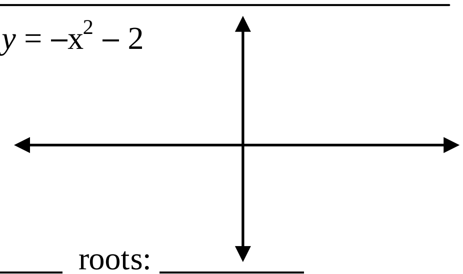
(4) Graph $y = x^2 + 5$



zeros: ________ roots: __________

Solve for x : $x^2 + 5 > 0$

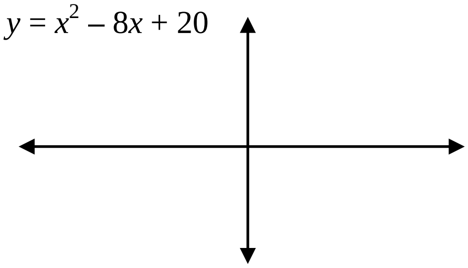
(5) Graph $y = -x^2 - 2$



zeros: ________ roots: __________

Solve for x: $-x^2 - 2 \geq 0$

(6) Graph $y = x^2 - 8x + 20$



zeros: ________ roots: __________

Solve for x: $x^2 - 8x \leq -20$

Unit 5, Activity 10, Solving Quadratic Inequalities by Graphing with Answers

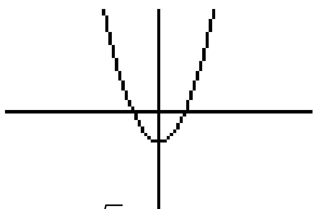
Name ___*Key*___________________________ Date______________________

SPAWN In your Bellringer, you found the zeros and end-behavior of the related graph to help you solve the inequality. What if your equation had only imaginary roots and no real zeros, how could you use the related graph?

Answers will vary, but hopefully will talk about end-behavior and the y–values always being positive or always negative, so the solution will be all reals or the empty set

Quadratic Inequalities Find the zeros and roots of the following quadratic equations and fast graph, paying attention only to the x–intercepts and the end-behavior. Use the graphs to help you solve the one-variable inequalities by looking at the positive and negative values of *y*.

(1) Graph $y = x^2 - 3$

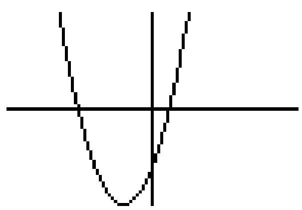


zeros: $x = \pm\sqrt{3}$ roots: $x = \pm\sqrt{3}$

Solve for *x*: $x^2 - 3 > 0$

$x < -\sqrt{3}$ *or* $x > \sqrt{3}$

(2) Graph $y = x^2 + 4x - 6$

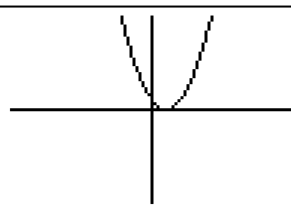


zeros: $x = -2 \pm \sqrt{10}$ roots: $x = -2 \pm \sqrt{10}$

Solve for *x* : $x^2 + 4x \leq 6$ (*Hint: isolate 0 first*)

$-2 - \sqrt{10} \leq x \leq -2 + \sqrt{10}$

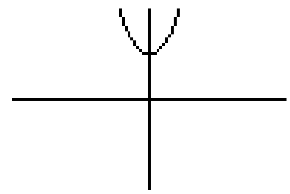
(3) Graph $y = 4x^2 - 4x + 1$



zeros: *x = ½* roots : ***double root at x = ½***

Solve for *x* : $4x^2 - 4x + 1 < 0$ ***empty set***

(4) Graph $y = x^2 + 5$

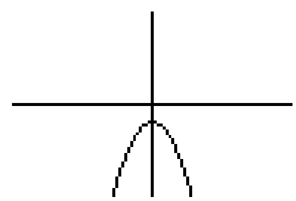


zeros: ***none*** roots: $x = \pm i\sqrt{5}$

Solve for *x* : $x^2 + 5 > 0$

All real numbers

(5) Graph $y = -x^2 - 4$

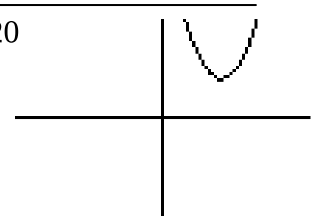


zeros: ***none*** roots: ***x = ±2i***

Solve for *x* : $-x^2 - 4 \geq 0$

empty set

(6) Graph $y = x^2 - 8x + 20$



zeros: ***none*** roots: $x = 4 \pm 2i$

Solve for *x*: $x^2 - 8x \leq -20$ ***empty set***

Unit 5, Activity 12, Factor Theorem Discovery Worksheet

Name______________________________________ Date________________________

Synthetic Division

(1) $(x^3 + 8x^2 - 5x - 84) \div (x + 5)$

(a) Use synthetic division to divide and write the answers in equation form as

$$\frac{dividend}{divisor} = quotient + \frac{remainder}{divisor}$$

$$\frac{x^3 + 8x^2 - 5x - 84}{x + 5} =$$

(b) Multiply both sides of the equation by the divisor (*do not expand*) and write in equation form as polynomial = (divisor)(quotient) + remainder) in other words
P(x) = (x – c)(Q(x)) + Remainder

P(x) = ____

(2) $(x^3 + 8x^2 - 5x - 84) \div (x - 3)$ *(Same directions as #1)*

(a) $$\frac{x^3 + 8x^2 - 5x - 84}{x - 3} =$$

(b) $P(x)$ = ____

Remainder Theorem

(3) What is the remainder in #1b above? _________ What is c? _____ Find $P(-5)$. ________

(4) What is the remainder in #2b above? _________ What is c? _____ Find $P(3)$. ________

(5) Complete the Remainder Theorem: If *P(x)* is a polynomial and *c* is a number, and if *P(x)* is

divided by $x - c$, then ____

(6) Use your calculators to verify the Remainder Theorem.

(a) Enter $P(x) = x^3 + 8x^2 - 5x - 84$ into y_1 and find *P(–5)* and *P(3)* on the home screen as $y_1(-5)$ and $y_1(3)$.

(b) Practice: $f(x) = 4x^3 - 6x^2 + 2x - 5$. Find *f(3)* using synthetic division and verify on the calculator.

(c) Explain why synthetic division is sometimes called *synthetic substitution*.

Unit 5, Activity 12, Factor Theorem Discovery Worksheet

Factor Theorem

(7) Define factor ≡ ____

____

(8) Factor the following:
(a) 12 (b) $x^2 - 9$ (c) $x^2 - 5$ (d) $x^2 + 4$

(e) $x^3 + 8x^2 - 5x - 84$ (*Hint: See #2b above.*) = ______________________________________

(9) Using 8(e) complete the Factor Theorem: If $P(x)$ is a polynomial, then $x - c$ is a factor of

$P(x)$ if and only if ____

(10) Work the following problem to verify the Factor Theorem: Factor $f(x) = x^2 + 3x + 2$ and find $f(-2)$ and $f(-1)$.

(11) In #1 and #2 above, you redefined the division problem as $P(x) = (x - c)(Q(x)) +$ Remainder. $Q(x)$ is called a *depressed polynomial* because the powers of x are one less than the powers of $P(x)$. The goal is to develop a quadratic depressed equation that can be solved by quadratic function methods.

(a) In #2b, you rewrote $\dfrac{x^3+8x^2-5x-84}{x-3} = x^2 - 11x + 28$ and $P(x) = (x^2 + 11x + 28)(x - 3)$
What is the depressed equation? ____

(b) Finish factoring $x^3 + 8x^2 - 5x - 84 =$ ____

List all the zeros: ____

(12) (a) Use synthetic division to determine if $(x - 2)$ is a factor of $y = x^3 + 2x^2 - 5x - 6$.

(b) What is the depressed equation? ____

(c) Factor y completely: ____

Unit 5, Activity 12, Factor Theorem Discovery Worksheet

Factor Theorem Practice

Given one factor of the polynomial, use synthetic division and the depressed polynomial to factor completely.

(1a) $x + 1$; $x^3 + x^2 - 16x - 16$,

(1b) $x + 6$; $x^3 + 7x^2 - 36$.

Given one factor of the polynomial, use synthetic division to find all the roots of the equation.

(2a) $x - 1$; $x^3 - x^2 - 2x + 2 = 0$,

(2b) $x + 2$; $x^3 - x^2 - 2x + 8 = 0$

Given two factors of the polynomial, use synthetic division and the depressed polynomials to factor completely. (*Hint: Use the second factor in the 3rd degree depressed polynomial to get a depressed quadratic polynomial, then factor.)*

(3a) $x - 1$, $x - 3$; $x^4 - 10x^3 + 35x^2 - 50x + 24$

(3b) $x + 3$, $x - 4$, $x^4 - 2x^3 - 13x^2 + 14x + 24$

Unit 5, Activity 12, Factor Theorem Discovery Worksheet with Answers

Name ***Key*** Date________________________

Synthetic Division

(1) $(x^3 + 8x^2 - 5x - 84) \div (x + 5)$

(a) Use synthetic division to divide and write the answers in equation form as $\dfrac{dividend}{divisor} = quotient + \dfrac{remainder}{divisor}$

$$\frac{x^3+8x^2-5x-84}{x+5} = x^2+3x-20+\frac{16}{x+5}$$

(b) Multiply both sides of the equation by the divisor (*do not expand*) and write in equation form as polynomial = (divisor)(quotient) + remainder) in other words
$P(x) = (x - c)(Q(x))$ + Remainder

$P(x) =$ ***$(x + 5)(x^2 + 3x - 20) + 16$***

(2) $(x^3 + 8x^2 - 5x - 84) \div (x - 3)$ *(Same directions as #1)*

(a) $$\frac{x^3+8x^2-5x-84}{x-3} = x^2+11x+28+\frac{0}{x-3}$$

(b) $P(x) =$ ***$(x - 3)(x^2 + 11x + 28) + 0$***

Remainder Theorem

(3) What is the remainder in #1b above? ***16*** What is c? ***-5*** Find $P(-5)$. ***16*** .

(4) What is the remainder in #2b above? ***0*** What is c? ***3*** Find P*(3)*. ***0*** .

(5) Complete the Remainder Theorem: If *P(x)* is a polynomial and *c* is a number, and if *P(x)* is divided by $x - c$, ***then the remainder equals P(c).***

(6) Use your calculators to verify the Remainder Theorem.

(a) Enter $P(x) = x^3 + 8x^2 - 5x - 84$ into y_1 and find *P(–5)* and *P(3)* on the home screen as $y_1(-5)$ and $y_1(3)$.

(b) Practice: $f(x) = 4x^3 - 6x^2 + 2x - 5$. Find *f(3)* using synthetic division and verify on the calculator.

3	4	–6	2	–5
		12	18	60
	4	6	20	55

$f(3) = 4(3)^3 - 6(3)^2 + 2(3) - 5 = 55$

(c) Explain why synthetic division is sometimes called *synthetic substitution*. ***See 6(b)***

Factor Theorem

(7) Define factor ≡ *two or more numbers or polynomials that are multiplied together to get a third number or polynomial.*

(8) Factor the following:

(a) 12 (b) $x^2 - 9$ (c) $x^2 - 5$ (d) $x^2 + 4$

12 = (3)(4) $(x-3)(x+3)$ $\left(x-\sqrt{5}\right)\left(x+\sqrt{5}\right)$ $(x+2i)(x-2i)$

(answers may vary for (a))

(e) $x^3 + 8x^2 - 5x - 84$ (*Hint: See #2b above.*) = $(x-3)(x^2+11x+28)$

(9) Using 8(e) complete the Factor Theorem: If *P(x)* is a polynomial, then $x - c$ is a factor of $P(x)$ if and only if *P(c) = 0. (The remainder is 0 therefore P(c) must be 0.)*

(10) Work the following problem to verify the Factor Theorem: Factor $f(x) = x^2 + 3x + 2$ and find $f(-2)$ and $f(-1)$.

$x^2 + 3x + 2 = (x+2)(x+1)$ $f(-2) = 0, f(-1) = 0$

(11) In #1 and #2 above, you redefined the division problem as P(x) = (x – c)(Q(x)) + Remainder. Q(x) is called a *depressed polynomial* because the powers of x are one less than the powers of *P(x)*. The goal is to develop a quadratic depressed equation that can be solved by quadratic function methods.

(a) In #2b, you rewrote $\dfrac{x^3+8x^2-5x-84}{x-3} = x^2+11x+28$ and $P(x) = (x^2 + 11x + 28)(x-3)$

What is the depressed equation? $Q(x) = x^2 - 11x + 28$

(b) Finish factoring $x^3 + 8x^2 - 5x - 84 = (x-3)(x-7)(x-4)$

List all the zeroes: $\{3, 7, 4\}$

(12) (a) Use synthetic division to determine if $(x - 2)$ is a factor of $y = x^3 + 2x^2 - 5x - 6$.

```
2 |  1   2   –5   –6
         2    8    6        Yes, (x–2) is a factor.
     1   4    3    0   ←   Coefficients of the depressed equation
```

(b) What is the depressed equation? $x^2 + 4x + 3$

(c) Factor y completely: $(x-2)(x+3)(x+1)$

Factor Theorem Practice

Unit 5, Activity 12, Factor Theorem Discovery Worksheet with Answers

Given one factor of the polynomial, use synthetic division and the depressed polynomial to factor completely.

(1a) $x + 1$; $x^3 + x^2 - 16x - 16$,

$(x + 1)(x - 4)(x + 4)$

(1b) $x + 6$; $x^3 + 7x^2 - 36$.

$(x+6)(x + 3)(x - 2)$

Given one factor of the polynomial, use synthetic division to find all the roots of the equation.

(2a) $x - 1$; $x^3 - x^2 - 2x + 2 = 0$,

$\left\{1, \sqrt{2}, -\sqrt{2}\right\}$

(2b) $x + 2$; $x^3 - x^2 - 2x + 8 = 0$

$\left\{-2, \frac{3}{2} + \frac{\sqrt{7}}{2}i, \frac{3}{2} - \frac{\sqrt{7}}{2}i\right\}$

Given two factors of the polynomial, use synthetic division and the depressed polynomials to factor completely. (*Hint: Use the second factor in the 3^rd^ degree depressed polynomial to get a depressed quadratic polynomial, then factor.)*

(3a) $x - 1$, $x - 3$; $x^4 - 10x^3 + 35x^2 - 50x + 24$

$(x - 1)(x - 3)(x - 2)(x - 4)$

(3b) $x + 3$, $x - 4$, $x^4 - 2x^3 - 13x^2 + 14x + 24$

$(x+3)(x - 4)(x - 2)(x+1)$

Unit 5, Activity 13, Exactly Zero

Name_______________________________________ Date________________________

Graph the following on your calculator and find all exact zeros and roots and factors:

(1) $f(x) = x^3 + 2x^2 - 10x + 4$

(2) $f(x) = x^4 + 2x^3 - 11x^2 - 18x + 9$

(3) $f(x) = x^4 + 8x^3 + 22x^2 + 48x + 96$

(4) $f(x) = 2x^3 + 7x^2 - x - 2$ *(Hint: Leading coefficient is 2; therefore, factors must multiply out to get that coefficient*)

(5) $f(x) = 3x^3 - 4x^2 - 28x - 16$

(6) Discuss the process used to find the exact answers.

Unit 5, Activity 13, Exactly Zero with Answers

Name ________Key________________________ Date______________________

Graph the following on your calculator and find all exact zeros and roots and factors:

(1) $f(x) = x^3 + 2x^2 - 10x + 4$

zeros/roots: $\left\{2, 2+\sqrt{6}, 2-\sqrt{6}\right\}$***, factors:*** $f(x) = (x-2)\left(x-2-\sqrt{6}\right)\left(x-2+\sqrt{6}\right)$

(2) $f(x) = x^4 + 2x^3 - 11x^2 - 18x + 9$

zeros/roots: $\left\{3, -3, -1+\sqrt{2}, -1-\sqrt{2}\right\}$,

factors: $f(x) = (x-3)(x+3)\left(x+1-\sqrt{2}\right)\left(x+1+\sqrt{2}\right)$

(3) $f(x) = x^4 + 8x^3 + 22x^2 + 48x + 96$

zeros; x = −4, roots: $\left\{-4, -4, i\sqrt{6}, -i\sqrt{6}\right\}$,

factors: $f(x) = (x+4)^2\left(x-i\sqrt{6}\right)\left(x+i\sqrt{6}\right)$

(4) $f(x) = 2x^3 + 7x^2 - x - 2$ *(Hint: Leading coefficient is 2; therefore, factors must multiply out to get that coefficient)*

zeros/roots: $\left\{-\frac{1}{2}, -\frac{3}{2}+\frac{\sqrt{17}}{2}, -\frac{3}{2}-\frac{\sqrt{17}}{2}\right\}$,

factors: $f(x) = (2x+1)\left(x-\frac{-3+\sqrt{17}}{2}\right)\left(x-\frac{-3-\sqrt{17}}{2}\right)$

(5) $f(x) = 3x^3 - 4x^2 - 28x - 16$

zeros/roots: $\left\{4, -2, -\frac{2}{3}\right\}$***, factors: $f(x) = (3x+2)(x-4)(x+2)$***

(6) Discuss the process used to find the exact answers. ***Find all rational roots on the calculator. Use these with synthetic division to find a depressed quadratic equation and solve with the quadratic formula.***

Unit 5, Activity 14, Rational Roots of Polynomials

Name______________________________________ Date________________________

Vocabulary Self–Awareness Chart

(1) Rate your understanding of each number system with either a "+" (understand well), a "✓" (limited understanding or unsure), or a "–" (don't know)

Complex Number System Terms		+	✓	–	Roots from Exact Zero BLM
1	integer				
2	rational number				
3	irrational number				
4	real number				
5	imaginary number				
6	complex number				

(2) List all of the roots found in the Exact Zero BLM completed in Activity #13.

(1) $f(x) = x^3 + 2x^2 - 10x + 4$ __________________________________
(2) $f(x) = x^4 + 2x^3 - 11x^2 - 18x + 9$ __________________________________
(3) $f(x) = x^4 + 8x^3 + 22x^2 + 48x + 96$ __________________________________
(4) $f(x) = 2x^3 + 7x^2 - x - 2$ __________________________________
(5) $f(x) = 3x^3 - 4x^2 - 28x - 16$ __________________________________

(3) Fill the roots in the chart in the proper classification.

Rational Root Theorem

(4) Define rational number: ____

____

(5) Circle all the rational roots in the equations above.
(6) What is alike about all the polynomials that have integer rational roots?
(7) What is alike about all the polynomials that have fraction rational roots?
(8) Complete the Rational Root Theorem: If a polynomial has integral coefficients, then any rational roots will be in the form $\frac{p}{q}$ where p is______________________________________ and q is____

(9) Identify the p = constant and the q = leading coefficient of the following equations from the Exact Zero BLM and list all possible rational roots:

	polynomial	factors of p	factors of q	possible rational roots
(1)	$f(x) = x^3 + 2x^2 - 10x + 4$			
(2)	$f(x) = x^4 + 2x^3 - 11x^2 - 18x + 9$			
(3)	$f(x) = x^4 + 8x^3 + 22x^2 + 48x + 96$			
(4)	$f(x) = 2x^3 + 7x^2 - x - 2$			
(5)	$f(x) = 3x^3 - 4x^2 - 28x - 16$			

Additional Theorems for Graphing Aids

(10) Fundamental Theorem of Algebra: Every polynomial function with complex coefficients has at least one root in the set of complex numbers.

(11) Number of Roots Theorem: Every polynomial function of degree n has exactly n complex roots. (Some may have multiplicity.)

(12) Complex Conjugate Root Theorem: If a complex number a + b*i* is a solution of a polynomial equation with real coefficients, then the conjugate a – b*i* is also a solution of the equation. (e.g. If 2 +3i is a root then 2 – 3i is a root.)

(13) Intermediate Value Theorem for Polynomials: (as applied to locating zeros). If $f(x)$ defines a polynomial function with real coefficients, and if for real numbers a and b the values of $f(a)$ and $f(b)$ are opposite signs, then there exists at least one real zero between a and b.

(a) Consider the following chart of values for a polynomial. Because a polynomial is continuous, in what intervals of x does the Intermediate Values Theorem guarantee a zero?

Intervals of Zeros: ____

x	–5	–4	–3	–2	–1	0	1	2	3	4
$f(x)$	–1482	–341	216	357	250	63	–36	121	702	1875

This is data for the polynomial $f(x) = 28x^3 + 44x^2 - 171x + 63$.

(b) List all the possible rational roots:

(c) Circle the ones that lie in the Intervals of Zeros.

(d) Use synthetic division with the circled possible rational roots to find a depressed equation to locate the remaining roots.

Unit 5, Activity 14, Rational Roots of Polynomials with Answers

Name *Key* ______________________________ Date ________________________

Vocabulary Self–Awareness Chart

(1) Rate your understanding of each number system with either a "+" (understand well), a "✓" (limited understanding or unsure), or a "–" (don't know)

	Complex Number System Terms	+	✓	–	Roots from Exact Zero BLM
1	integer				*2, –2, 3, –3, 4, –4*
2	rational number				*2, –2, 3, –3, 4, –4,* $-\frac{1}{2}, -\frac{2}{3}$
3	irrational number				$2+\sqrt{6}, 2-\sqrt{6}$, $-1+\sqrt{2}, -1-\sqrt{2}$, $-\frac{3}{2}+\frac{\sqrt{17}}{2}, -\frac{3}{2}-\frac{\sqrt{17}}{2}$
4	real number				*all the answers in #1 – 3 above*
5	imaginary number				$i\sqrt{6}, -i\sqrt{6}$
6	complex number				*all the answers in #1 – 5 above*

(2) List all of the roots found in the Exact Zero BLM completed in Activity #13.

(1) $f(x) = x^3 + 2x^2 - 10x + 4$ $\{2, 2+\sqrt{6}, 2-\sqrt{6}\}$

(2) $f(x) = x^4 + 2x^3 - 11x^2 - 18x + 9$ $\{3, -3, -1+\sqrt{2}, -1-\sqrt{2}\}$

(3) $f(x) = x^4 + 8x^3 + 22x^2 + 48x + 96$ $\{-4, -4, i\sqrt{6}, -i\sqrt{6}\}$

(4) $f(x) = 2x^3 + 7x^2 - x - 2$ $\left\{-\frac{1}{2}, -\frac{3}{2}+\frac{\sqrt{17}}{2}, -\frac{3}{2}-\frac{\sqrt{17}}{2}\right\}$

(5) $f(x) = 3x^3 - 4x^2 - 28x - 16$ $\left\{4, -2, -\frac{2}{3}\right\}$

(3) Fill the roots in the chart in the proper classification.

Rational Root Theorem

(4) Define rational number: $\frac{p}{q}$ *where p and q are integers and q ≠ 0. All terminating and repeating decimals can be expressed as fractions in this form*

(5) Circle all the rational roots in the equations above.

(6) What is alike about all the polynomials that have integer rational roots? *The leading coefficient = 1*

(7) What is alike about all the polynomials that have fraction rational roots? *the leading coefficient ≠ 1*

(8) Complete the Rational Root Theorem: If a polynomial has integral coefficients, then any rational roots will be in the form $\frac{p}{q}$ where p is *a factor of the constant* and q is *a factor of the leading coefficient.*

(9) Identify the p = constant and the q = leading coefficient of the following equations from the Exact Zero BLM and list all possible rational roots:

	polynomial	factors of p	factors of q	possible rational roots
(1)	$f(x) = x^3 + 2x^2 - 10x + 4$	±1, ±2, ±4	±1	±1, ±2, ±4
(2)	$f(x) = x^4 + 2x^3 - 11x^2 - 18x + 9$	±1, ±3, ±9	±1	±1, ±3, ±9
(3)	$f(x) = x^4 + 8x^3 + 22x^2 + 48x + 96$	±1, ±2, ±7, ±14, ±49, ±96	±1	±1, ±2, ±7, ±14, ±49, ±96
(4)	$f(x) = 2x^3 + 7x^2 - x - 2$	±1, ±2	±1, ±2	±1, ± ½ , ±2
(5)	$f(x) = 3x^3 - 4x^2 - 28x - 16$	±1, ±2, ±4, ±8, ±16	±1, ±3	±1, ±2, ±4, ±8, ±16, $\pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}, \pm\frac{16}{3}$

Additional Theorems for Graphing Aids

(10) Fundamental Theorem of Algebra: Every polynomial function with complex coefficients has at least one root in the set of complex numbers.

(11) Number of Roots Theorem: Every polynomial function of degree n has exactly n complex roots. (Some may have multiplicity.)

(12) Complex Conjugate Root Theorem: If a complex number a + bi is a solution of a polynomial equation with real coefficients, then the conjugate a – bi is also a solution of the equation. (e.g. If 2 +3i is a root then 2 – 3i is a root.)

(13) Intermediate Value Theorem for Polynomials: (as applied to locating zeros). If $f(x)$ defines a polynomial function with real coefficients, and if for real numbers a and b the values of $f(a)$ and $f(b)$ are opposite signs, then there exists at least one real zero between a and b.

(a) Consider the following chart of values for a polynomial. Because a polynomial is continuous, in what intervals of x does the Intermediate Values Theorem guarantee a zero?

Intervals of Zeroes: ***(–4, –3), (0, 1), (1, 2)***

x	–5	–4	–3	–2	–1	0	1	2	3	4
$f(x)$	–1482	–341	216	357	250	63	–36	121	702	1875

This is data for the polynomial $f(x) = 28x^3 + 44x^2 - 171x + 63$.

(b) List all the possible rational roots:

factors of 63: {±1, ±3, ±7, ±9, ±21, ±63}, factors of 28: {±1, ±2, ±4, 7, ±14, ±28}

possible rational roots:

$$\pm\left\{1, 3, 7, 9, 21, 63, \frac{1}{2}, \frac{1}{4}, \frac{1}{7}, \frac{1}{14}, \frac{1}{28}, \frac{3}{2}, \frac{3}{4}, \frac{3}{7}, \frac{3}{14}, \frac{3}{28}, \frac{7}{2}, \frac{7}{4}, \frac{9}{2}, \frac{9}{4}, \frac{9}{7}, \frac{9}{14}, \frac{21}{2}, \frac{21}{4}, \frac{63}{2}, \frac{63}{4}\right\}$$

(c) Circle the ones that lie in the Intervals of Zeroes.

$\left\{-\frac{7}{2}, \frac{1}{2}, \frac{1}{4}, \frac{1}{7}, \frac{1}{14}, \frac{1}{28}, \frac{3}{2}, \frac{3}{4}, \frac{3}{7}, \frac{3}{14}, \frac{3}{28}, \frac{7}{4}, \frac{9}{4}, \frac{9}{7}, \frac{9}{14}\right\}$ ***Try*** $-\frac{7}{2}$ ***first because it is the only one in that interval.***

(d) Use synthetic division with the circled possible rational roots to find a depressed equation to locate the remaining roots. $\left\{-\frac{7}{2}, \frac{3}{7}, \frac{3}{2}\right\}$

Unit 5, Activity 15, Solving the Polynomial Mystery

Name______________________________________ Date________________________

Answer #1 – 8 below concerning this polynomial:

$$f(x) = 4x^4 - 4x^3 - 11x^2 + 12x - 3$$

(1) How many roots does the Fundamental Theorem of Algebra guarantee this equation has? ___

(2) How many roots does the Number of Roots Theorem say this equation has? _______

(3) List all the possible rational roots:____

(4) Use the chart below and the Intermediate Value Theorem to locate the interval/s of the zeroes.

____

x	–2	$-\frac{3}{2}$	–1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3
y	25	–12	–18	–11	–3	0	–2	–3	9	52	150

(5) If you have one root, use synthetic division to find the depressed equation and rewrite y as a factored equation with one binomial root and the depressed equation.

y = (____________) (____)
factor depressed equation

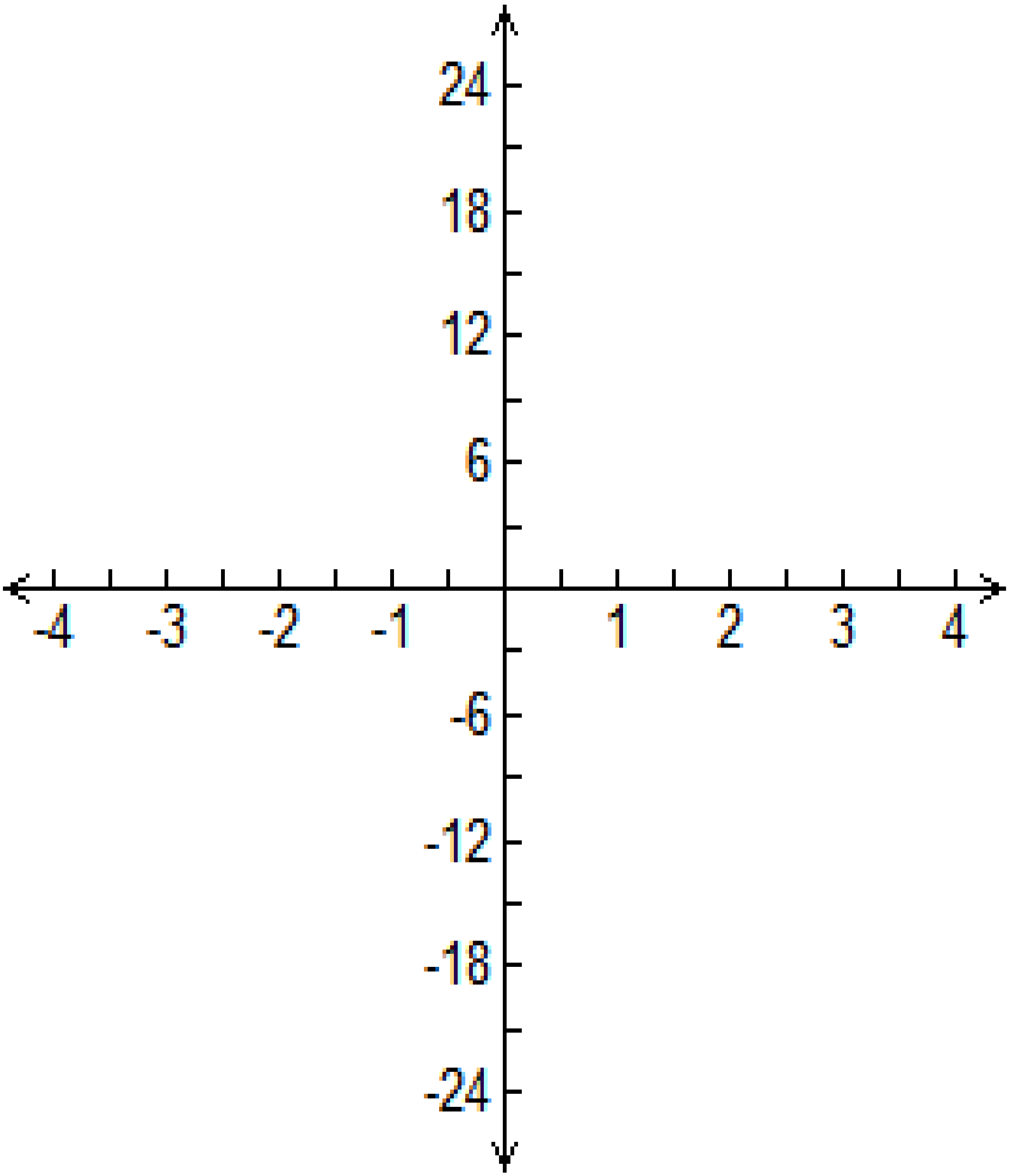
(6) Use synthetic division on the depressed equation to find all the other roots.

List all the roots repeating any roots that have multiplicity. {_____________________}

(7) Write the equation factored with no fractions and no exponents greater than one.

y = (____________)(_____________)(_____________)(_____________)

(8) Graph *f(x)* without a calculator using all the available information in questions #1–7 on the previous page.



Name Key Date

Answer all the questions on this page concerning this polynomial:

$$f(x) = 4x^4 - 4x^3 - 11x^2 + 12x - 3$$

(1) How many roots does the Fundamental Theorem of Algebra guarantee this equation has? 1

(2) How many roots does the Number of Roots Theorem say this equation has? 4

(3) List all the possible rational roots: $\{\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{4}, \pm \frac{3}{4}\}$

$p = 3 \quad \{\pm 1, \pm 3\}$

$q = 4 \quad \{\pm 1, \pm 2, \pm 4\}$

(4) Use the chart at below and the Intermediate Value Theorem to locate the interval/s of the zeroes.

$(-2, -\frac{3}{2})$, root at $x = \frac{1}{2}$, $(\frac{3}{2}, 2)$

x	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3
y	25	-12	-18	-11	-3	0	-2	-3	9	52	150

(5) If you have one root, use synthetic division to find the depressed equation and rewrite y as a factored equation with one binomial root and the depressed equation.

$y = (x - \frac{1}{2})(4x^3 - 2x^2 - 12x + 6)$

factor depressed equation

```
1/2 | 4   -4   -11   12   -3
    |      2    -1   -6    3
    ---------------------------
      4   -2   -12    6    0
```

$\frac{1}{2}$ may be a double root so try it again

(6) Use synthetic division on the depressed equation to find all the other roots.

List all the roots repeating any roots that have multiplicity. $\{\frac{1}{2}, \frac{1}{2}, \sqrt{3}, -\sqrt{3}\}$

```
1/2 | 4   -2   -12    6
    |      2     0   -6
    ----------------------
      4    0   -12    0
```

Depressed equation: $4x^2 - 12 = 0$

$4x^2 = 12$

$x^2 = 3 \Rightarrow x = \pm\sqrt{3}$

(7) Write the equation factored with no fractions and no exponents greater than one.

$y = (2x - 1)(2x - 1)(x - \sqrt{3})(x + \sqrt{3})$

(8) Graph *f(x)* without a calculator using all the available information in questions #1–7 on the previous page.