Little Black Book of Algebra II Properties
Unit 4 - Radicals & the Complex Number System

4.1 Radical Terminology – define radical sign, radicand, index, like radicals, root, n^{th} root, principal root, conjugate.

4.2 Rules for Simplifying $\sqrt[n]{b^n}$ – identify and give examples of the rules for even and odd values of *n*.

4.3 Product and Quotient Rules for Radicals – identify and give examples of the rules.

4.4 Rationalizing the Denominator – explain: what does it mean and why do it – the process for rationalizing a denominator of radicals with varying indices and a denominator that contains the sum of two radicals.

4.5 Radicals in Simplest Form - list what to check to make sure radicals are in simplest form.

4.6 Addition and Subtraction Rules for Radicals – identify and give examples.

4.7 Graphing Simple Radical Functions – show the effect of constant both inside and outside of a radical on the domain and range.

4.8 Steps to Solve Radical Equations – identify and give examples.

4.9 Complex Numbers – define: $a + bi$ form, i, i^2, i^3, and i^4; explain how to find the values of i^{4n}, i^{4n+1}, i^{4n+2}, i^{4n+3}, explain how to conjugate, and how to find the absolute value of $a + bi$.

4.10 Properties of Complex Number System – provide examples of the Equality Property, the Commutative Property Under Addition/Multiplication, the Associative Property Under Addition/Multiplication, and the Closure Property Under Addition/Multiplication.

4.11 Operations on Complex Numbers in $a + bi$ form – provide examples of addition, additive identity, additive inverse, subtraction, multiplication, multiplicative identity, squaring, division, absolute value, reciprocal, raising to a power, and factoring the sum of two perfect squares.

4.12 Root vs. Zero – explain the difference between a root and a zero and how to determine the number of roots of a polynomial.

Algebra II – Date

Graph on the graphing calculator and find the points of intersection:

(1) $y_1 = x^2$ and $y_2 = 9$

(2) $y_1 = x^2$ and $y_2 = -9$

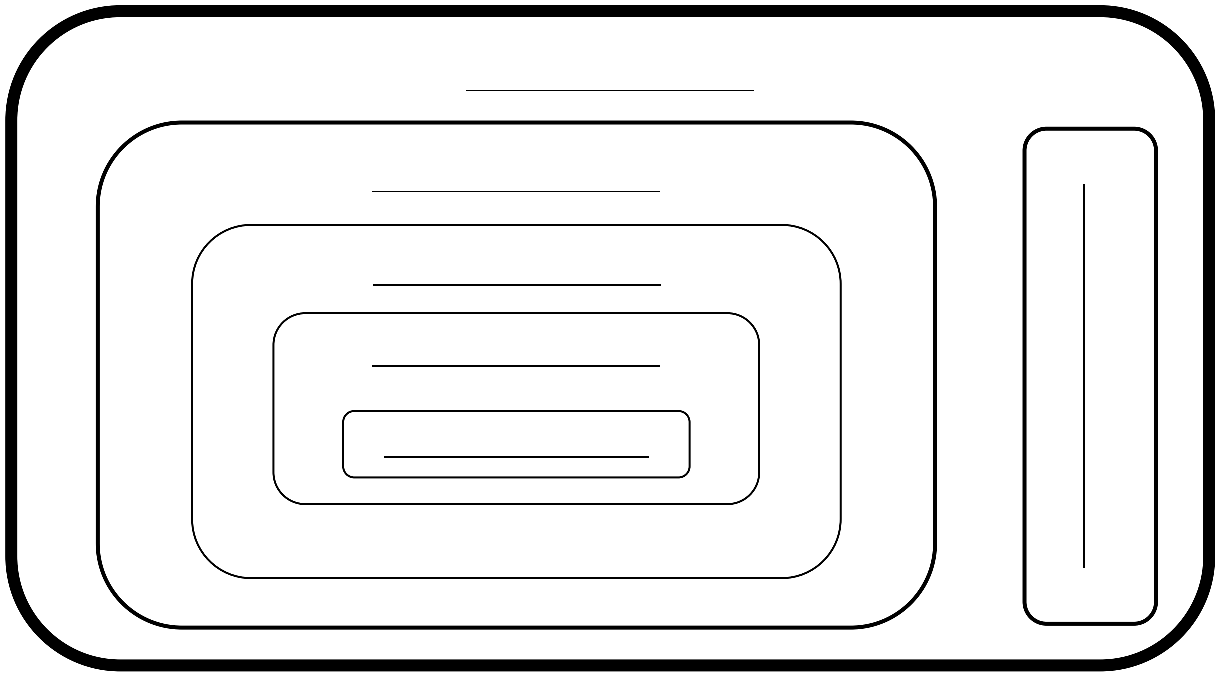
(3) $y_1 = x^2$ and $y_2 = 0$

(4) Discuss the number of points of intersection each set of equations has.

Name_______________________________________ Date_________________________

Reviewing Sets of Numbers

Fill in the following sets of numbers in the Venn diagram: natural numbers, whole numbers, integers, rational numbers, irrational numbers, real numbers.



1. Write the symbol for the set and list its elements in set notation:

   natural numbers: _________________ What is another name for natural numbers?__________

   whole numbers: ____

   integers: ____

2. Define rational numbers. What is its symbol and why? Give some examples.______________

   ____

   ____

3. Are your Bellringers rational or irrational? Why? ______________________________________

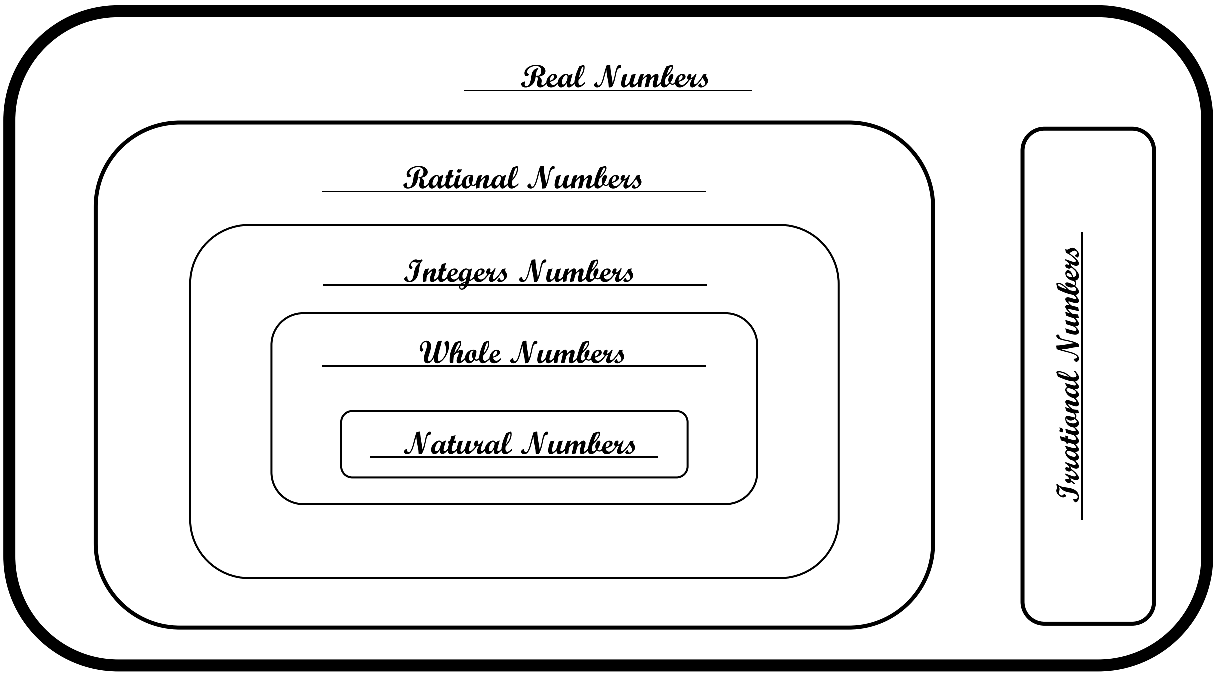
   ____

Unit 4, Activity 2, Sets of Numbers with Answers

Name_______________________________________ Date________________________

Reviewing Sets of Numbers

Fill in the following sets of numbers in the Venn diagram: natural numbers, whole numbers, integers, rational numbers, irrational numbers, real numbers.



1. Write the symbol for the set and list its elements in set notation:

   natural numbers: *N={1, 2, 3, …}* What is another name for natural numbers? *Counting*

   whole numbers: *W = {0, 1, 2, 3, …}*

   integers: *J or Z = {. . . , –3, –2, –1, 0, 1, 2, 3, …}*

2. Define rational numbers. What is its symbol? Give some examples. *Any number in the form p/q where p and q are integers, q ≠ 0. The symbol is Q for quotient. Ex. All repeating and terminating decimals and fractions of integers. 7, 7.5, 7.6666…, ½ , – 1/3*

3. Are your Bellringer problems rational or irrational? Why? *Irrational because they cannot be expressed as fractions of integers. Their decimal representations do not repeat or terminate.*

Unit 4, Activity 2, Multiplying & Dividing Radicals

Name________________________________ Date___________________________________

Multiplying and Dividing Radicals

1. Can the product of two irrational numbers be a rational number? Give an example.________

____

2. What does "rationalizing the denominator" mean and why do we rationalize the

denominator? ____

____

3. Rationalize the following denominators and simplify:

(1) $\frac{1}{\sqrt{5}}$ (2) $\frac{1}{\sqrt[3]{5}}$ (3) $\frac{1}{\sqrt{8}}$

4. List what should be checked to make sure a radical is in simplest form:

a. ____

b. ____

c. ____

5. Simplify the following expressions applying rules to radicals with variables in the radicand.

(1) $\sqrt{72x^3y^4}$

(2) $\sqrt[3]{80s^4t^6}$

(3) $\sqrt{2xy}\cdot\sqrt{6x^3y}$

(4) $5\sqrt[3]{4x^2y^5}\cdot 7\sqrt[3]{2x^2y}$

(5) $\frac{\sqrt{162x^6}}{\sqrt{10x^7}}$

(6) $\frac{\sqrt[3]{2s^2}}{\sqrt[3]{18s^3}}$

Application

The time in seconds, *t*(*L*), for one complete swing of a pendulum is dependent upon the length of the pendulum in feet, *L*, and gravity which is 32 ft/sec^2 on earth. It is modeled by the function $t(L) = 2\pi\sqrt{\frac{L}{32}}$. Find the time for one complete swing of a 4-foot pendulum. Express the exact simplified answer in function notation and express the answer in a sentence rounding to the nearest tenth of a second.

Unit 4, Activity 2, Multiplying & Dividing Radicals with Answers

Name________________________________ Date____________________________________

Multiplying and Dividing Radicals

1. Can the product of two irrational numbers be a rational number? Give an example.

   Yes, $\sqrt{2}\sqrt{32}=8$

2. What does "rationalizing the denominator" mean and why do we rationalize the denominator? ***Rationalizing the denominator means making sure that the number in the denominator is a rational number and not an irrational number with a radical. We rationalize denominators because we do not want to divide by a nonrepeating, nonterminating decimal.***

3. Rationalize the following denominators and simplify:

   (1) $\frac{1}{\sqrt{5}}=\frac{\sqrt{5}}{5}$ (2) $\frac{1}{\sqrt[3]{5}}=\frac{\sqrt[3]{25}}{5}$ (3) $\frac{1}{\sqrt{8}}=\frac{\sqrt{2}}{4}$

4. List what should be checked to make sure a radical is in simplest form:

   a. ***The radicand contains no exponent greater than or equal to the index***

   b. ***The radicand contains no fractions***

   c. ***The denominator contains no radicals***

5. Simplify the following expressions applying rules to radicals with variables in the radicand.

   (1) $\sqrt{72x^3y^4}=6y^2|x|\sqrt{2x}$

   (2) $\sqrt[3]{80s^4t^6}=2st^2\sqrt[3]{10s}$

   (3) $\sqrt{2xy}\cdot\sqrt{6x^3y}=2x^2|y|\sqrt{3}$

   (4) $5\sqrt[3]{4x^2y^5}\cdot 7\sqrt[3]{2x^2y}=70xy^2\sqrt[3]{x}$

   (5) $\frac{\sqrt{162x^6}}{\sqrt{10x^7}}=\frac{9\sqrt{5x}}{5x}$

   (6) $\frac{\sqrt[3]{2s^2}}{\sqrt[3]{18s^3}}=\frac{\sqrt[3]{3s^2}}{3s}$

Application

The time in seconds, *t*(*L*), for one complete swing of a pendulum is dependent upon the length of the pendulum in feet, *L*, and gravity which is 32 ft/sec^2 on earth. It is modeled by the function $t(L)=2\pi\sqrt{\frac{L}{32}}$. Find the time for one complete swing of a 4-foot pendulum.

Express the exact simplified answer in function notation and express the answer in a sentence rounding to the nearest tenth of a second. $t(L)=\frac{\pi\sqrt{2}}{2}$. ***One complete swing of a 4-foot pendulum takes approximately 2.2 seconds.***

Unit 4, Activity 4, Graphing Radical Functions Discovery Worksheet

Name________________________________ Date____________________________________

Radical Graph Translations

	Equation	Sketch	Domain	Range	*x*-intercept	*y*-intercept
1	$y=\sqrt{x}+3$					
2	$y=\sqrt{x}-3$					
3	$y=\sqrt[3]{x}+2$					
4	$y=\sqrt[3]{x}-2$					
5	$y=\sqrt{x-4}$					
6	$y=\sqrt{x+4}$					
7	$y=\sqrt[3]{x+5}$					
8	$y=\sqrt[3]{x-5}$					
9	$y=\sqrt{x-3}+5$					

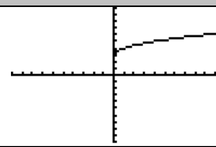
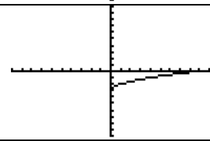
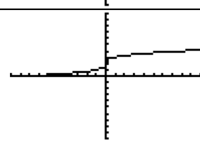
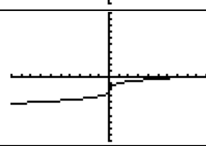
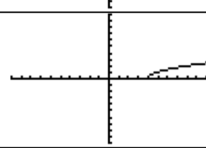
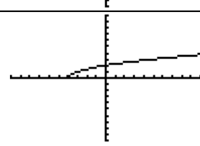
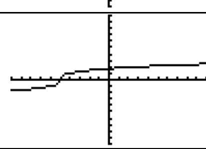
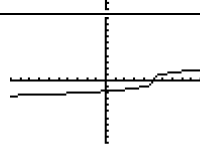
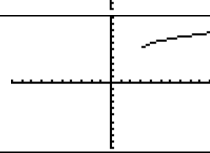
(10) What is the difference in the graph when a constant is added outside of the radical, *f(x) + k*, or inside of the radical, *f(x + k)*?

(11) What is the difference in the domains and ranges of $f(x)=\sqrt{x}$ and $g(x)=\sqrt[3]{x}$? Why is the domain of one of the functions restricted and the other not?

Unit 4, Activity 4, Graphing Radicals Functions Discovery Worksheet with Answers

Name________________________________ Date____________________________________

Radical Graph Translations

	Equation	Sketch	Domain	Range	*x*-intercept	*y*-intercept
1	$y=\sqrt{x}+3$		$x \geq 0$	$y \geq 3$	none	$(0, 3)$
2	$y=\sqrt{x}-3$		$x \geq 0$	$y \geq -3$	$(9, 0)$	$(0, -3)$
3	$y=\sqrt[3]{x}+2$		all reals	all reals	$(-8, 0)$	$(0, 2)$
4	$y=\sqrt[3]{x}-2$		all reals	all reals	$(8, 0)$	$(0, -2)$
5	$y=\sqrt{x-4}$		$x \geq 4$	$y \geq 0$	$(4, 0)$	none
6	$y=\sqrt{x+4}$		$x \geq -4$	$y \geq 0$	$(-4, 0)$	$(0, 2)$
7	$y=\sqrt[3]{x+5}$		all reals	all reals	$(-5, 0)$	$\left(0, \sqrt[3]{5}\right)$
8	$y=\sqrt[3]{x-5}$		all reals	all reals	$(5, 0)$	$\left(0, \sqrt[3]{-5}\right)$
9	$y=\sqrt{x-3}+5$		$x \geq 3$	$y \geq 5$	none	none

(10) What is the difference in the graph when a constant is added outside of the radical, *f(x) + k*, or inside of the radical, *f(x + k)*? *Outside the radical changes the vertical shift, + up and – down. A constant inside the radical, changes the horizontal shift, + left and n right.*

(11) What is the difference in the domains and ranges of $f(x)=\sqrt{x}$ and $g(x)=\sqrt[3]{x}$? Why is the domain of one of the functions restricted and the other not? *Even index radicals have a restricted domain $x \geq 0$ and, therefore, a resulting restricted range $y \geq 0$. The domain and range of odd index radicals are both all reals. You cannot take an even index radical of a negative number.*

Name________________________________ Date____________________________________

Complex Number System Word Grid

Place an "X" in the box corresponding to the set to which the number belongs:

	-5	3	0	28	5.7	$4.1\overline{6}$	π	½	$\sqrt{5}$	$3\sqrt{5}$	$6+3\sqrt[3]{5}$	i	$\sqrt{-4}$	$6i\sqrt{2}$	$2+3i$			
Natural #																		
Whole #																		
Integer																		
Rational #																		
Irrational #																		
Real #																		
Imaginary #																		
Complex #																		

Properties of the Complex Number System

When creating any new number system, certain mathematical properties and operations must be defined. Your team will be assigned some of the following properties. On the transparency or chart paper, define the property for the Complex Number System in words (verbally) and using $a + bi$ (symbolically) and give a complex number example without using the book. Each member of your team will present one of the properties to the class, and the class will decide if it is correct. The team with the most Best Properties wins a bonus point (or candy, etc.).

Sample:

Properties/Operations	**Defined verbally and symbolically with a complex number example**
Equality of Complex Numbers	– Two complex numbers are equal if the real parts are equal and the imaginary parts are equal. – $a + bi = c + di$ if and only if $a = c$ and $b = d$. – $6+3i=\sqrt{36}+\sqrt{-9}$

<u>Properties and operations</u>: addition, additive identity, additive inverse, subtraction, multiplication, multiplicative identity, squaring, dividing, absolute value, reciprocal (multiplicative inverse), commutative under addition and multiplication, associative under addition/multiplication, closed under addition and multiplication, factoring the difference in two perfect squares, factoring the sum of two perfect squares

*Unit 4, Activity 7, **Complex Number System with Answers***

Name_________________________________ Date____________________________________

Complex Number System Word Grid

Place an "X" in the box corresponding to the set to which the number belongs:

	-5	3	0	28	5.7	$4.1\overline{6}$	π	½	$\sqrt{5}$	$3\sqrt{5}$	$6+3\sqrt[3]{5}$	i	$\sqrt{-4}$	$6i\sqrt{2}$	$2+3i$			
Natural #		X		X														
Whole #		X	X	X														
Integer	X	X	X	X														
Rational #	X	X	X	X	X	X		X										
Irrational #							X		X	X	X							
Real #	X	X	X	X	X	X	X	X	X	X	X							
Imaginary #												X	X	X				
Complex #	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X			
Algebraic #	X	X	X	X	X	X		X	X	X	X	X	X	X	X			
Transcendental							X											
Perfect #				X														
Prime #		X																
Composite #				X														
Surd									X	X	X							

Optional sets of numbers for discussion:

- ***algebraic number ≡ real # that occurs as root of a polynomial equation that have integer coefficients.***
- ***transcendental number ≡ not algebraic***
- ***perfect number ≡ any natural number that is equal to the sum of its divisors < itself such as 6 = 1 + 2 + 3***
- ***prime number ≡ any number that can be divided, without a remainder, only by itself and 1***
- ***composite number ≡ a natural number that is a multiple of two numbers other than itself and 1***
- ***surd ≡ an irrational number that can be expressed as a radical***

Properties of the Complex Number System

When creating any new number system, certain mathematical properties and operations must be defined. Your team will be assigned some of the following properties. On the transparency or chart paper, define the property for the Complex Number System in words (verbally) and using $a + bi$ (symbolically) and give a complex number example without using the book. Each member of your team will present one of the properties to the class, and the class will decide if it is correct. The team with the most Best Properties wins a bonus point (or candy, etc.).

Sample:

Properties/Operations	**Defined verbally and symbolically with a complex number example**
Equality of Complex Numbers	– Two complex numbers are equal if the real parts are equal and the imaginary parts are equal. – $a + bi = c + di$ if and only if $a = c$ and $b = d$. – $6+3i=\sqrt{36}+\sqrt{-9}$

Properties and operations: addition, additive identity, additive inverse, subtraction, multiplication, multiplicative identity, squaring, dividing, absolute value, reciprocal (multiplicative inverse), commutative under addition and multiplication, associative under addition/multiplication, closed under addition and multiplication, factoring the difference in two perfect squares, factoring the sum of two perfect squares

Name________________________________ Date___________________________________

Do You Really Know the Difference?

State whether the following numbers are strictly real (R), strictly imaginary (I) or complex with real and imaginary parts (C) and discuss why.

(1) i
(2) i^2
(3) $\sqrt{-9}$
(4) $\sqrt{-2}\sqrt{-5}$
(5) i^n if n is even
(6) the sum of a complex number *(a+bi)* and its conjugate
(7) the difference of complex number *(a+bi)* and its conjugate
(8) the product of complex number *(a+bi)* and its conjugate
(9) the conjugate of an imaginary number
(10) the conjugate of a real number
(11) the reciprocal of an imaginary number
(12) the additive inverse of an imaginary number
(13) the multiplicative identity of an imaginary number
(14) the additive identity of an imaginary number

Answers:

(1) ___ ____

(2) ___ ____

(3) ___ ____

(4) ___ ____

(5) ___ ____

(6) ___ ____

(7) ___ ____

(8) ___ ____

(9) ___ ____

(10) ___ ____

(11) ___ ____

(12) ___ ____

(13) ___ ____

(14) ___ ____

Name_________________________________ Date____________________________________

Do You Really Know the Difference?

State whether the following numbers are strictly real (R), strictly imaginary (I) or complex with real and imaginary parts (C) and discuss why.

(1) i
(2) i^2
(3) $\sqrt{-9}$
(4) $\sqrt{-2}\sqrt{-5}$
(5) i^n if n is even

(6) the sum of a complex number $(a+bi)$ and its conjugate
(7) the difference of complex number $(a+bi)$ and its conjugate
(8) the product of complex number $(a+bi)$ and its conjugate
(9) the conjugate of an imaginary number
(10) the conjugate of a real number
(11) the reciprocal of an imaginary number
(12) the additive inverse of an imaginary number
(13) the multiplicative identity of an imaginary number
(14) the additive identity of an imaginary number

Answers:

(1) $\mathcal{I}$ This is the imaginary number equal to $\sqrt{-1}$.

(2) $\mathcal{R}$ $i^2 = -1$ which is real.

(3) $\mathcal{I}$ $\sqrt{-9} = 3i$ which is imaginary

(4) $\mathcal{R}$ $\sqrt{-2}\sqrt{-5} = \left(i\sqrt{2}\right)\left(i\sqrt{5}\right) = i^2\sqrt{10} = -\sqrt{10}$ which is real.

(5) $\mathcal{R}$ If n is even then i^n will either be 1 or –1 which are real.

(6) $\mathcal{R}$ $(a + bi) + (a - bi) = 2a$ which is real.

(7) $\mathcal{I}$ $(a + bi) - (a - bi) = 2bi$ which is imaginary

(8) $\mathcal{R}$ $(a + bi)(a - bi) = a^2 + b^2$ which is real.

(9) $\mathcal{I}$ The conjugate of $(0 + bi)$ is $(0 - bi)$ which is imaginary.

(10) $\mathcal{R}$ The conjugate of $(a + 0i)$ is $(a - 0i)$ which is real.

(11) $\mathcal{I}$ The reciprocal if i is $\frac{1}{i}$ which equals $-i$ when you rationalize the denominator – imaginary.

(12) $\mathcal{I}$ The additive inverse of $(0 + bi)$ is $(0 - bi)$ which is imaginary

(13) $\mathcal{R}$ The multiplicative identity of $(0 + bi)$ is $(1 + 0i)$ which is real.

(14) $\mathcal{R}$ The additive identity of $(0 + bi)$ is $(0 + 0i)$ which is real