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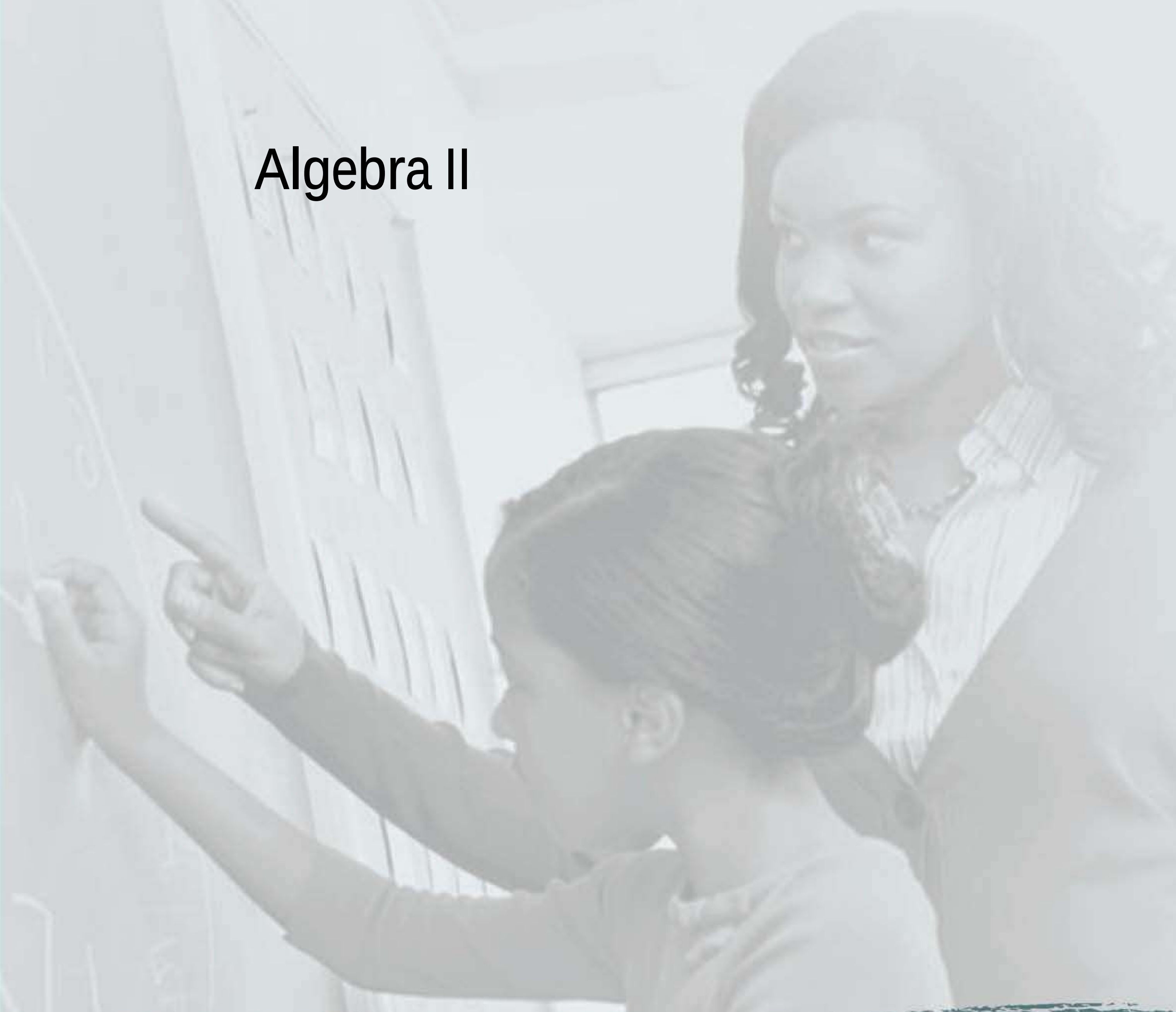
STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

Algebra II



Transitional Curriculum

REVISED 2012

BLACKLINE MASTERS

LOUISIANA DEPARTMENT OF EDUCATION

Little Black Book of Algebra II Properties
Unit 1 - Functions

1.1 *Function of x* – define function, how to identify equations as functions of *x*, how to identify graphs as functions of *x*, how to determine if sets of ordered pairs are functions of *x*, how to explain the meaning of $f(x)$ (e.g., If $f(x) = 3x^2 - 4$, find $f(3)$ and how to explain the process used in terms of a function machine.)

1.2 *Four Ways to Write Solution Sets* – explain/define roster notation, interval notation using $\cup$ and $\cap$, number line, set notation using "and" or "or".

1.3 *Absolute Value Equations* and *Inequalities as Solution Sets* – write solutions in terms of "distance," change absolute value notation to other notations and vice versa (e.g., write $|x| < 4$, $|x - 5| \leq 6$, $|x| \geq 9$ as number lines, as words in terms of distance, as intervals, and in set notation; write : $[-8, 8]$, $(-4, 6)$ in absolute value notation.).

1.4 *Domain* and *Range* – write the definitions, give two possible restrictions on domains based on denominators and radicands, determine the domain and range from sets of ordered pairs, graphs, equations, and inputs and outputs of the function machine; define abscissa, ordinate, independent variable, and dependent variables.

1.5 *Slope of a Line* – define slope, describe lines with positive, negative, zero and no slope, state the slopes of perpendicular lines and parallel lines.

1.6 *Equations of Lines* – write equations of lines in slope-intercept, point-slope, and standard forms, and describe the process for finding the slope and *y*-intercept for each form.

1.7 *Distance between Two Points and Midpoint of a Segment* – write and explain the formula for each.

1.8 *Piecewise Linear Functions* – define and explain how to find domain and range for these functions. (e.g., Graph and find the domain and range of $f(x) = \begin{cases} 2x+1 & if\ x > -3 \\ -x-5 & if\ x \leq -3 \end{cases}$)

1.9 *Absolute Value Function* – define $y = |x|$ as a piecewise function and demonstrate an understanding of the relationships between the graphs of $y = |x|$ and $y = a|x - h| + k$ (i.e., domains and ranges, the effects of changing a, h, and k). Write $y = 2|x-3| +5$ as a piecewise function, explain the steps for changing the absolute value equation to a piecewise function, and determine what part of the function affects the domain restrictions.

1.10 *Step Functions* and *Greatest Integer Function* – define each and relate to the piecewise function. Graph the functions and find the domains and ranges. Work and explain how to work the following examples: (1) Solve for *x*: $\left[\!\left[\frac{1}{2}x\right]\!\right] = 7$.

(2) If $f(x) = [\![2x - 5]\!] + 3$, find $f(0.6)$ and $f(10.2)$.

1.11 *Composite Functions* – define, find the rules of $f(g(x))$ and $g(f(x))$ using the example, $f(x) = 3x + 5$ and $g(x) = x^2$, interpret the meaning of $f \circ g$, explain composite functions in terms of a function machine, explain how to find the domain of composite functions, and how to graph composite functions with the graphing calculator.

1.12 *Inverse Functions* – define, write proper notation, find compositions, use symmetry to find the inverse of a set of ordered pairs or an equation, determine how to tell if the inverse relation of a set of ordered pairs is a function, explain how to tell if the inverse of an equation is a function, and explain how to tell if the inverse of a graph is a function.

Algebra II – Date

Determine if each of the following is a function of x. Explain both yes and no answers.

(1) the set of ordered pairs
$\{(x, y) : (1, 2), (3, 5), (3, 6), (7, 5), (8, 2)\}$

(2) $\{(x, y): (1,1), (2,4), (3,9), (-1,1), (-2,4), (-3,9)\}$

(3) the relationship "x is a student of y"

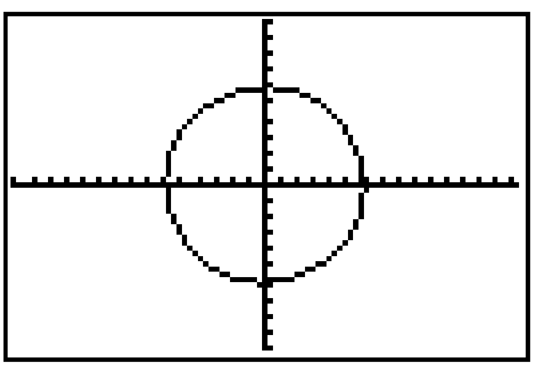
(4) the relationship "x is the biological daughter of mother y"

(5) the equation $2x + 3y = 6$

(6) the equation $x + y^2 = 9$

(7) the equation $y = x^2 + 4$

(8)



Unit 1, Activity 3, Domain & Range Discovery Worksheet

Name_______________________________________ Date________________________

Domain & Range in Real-World Applications

Complete the following chart:

Application	Function Notation	Independent Variable	Allowable Values of the Independent Variable	Dependent Variable	Resulting Values of the Dependent Variable
(1) The area of a circle depends on its radius	$A(r)=\pi r^2$				
(2) The length of the box is twice the width, thus it depends on the width	$l(w) = 2w$				
(3) The state tax on food is 5%, and the amount of tax you pay depends on the cost of the food bought	$t(c) = 0.05c$				
(4) d depends on s in a set of ordered pairs, {(s, d): (1,1), (2, 4), (3, 9), (-1, 1), (-2, 4), (-3, 9)}	$d(s) = s^2$				

- Define **<u>domain</u>**: ____

Specify the <u>domains</u> of the above functions in <u>interval notation</u> and explain why they are restricted:

(1) Domain: ________________ Why restricted? ____

(2) Domain: ________________ Why restricted? ____

(3) Domain: ________________ Why restricted? ____

(4) Domain: ________________ Why restricted? ____

- Define **<u>range</u>**: ____

Specify the <u>ranges</u> of the above functions in <u>interval notation</u> and explain why they are restricted:

(1) Range: _________________ Why restricted? ____

(2) Range: _________________ Why restricted? ____

(3) Range: _________________ Why restricted? ____

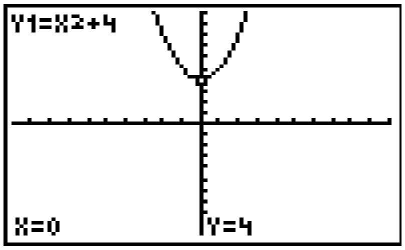
(4) Range: _________________ Why restricted? ____

Unit 1, Activity 3, Domain & Range Discovery Worksheet

Domain & Range from Graphs

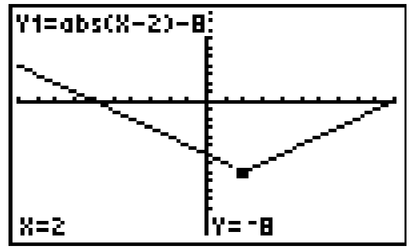
In the following graphs, what is the independent variable? ____ the dependent variable? _____

State the domain and range of the following graphs using <u>interval notation</u>. Assume the graphs continue to infinity as the picture leaves the screen.



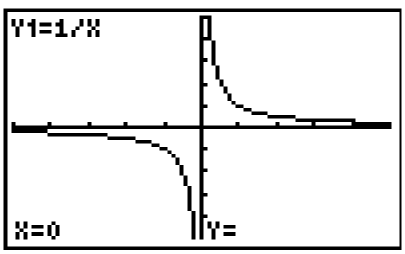
(5) Domain: ________________

Range: __________________



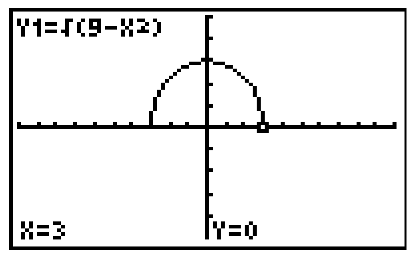
(8) Domain: ________________

Range: __________________



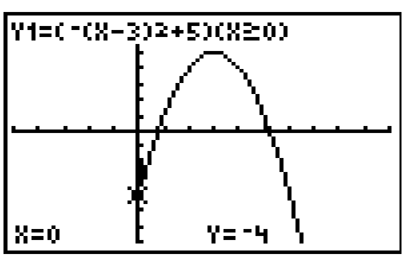
(6) Domain: ________________

Range: __________________



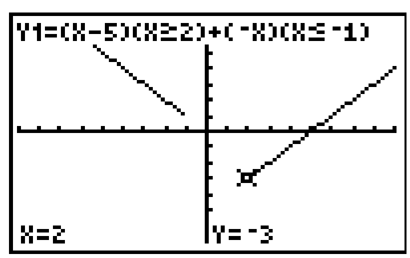
(9) Domain: ________________

Range: __________________



(7) Domain: ________________

Range: __________________



(10) Domain: ________________

Range: __________________

Reviewing Absolute Value Notation: State the domain and range of #6 and #9 above in absolute value notation:

(11) Domain of #6: ____________________ Range: ______________________

(12) Domain of # 9: ___________________ Range: ______________________

Unit 1, Activity 3, Domain & Range Discovery Worksheet

Domain & Range from Algebraic Equations

Consider the following functions.

- Decide if there are any values of *x* that are not allowed, therefore, creating a restricted domain. State the domain of each function in set notation and why it is restricted.
- Then consider if this restricted domain results in a restricted range. State the range of each function in set notation and why it is restricted.

Function	Domain *and* Why Restricted	Range *and* Why Restricted
(13) $f(x) = 3x + 1$		
(14) $f(x) = \frac{1}{x}$		
(15) $g(x) = \sqrt{x}$		
(16) $f(x) = \frac{1}{\sqrt{2x-6}}$		
(17) $g(x) = -\sqrt{x-2}$		

(18) Explain two types of domain restrictions in the real number system demonstrated by the examples above:

I. ____

II. ____

Combinations of Functions When a third function is created from the combination of two functions, the domain of the combination must include the domains of the original function further restricted by the new combination function. State the domain of the following functions using set notation.

$f(x) = \sqrt{x-2}$ What is the domain of *f(x)*?________________________

$g(x) = \frac{1}{x-3}$ What is the domain of *g(x)*? ________________________

Find the equation for the following combinations and determine the new domain in set notation:

(19) *(f + g)(x)* = __________________________________ Domain*:* _____________________

(20) *(fg)(x)* = __________________________________ Domain*:* _____________________

(21) $\frac{g}{f}(x)$ = __________________________________ Domain*:* _____________________

Unit 1, Activity 3, Domain & Range Discovery Worksheet with Answers

Name_______________________________________ Date________________________

Domain & Range in Real-World Applications

Complete the following chart:

Application	Function Notation	Independent Variable	Allowable Values of the Independent Variable	Dependent Variable	Resulting Values of the Dependent Variable
(1) The area of a circle depends on its radius	$A(r)=\pi r^2$	*r*	$r \geq 0$	*a*	$a \geq 0$
(2) The length of the box is twice the width thus it depends on the width	$l(w) = 2w$	*w*	$w \geq 0$	*ℓ*	$\ell \geq 0$
(3) The state tax on food is 5%, and the amount of tax you pay depends on the cost of the food bought	$t(c) = 0.05c$	*c*	$c \geq 0$	*t*	$t \geq 0$
(4) d depends on s in a set of ordered pairs, {(s, d): (1,1), (2, 4), (3, 9), (-1, 1), (-2, 4), (-3, 9)}	$d(s) = s^2$	*s*	*{1, 2, 3, -1,-2, –3}*	*d*	*(1, 4, 9}*

- Define **domain**: *the allowable values of the independent variable* .

Specify the domains of the above functions in interval notation and explain why they are restricted:

(1) Domain: ____$[0,\infty)$______ Why restricted? *because all radii are greater than or = 0*

(2) Domain: ____$[0,\infty)$______ Why restricted? *because all widths are greater than or = 0*

(3) Domain: ____$[0,\infty)$______ Why restricted? *because all costs are greater than or = 0*

(4) Domain: *{1, 2, 3, -1, -2, -3}* Why restricted? *cannot write as interval only roster first terms*

- Define **range**: *resulting values of the dependent variable*

Specify the ranges of the above functions in interval notation and explain why they are restricted:

(1) Range: $[0,\infty)$_________ Why restricted? *because all areas > 0,*

(2) Range: $(0,\infty)$_________ Why restricted? *because all lengths > 0,*

(3) Range: $[0,\infty)$_________ Why restricted? *because all taxes > 0,*

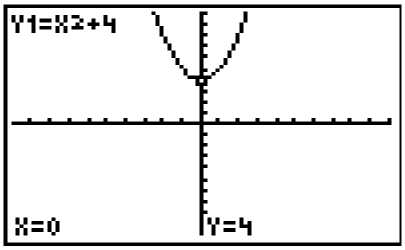
(4) Range: *{1, 4, 9}*_________ Why restricted? *cannot write as interval only roster second terms*

Unit 1, Activity 3, Domain & Range Discovery Worksheet with Answers

Domain & Range from Graphs

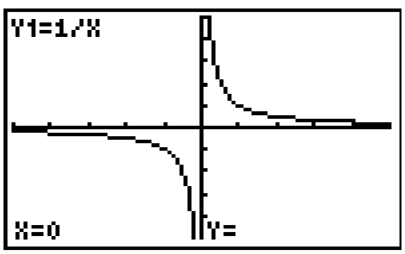
In the following graphs, what is the independent variable? __*x*__ the dependent variable? __*y*__

State the domain and range of the following graphs using <u>interval notation</u>. Assume the graphs continue to infinity as the picture leaves the screen.



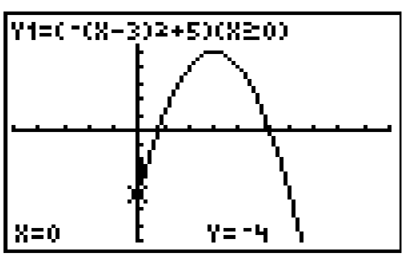
(5) Domain: $(-\infty,\infty)$

Range: $[4,\infty)$



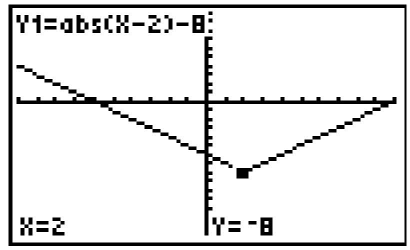
(6) Domain: $(-\infty,0)\cup(0,\infty)$

Range: $(-\infty,0)\cup(0,\infty)$



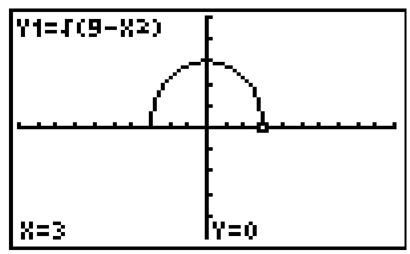
(7) Domain: $[0,\infty)$

Range: $(-\infty,5]$



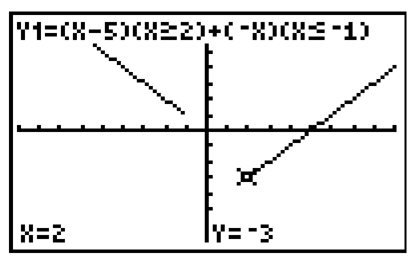
(8) Domain: $(-\infty,\infty)$

Range: $[-8,\infty)$



(9) Domain: *[–3, 3]*

Range: *[0, 3]*



(10) Domain: $(-\infty,-1]\cup[2,\infty)$

Range: $[-3,\infty)$

Reviewing Absolute Value Notation: State the domain and range of #6 and #9 above in absolute value notation:

(11) Domain of #6: $|x| > 0$ Range: $|y| > 0$

(12) Domain of # 9: $|x| \le 3$ Range: $|y - 1.5| \le 1.5$

Domain & Range from Algebraic Equations

Consider the following functions.

- Decide if there are any values of *x* that are not allowed, therefore, creating a restricted domain. State the domain of each function in set notation and why it is restricted.
- Then consider if this restricted domain results in a restricted range. State the range of each function in set notation and why it is restricted.

Function	Domain *and* Why Restricted	Range *and* Why Restricted
(13) $f(x)=3x+1$	$\{x : x \in Reals\}$ no restrictions	$\{y : y \in Reals\}$ no restrictions
(14) $f(x)=\frac{1}{x}$	$\{x : x \neq 0\}$ Division by zero is undefined.	$\{y : y \neq 0\}$ Because the numerator is a constant, y will never result in the value 0.
(15) $g(x)=\sqrt{x}$	$\{x : x \geq 0\}$ You cannot take a square root of a negative number	$\{y : y \geq 0\}$, A radical is always the principal square root therefore always positive or zero.
(16) $f(x)=\frac{1}{2x-6}$	$\{x : x \neq 3\}$ Division by zero is undefined	$\{y : y \neq 0\}$ Because the numerator is a constant, y will never result in the value 0.
(17) $g(x)=-\sqrt{x-2}$	$\{x : x \geq 2\}$, You cannot take a square root of a negative number.	$\{y : y \leq 0\}$, A radical is always the principal square root therefore always positive or zero. The negative in front of the radical makes it always negative or zero

(18) Explain two types of domain restrictions in the real number system demonstrated by the examples above:

I. Division by zero is undefined,

II. The value under the square root (or any even root) must be > 0.

Combinations of Functions When a third function is created from the combination of two functions, the domain of the combination must include the domains of the original functions further restricted by the new combination function. State the domain of the following functions using set notation.

$f(x)=\sqrt{x-2}$ What is the domain of *f(x)*? $\{x: x \geq 2\}$

$g(x)=\frac{1}{x-3}$ What is the domain of *g(x)*? $\{x: x \neq 3\}$

Find the equation for the following combinations and determine the new domain in set notation:

(19) *(f + g)(x)* = $(f+g)(x)=\sqrt{x-2}+\frac{1}{x-3}$ Domain: $\{x: x \geq 2 \text{ and } x \neq 3\}$

(20) *(f g)(x)* = $(fg)(x)=\frac{\sqrt{x-2}}{x-3}$ Domain: $\{x: x \geq 2 \text{ and } x \neq 3\}$

(21) $\frac{g}{f}(x)$ = $\frac{g}{f}(x)=\frac{1}{(x-3)\sqrt{x-2}}$ Domain: $\{x: x > 2 \text{ and } x \neq 3\}$

Name_______________________________________ Date________________________

Vocabulary Self–Awareness Chart

Complete the following with a partner.

- Rate your understanding of each concept with either a "+" (understand well), "✓" (limited understanding or unsure), or a "–" (don't know)
- Write the formula or description

Mathematical Terms		+	✓	–	Formula or description
1	slope of a line				
2	slope of horizontal line				
3	equation of a horizontal line				
4	slope of a line that starts in Quadrant III and ends in Quadrant I				
5	slope of a line that starts in Quadrant II and ends in Quadrant IV				
6	slope of a vertical line				
7	equation of a vertical line				
8	slopes of parallel lines				
9	slopes of perpendicular lines				
10	point-slope form of equation of line				
11	y-intercept form of equation of line				
12	standard form of equation of line				
13	distance formula				
14	midpoint formula				

Unit 1, Activity 5, Linear Equation Terminology

Sample Problems

Create a sample problem for each concept and solve it: *(The first one has been created for you as an example, but you still have to solve it.)*

1) slope of a line *Find the slope of the line between the two points (–2, 6) and (9, 4)*	8) slopes of parallel lines
2) slope of horizontal line	9) slopes of perpendicular lines
3) equation of a horizontal line	10) point-slope form of equation of line
4) slope of a line that starts in Quadrant III and ends in Quadrant I	11) *y*-intercept form of equation of line
5) slope of a line that starts in Quadrant II and ends in Quadrant IV	12) standard form of equation of line
6) slope of a vertical line	13) distance formula
7) equation of a vertical line	14) midpoint formula

Unit 1, Activity 5, Linear Equation Terminology with Answers

Name____ Date_________________________

Vocabulary Self–Awareness Chart

Complete the following with a partner.

- Rate your understanding of each concept with either a "+" (understand well), "✓" (limited understanding or unsure), or a "–" (don't know)
- Write the formula or description

	Mathematical Terms	+	✓	–	Formula or description
1	slope of a line				$m=\frac{y_2-y_1}{x_2-x_1}$ or $\frac{\Delta y}{\Delta x}$
2	slope of horizontal line				$m = 0$
3	equation of a horizontal line				$y = k$, where k is some constant
4	slope of a line that starts in Quadrant III and ends in Quadrant I				positive
5	slope of a line that starts in Quadrant II and ends in Quadrant IV				negative
6	slope of a vertical line				undefined
7	equation of a vertical line				$x = k$, where k is some constant
8	slopes of parallel lines				same slopes
9	slopes of perpendicular lines				opposite reciprocal slopes
10	point-slope form of equation of line				$y - y_1 = m(x - x_1)$
11	*y*-intercept form of equation of line				$y = mx + b$
12	standard form of equation of line				$Ax + By = C$ no fractions
13	distance formula				$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ or $\sqrt{(\Delta x)^2+(\Delta y)^2}$
14	midpoint formula				$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

Unit 1, Activity 5, Linear Equation Terminology with Answers

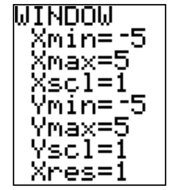
Sample Problems

Create a sample problem for each concept and solve it: *(The first one has been created for you as an example, but you still have to solve it.)*

1) slope of a line *Find the slope of the line between the two points (–2, 6) and (9, 4)* ***(Solution:*** $-\frac{2}{11}$ ***)***	8) slopes of parallel lines ***Find the slope of a line parallel to the line*** $y = 3x + 5$ ***(Solution:*** $m = 3$***)***
2) slope of horizontal line ***Find the slope of the line*** $y = 3$ ***(Solution:*** $m = 0$***)***	9) slopes of perpendicular lines ***Find the slope of a line perpendicular to the line*** $y = 3x + 5$ ***(Solution:*** $m = -\frac{1}{3}$ ***)***
3) equation of a horizontal line ***Find the equation of the horizontal line through the point (2, 4)*** ***(Solution:*** $y = 4$***)***	10) point-slope form of equation of line ***Find the equation of the line in point–slope form through the point (4, –5) and has a slope*** $m = ½$ ***(Solution:*** $y + 5 = ½ (x – 4)$***)***
4) slope of a line that starts in Quadrant III and ends in Quadrant I ***Find the slope of a line between the points (–2, –5) and (4, 13)*** ***(Solution:*** $m = 3$***)***	11) *y*-intercept form of equation of line ***Find the equation of the line in y–intercept form through the point (4, –5) and has a slope*** $m = ½$ ***(Solution:*** $y = ½ x – 7$***)***
5) slope of a line that starts in Quadrant II and ends in Quadrant IV ***Find the slope of a line between the points (–2, 5) and (4, –13)*** ***(Solution:*** $m = –3$***)***	12) standard form of equation of line ***Find the equation of the line in standard form through the point (4, –5) and has a slope*** $m = ½$ ***(Solution:*** $x – 2y = 14$***)***
6) slope of a vertical line ***Find the slope of the line*** $x = 8$ ***(Solution: m is undefined)***	13) distance formula ***Find the distance between the two points (2, 4) and (–3, 7) (Solution:*** $\sqrt{34}$ ***)***
7) equation of a vertical line ***Find the equation of the vertical line through the point (2, 4)*** ***(Solution:*** $x = 2$***)***	14) midpoint formula ***Find the midpoint between the two points (2, 4) and (–3, 7)*** ***(Solution:*** $\left(-\frac{1}{2}, \frac{11}{2}\right)$ ***)***

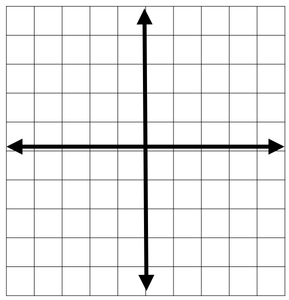
Unit 1, Activity 5, Translating Graphs of Lines Discovery Worksheet

Name_____________________________________ Date_______________________

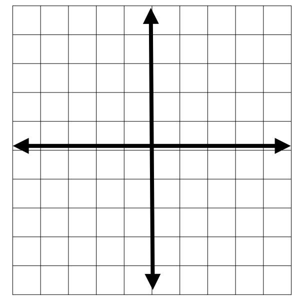


Translating Graphs of Lines The following graphs are transformations of the parent function $f(x) = x$ in the form $f(x) = a(x \pm h) \pm k$. Set your calculator window as shown at the right, graph each set of lines on the same screen, and sketch below. Discuss the changes in the equation and what effect the change has on the graph.

(1) $f(x) = 2(x - 0) + 0$
$g(x) = 2(x + 3) + 0$
$h(x) = 2(x - 4) + 0$



(2) $f(x) = 2(x - 0) + 0$
$g(x) = 2(x - 0) + 3$
$h(x) = 2(x - 0) - 4$

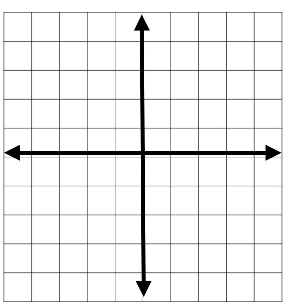


(3) What happens to the graph when you <u>add</u> a number <u>*in*</u> the function? (i.e. $f(x + h)$)

(4) What happens to the graph when you <u>subtract</u> a number <u>*in*</u> the function? (i.e. $f(x - h)$)

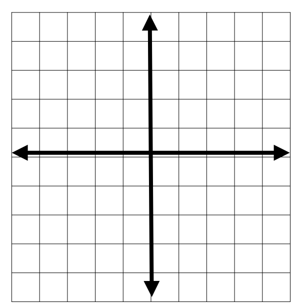
(5) What happens to the graph when you <u>add</u> a number to the function? (i.e. $f(x) + k$)

(6) What happens to the graph when you <u>subtract</u> a number from the function? (i.e. $f(x) - k$)



(7) $f(x) = 1(x - 0) - 2$
$g(x) = \frac{1}{4}(x - 0) - 2$
$h(x) = 4(x - 0) - 2$

(8) $f(x) = 2(x - 0) + 3$
$g(x) = 2(-x - 0) + 3$



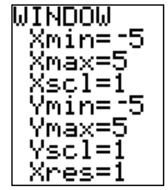
(9) What happens to the graph when the function is <u>multiplied</u> by a number between 0 and1? (i.e. $k\,f(x)$ where $0 < k < 1$)

(10) What happens to the graph when the function is <u>multiplied</u> by a number greater than 1? (i.e. $k\,f(x)$ where $k > 1$)

(11) What happens to the graph when you take the opposite of the *x* in the function? (i.e. $f(-x)$)

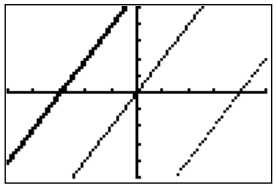
Unit 1, Activity 5, Translating Graphs of Lines Discovery Worksheet with Answers

Name_____________________________________ Date_______________________

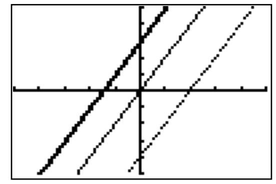


Translating Graphs of Lines The following graphs are transformations of the parent function $f(x) = x$ in the form $f(x) = a(x \pm h) \pm k$. Set your calculator window as shown and graph each set of lines on the same screen and sketch below. Discuss the changes in the equation and what effect the change has on the graph.

(1) $f(x) = 2(x - 0) + 0$
$g(x) = 2(x + 3) + 0$
$h(x) = 2(x - 4) + 0$



(2) $f(x) = 2(x - 0) + 0$
$g(x) = 2(x - 0) + 3$
$h(x) = 2(x - 0) - 4$



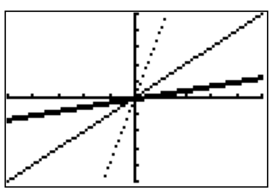
(3) What happens to the graph when you <u>add</u> a number <u>*in*</u> the function? (i.e. $f(x + h)$)
The x-value of the point where you start graphing moves to the left.

(4) What happens to the graph when you <u>subtract</u> a number <u>*in*</u> the function? (i.e. $f(x - h)$)
The x-value of the point where you start graphing moves to the right.

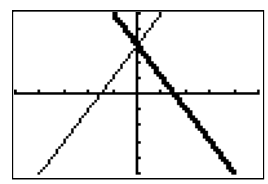
(5) What happens to the graph when you add a number to the function? (i.e. $f(x) + k$)
The y-value of the point where you start graphing moves up.

(6) What happens to the graph when you subtract a number from the function? (i.e. $f(x) - k$)
The y-value of the point where you start graphing moves down.

(7) $f(x) = 1(x - 0)$
$g(x) = \frac{1}{4}(x - 0)$
$h(x) = 4(x - 0)$



(8) $f(x) = 2(x - 0) + 3$
$g(x) = 2(-x - 0) + 3$



(9) What happens to the graph when the function is multiplied by a number between 0 and 1? (i.e. $k\,f(x)$ where $0 < k < 1$)
The graph becomes less steep.

(10) What happens to the graph when the function is multiplied by a number greater than 1? (i.e. $k\,f(x)$ where $k > 1$)
The graph becomes steeper.

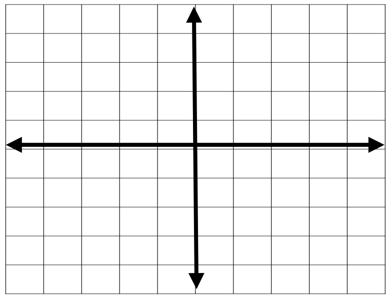
(11) What happens to the graph when you take the opposite of the x in the function? (i.e. $f(-x)$)
The graph rotates through space around the y–axis.

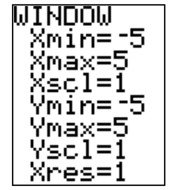
Unit 1, Activity 7, Translating Absolute Value Functions Discovery Worksheet

Name______________________________________ Date________________________

Graphing Absolute Value Functions Graph the following piecewise function by hand:

(1) $f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$





(2) On your graphing calculator graph the function $f(x) = |x|$ with this WINDOW and answer the following questions. *(Note: Absolute value is under MATH > NUM > 1: abs(, so in your calculator you will type* $y_1 = abs(x)$*)*

a. Compare the graph to the graph in #1 above. What is the relationship between the two?

b. What is the shape of the graph?

c. What is the slope of the two lines that create the graph?

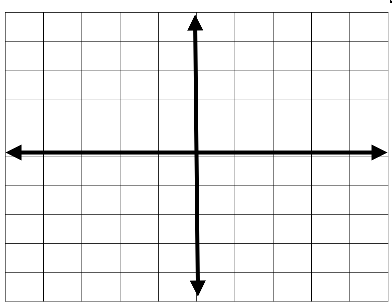
d. What is the vertex of the graph?

e. What is the domain and range?

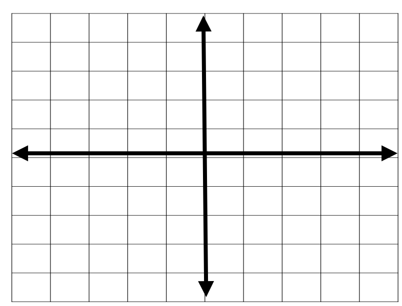
f. What is the axis of symmetry?

Translating Graphs of Absolute Value Functions The following graphs are transformations of the parent function $f(x) = |x|$ in the form $f(x) = a|x - h| + k$. Graph each on your calculator and sketch below and observe the type of transformation.

(3) $f(x) = |x| - 4$



(4). $f(x) = |x| + 2$

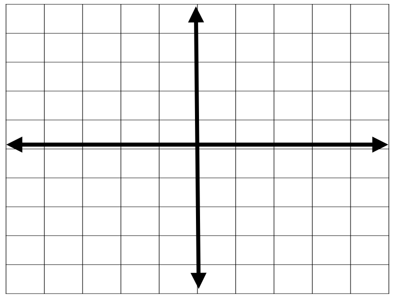


(5) What happens to the graph when you subtract a number from the function? (i.e. $f(x) - k$)

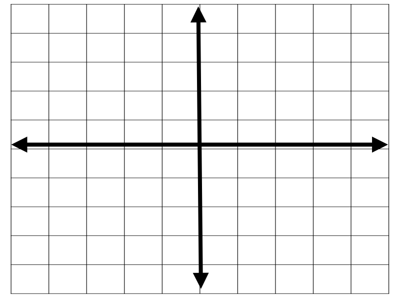
(6) What happens to the graph when you add a number to the function? (i.e. $f(x) + k$)

(7) $f(x)=|x-4|$

$f(x)=|x+2|$



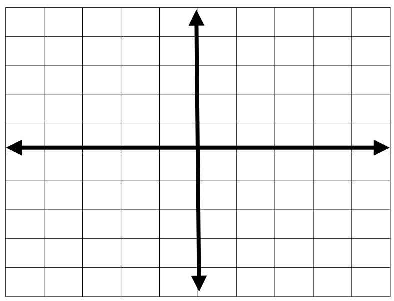
(8)



(9) What happens to the graph when you subtract a number <u>*in*</u> the function? (i.e. *f*(*x* − *h*))

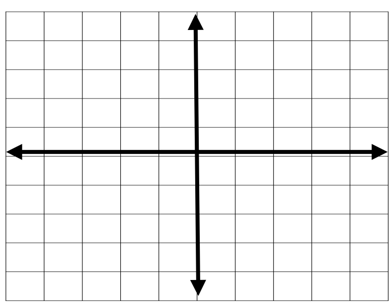
(10) What happens to the graph when you add a number <u>*in*</u> the function? (i.e. *f*(*x* + *h*))

(11) $f(x)=-|x|$

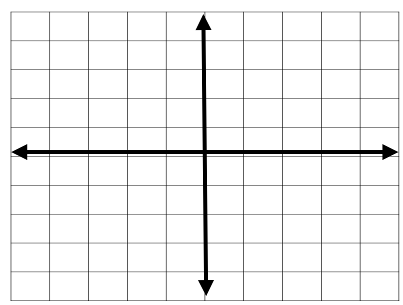


(12) What happens to the graph when you take the opposite of the function? (i.e. −*f*(*x*))

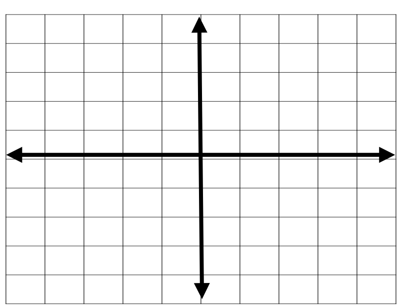
(13) $f(x)=2|x|$



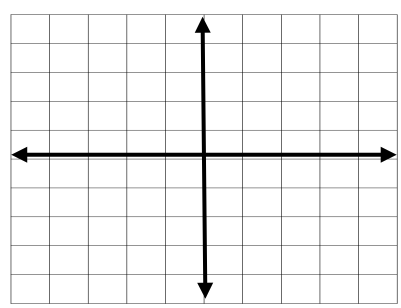
(14) $f(x)=\frac{5}{2}|x|$



(15) $f(x)=\frac{1}{2}|x|$



(16) $f(x)=\frac{2}{5}|x|$



17) What happens to the graph when the function is multiplied by a number greater than 1?

(18) What happens to the graph when the function is multiplied by a number between 0 and 1?

(19) These graphs are in the form *af(x)*. What does the "*a*" represent in these graphs?

Synthesis Write an equation for each described transformation.

(20) a V-shape shifted down 4 units: *f(x)* = ___________________

(21) a V-shape shifted left 6 units: *f(x)* = ___________________

(22) a V-shape shifted right 2 units and up 1 unit: *f(x)* = ___________________

(23) a V-shape rotated through space around the *x*-axis and shifted left 5 units: *f(x)* = ___________________

Analysis Describe the transformation that has taken place for the parent function $f(x) = |x|$.

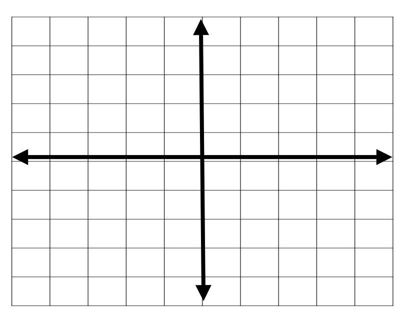
(24) $f(x) = |x| - 5$ ____

(25) $f(x) = 5|x+7|$ ____

(26) $f(x) = -\frac{1}{4}|x|$ ____

(27) $f(x) = |x-4| + 3$ ____

(28) Graph the function $f(x) = 2|x-1| - 3$ without a calculator and answer the following questions:



a. What is the shape of the graph?

b. What is the vertex of the graph and how do you know?

c. Does it open up or down and how do you know?

d. What are the slopes of the two lines that create the graph?

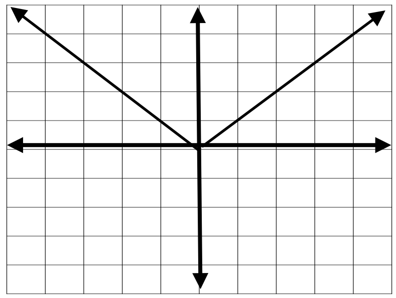
e. What is the domain and range?

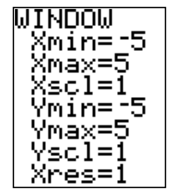
f. What is the axis of symmetry?

Name______________________________________ Date________________________

Graphing Absolute Value Functions Graph the following piecewise function by hand:

(1) $f(x)=\begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$





(2) On your graphing calculator graph the function $f(x)=|x|$ with this WINDOW and answer the following questions. *(Note: Absolute value is under MATH > NUM > 1: abs(so in your calculator you will type y_1 = abs(x))*

a. Compare the graph to the graph in #1 above. What is the relationship between the two?
the graphs are the same

b. What is the shape of the graph? ***Two rays with a common endpoint that form a V.***

c. What is the slope of the two lines that create the graph? $m = \pm 1$

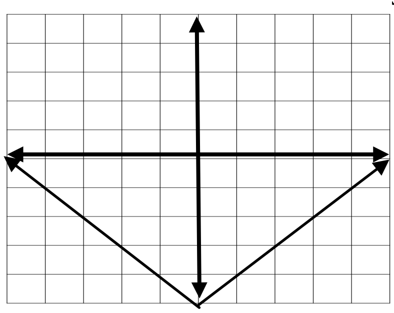
d. What is the vertex of the graph? ***(0, 0)***

e. What is the domain and range? ***Domain: all reals, Range:*** $y \ge 0$

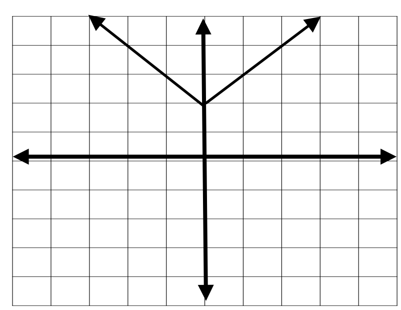
f. What is the axis of symmetry? $x = 0$

Translating Graphs of Absolute Value Functions The following graphs are transformations of the parent function *f(x)* = |*x*| in the form *f(x)* = *a*|*x* – *h*| + *k*. Graph each on your calculator and sketch below and observe the type of transformation.

(3) $f(x)=|x|-4$



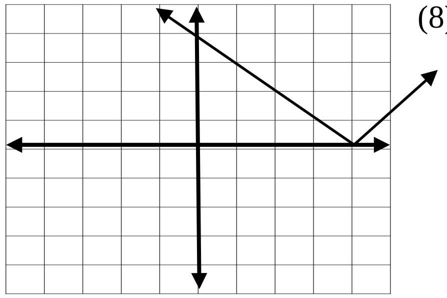
(4). $f(x)=|x|+2$



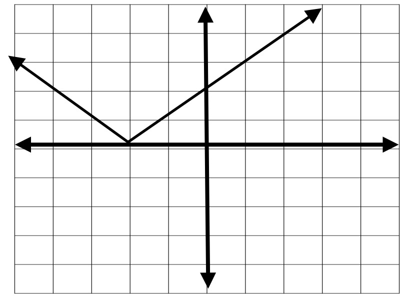
(5) What happens to the graph when you subtract a number from the function? (i.e. *f(x)* – *k*)
The graph shifts down.

(6) What happens to the graph when you add a number to the function? (i.e. *f(x)* + *k*)
The graph shifts up.

(7) $f(x) = |x - 4|$



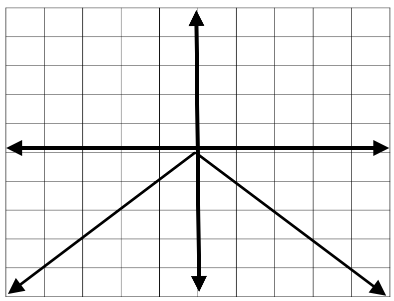
(8) $f(x) = |x + 2|$



(9) What happens to the graph when you subtract a number *in* the function? (i.e. *f(x – h)*)
The graph shifts to the right.

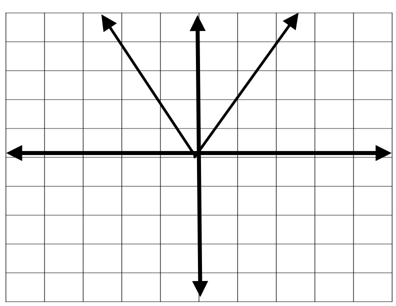
(10) What happens to the graph when you add a number *in* the function? (i.e. *f(x + h)*)
The graph shifts to the left.

(11) $f(x) = -|x|$

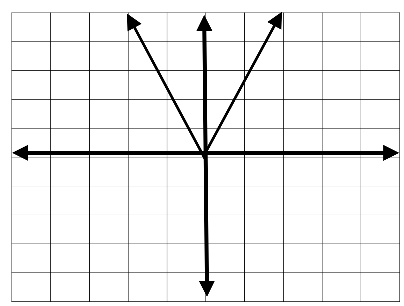


(12) What happens to the graph when you take the opposite of the function? (i.e. *–f(x)*)
The graph rotates through space around the x–axis.

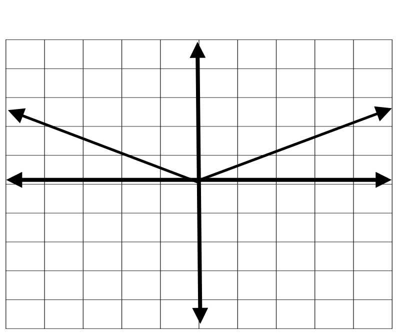
(13) $f(x) = 2|x|$



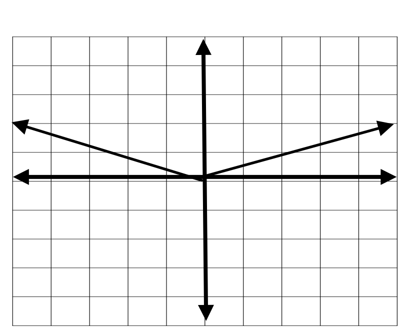
(14) $f(x) = \frac{5}{2}|x|$



(15) $f(x) = \frac{1}{2}|x|$



(16) $f(x) = \frac{2}{5}|x|$



(17) What happens to the graph when the function is multiplied by a number greater than 1?
The graph is stretched vertically thus gets steeper.

(18) What happens to the graph when the function is multiplied by a number between 0 and 1?
The graph is compressed vertically thus gets less steep.

(19) These graphs are in the form *af(x)*. What does the "*a*" represent in these graphs?
The slopes of the two rays are ±a.

Synthesis Write an equation for each described transformation.

(20) a V-shape shifted down 4 units: $f(x) = |x| - 4$

(21) a V-shape shifted left 6 units: $f(x) = |x + 6|$

(22) a V-shape shifted right 2 units and up 1 unit: $f(x) = |x - 2| + 1$

(23) a V-shape rotated through space around the *x*-axis and shifted left 5 units: $f(x) = -|x + 5|$

Analysis Describe the transformation that has taken place for the parent function $f(x) = |x|$.

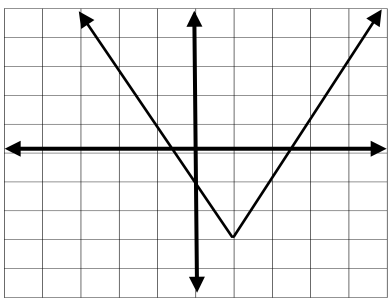
(24) $f(x) = |x| - 5$ ***a V–shaped graph shifted down 5 units***

(25) $f(x) = 5|x + 7|$ ***a steeper (slopes of ±5) V–shaped graph shifted left 7 units***

(26) $f(x) = -\frac{1}{4}|x|$ ***an upside down V–shaped graph not very steep with slopes of ± ¼***

(27) $f(x) = |x - 4| + 3$ ***a V–shaped graph shifted right 4 and up 3***

(28) Graph the function $f(x) = 2|x - 1| - 3$ without a calculator and answer the following questions:



a. What is the shape of the graph? ***V–shaped***

b. What is the vertex of the graph and how do you know?
(1, –3) because it shifted right 1 and down 3.

c. Does it open up or down and how do you know?
up because the leading coefficient is positive.

d. What are the slopes of the two lines that create the graph? ***m = ±2***

e. What is the domain and range? ***Domain: all reals, Range:*** $y \geq -3$

f. What is the axis of symmetry? ***x = 1***

Name_______________________________________ Date________________________

Absolute Value Inequalities *You previously learned how to solve the one variable absolute value inequality |x – h| > d using the concept that x is greater than a distance of d from the center h, and also to write the answer as an "and" or "or" statement. In this activity you will discover how to use the graph of a two variable absolute value function y = |x – h| + k to help you solve a one variable absolute value inequality.*

1. Solve the inequality $|x – 4| > 5$. Write the solution in terms of "distance" and in interval notation.

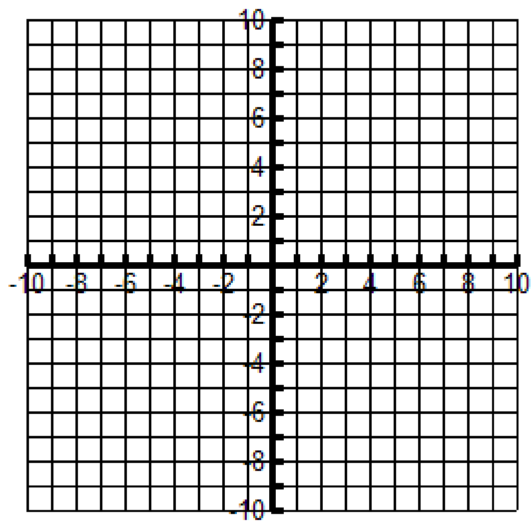
____

____

2. Isolate zero in the equation $|x – 4| > 5$. ____

3. Graph the function $f(x) = |x – 4| – 5$

4. Write the function as a piecewise function.



5. Find the x-intercepts of each piece of the piecewise function.

6. Use the graph of $f(x)$ to determine the interval/s where $f(x) > 0$ and explain how you got the answer looking at the graph. Does your answer match the answer to #1?

   solution in interval notation: _______________________ Explanation: __________________

   ____

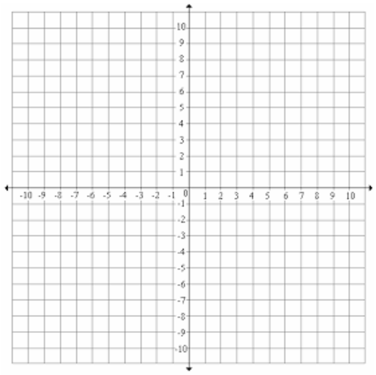
7. Write the solution in #6 in set notation. Using the piecewise function for *f(x) in #4,* explain why the solution to $|x – 4| > 5$ is an "or" statement instead of an "and" statement.

   solution in set notation: ______________ Explanation: ______________________________

   ____

Practice

(8) Graph $f(x) = |x + 2| - 6$ and write $f(x)$ as a piecewise function and find the x-intercepts.

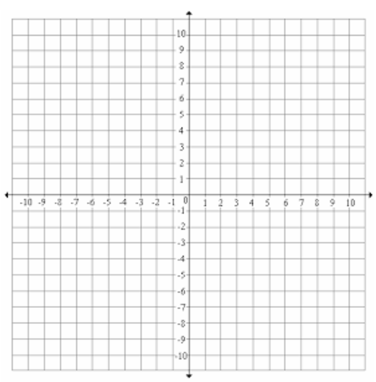


$f(x) = \begin{cases} \\ \\ \end{cases}$

x-intercepts: x =________ and x =_______

Solve $|x + 2| - 6 \le 0$ using the graph above (interval notation)____________________________

(9) Graph $f(x) = 3|x - 4| - 6$ and write $f(x)$ as a piecewise function and find the x-intercepts.

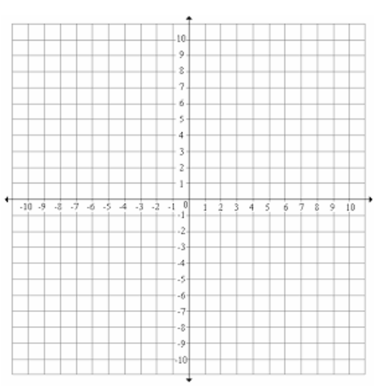


$f(x) = \begin{cases} \\ \\ \end{cases}$

x-intercepts: x =________ and x =_______

Solve $3|x - 4| - 6 > 0$ using the graph above (interval notation)___________________________

(10) Graph $f(x) = -2|x - 3| + 8$ and write $f(x)$ as a piecewise function and find the x-intercepts.



$f(x) = \begin{cases} \\ \\ \end{cases}$

x-intercepts: x =________ and x =_______

Solve $-2|x - 3| + 8 \ge 0$ using the graph above (interval notation)_________________________

Unit 1, Activity 9, Absolute Value Inequalities Discovery Worksheet with Answers

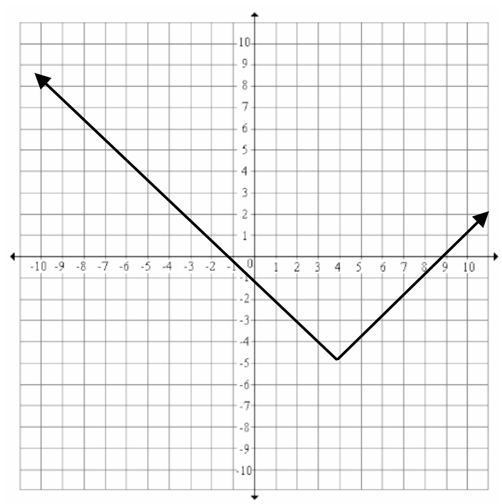
Name______________________________________ Date________________________

Absolute Value Inequalities *You previously learned how to solve the one variable absolute value inequality $|x - h| > d$ using the concept that x is greater than a distance of d from the center h, and also to write the answer as an "and" or "or" statement. In this activity you will discover how to use the graph of a two variable absolute value function $y = |x - h| + k$ to help you solve a one variable absolute value inequality.*

1. Solve the inequality $|x - 4| > 5$. Write the solution in terms of "distance" and in interval notation.

   x is a distance greater than 5 from 4 so the interval solution is $(-\infty, -1) \cup (9, \infty)$.

2. Isolate zero in the equation $|x - 4| > 5$. ***$|x - 4| - 5 > 0$***



3. Graph the function $f(x) = |x - 4| - 5$

4. Write the function as a piecewise function.

$$f(x) = \begin{cases} x - 9 & \text{if } x \geq 4 \\ -x - 1 & \text{if } x < 4 \end{cases}$$

5. Find the *x*-intercepts of each piece of the piecewise function.

   x-intercepts at $x = -1$ and 9

6. Use the graph of *f(x)* to determine the interval/s where *f(x)* > 0 and explain how you got the answer looking at the graph. Does your answer match the answer to #1?

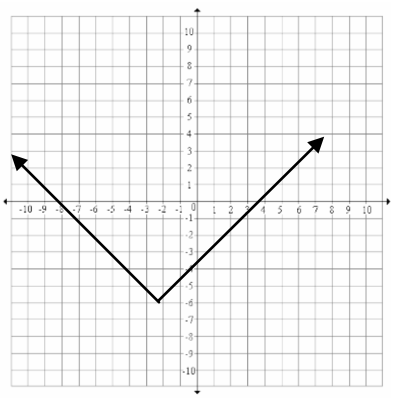
   solution in interval notation: ***$(-\infty, -1) \cup (9,\infty)$.*** Explanation: ***By looking at the graph and the x-intercepts, you can find the values of x for which the y values are > 0***

7. Write the solution in #6 in set notation. Using the piecewise function for *f(x) in* #4, explain why the solution to $|x - 4| > 5$ is an "or" statement instead of an "and" statement.

   solution in set notation: ***$\{x : x < -1 \text{ or } x > 9\}$*** Explanation: ***Since the intervals are in different equations of the piecewise function and do not intersect, you must use union which is an "or" statement.***

Practice

(8) Graph $f(x) = |x + 2| - 6$ and write $f(x)$ as a piecewise function and find the x-intercepts.

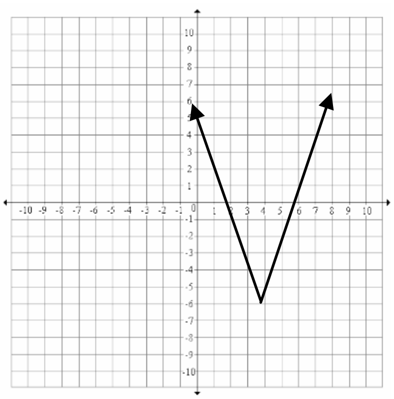


$$f(x) = \begin{cases} x - 4 & \text{if } x \geq -2 \\ -x - 8 & \text{if } x < -2 \end{cases},$$

x-intercepts: x = 4 and –8,

Solve |x + 2| – 6 ≤ 0 using the graph above (interval notation) ***[–8, 4]***

(9) Graph $f(x) = 3|x - 4| - 6$ and write $f(x)$ as a piecewise function and find the x-intercepts.

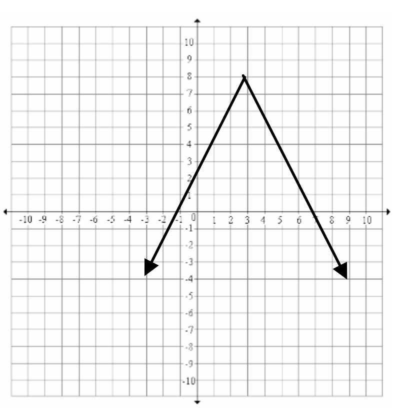


$$f(x) = \begin{cases} 3x - 18 & \text{if } x \geq 4 \\ -3x + 6 & \text{if } x < 4 \end{cases}$$

x-intercepts: x = 6 and 2

Solve $3|x - 4| - 6 > 0$ using the graph above (interval notation) ***(–∞, 2) ∪ (6,∞)***

(10) Graph $f(x) = -2|x - 3| + 8$ and write $f(x)$ as a piecewise function and find the x-intercepts.



$$f(x) = \begin{cases} -2x + 14 & \text{if } x \geq 3 \\ 2x + 2 & \text{if } x < 3 \end{cases}$$

x-intercepts: x = –1 and 7

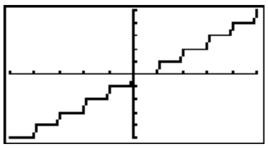
Solve $-2|x - 3| + 8 \geq 0$ using the graph above (interval notation) ***[–1, 7]***

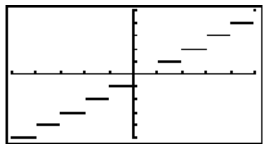
Unit 1, Activity 10, Greatest Integer Discovery Worksheet

Name______________________________________ Date________________________

Graphing the Greatest Integer Function

(1) On the graphing calculator, graph y = int(x). *(Note: On the TI calculator, the greatest integer function is under MATH, NUM, 5: int(In your calculator you will type y_1 = int(x).)* If it looks like the first graph below, the calculator is in connected mode. Change the mode to dot mode under MODE, DOT. Trace to find the indicated values.





f(1)=___, f(1.8)=___, f(2)=___, f(2.1)=___.

Which sides of the line segments have open or closed dots?

(2) Write a piecewise function for the graph above on the domain $-3 \le x \le 3$ and state the range.

$$f(x) = \begin{cases} \\ \\ \\ \end{cases}$$

Range: ________________________

The above piecewise function is defined symbolically as $f(x) = \lfloor x \rfloor$ *and verbally as "the greatest integer less than or equal to x" or, in other words, a "round down" function. It is a step function, and the graph is said to have "jump discontinuities" at the integers.*

Evaluating Greatest Integer Expressions Evaluate the following:

(3) $\lfloor 7.1 \rfloor$ = _________ (4) $\lfloor 1.8 \rfloor$ = _________ (5) $\lfloor \pi \rfloor$ = __________

(6) $\lfloor -6.8 \rfloor$ = _________ (7) $\lfloor -2.1 \rfloor$ = _________ (8) $\lfloor 0 \rfloor$ = __________

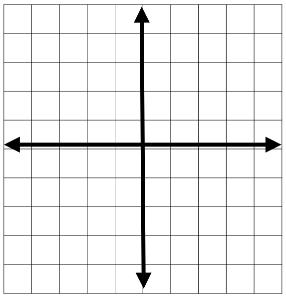
Solving Greatest Integer Equations

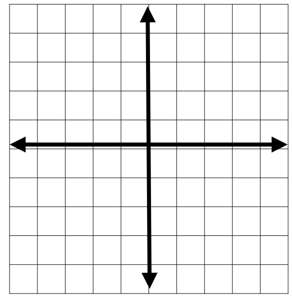
Solve the following equations for x and write the answers in set notation:

(9) $\left\lfloor \frac{2x}{7} \right\rfloor = 1$ (10) $\lfloor 3x \rfloor = 12$

Translating Graphs of Greatest Integer Functions Using what you learned about the translations of $y = a|x - h| + k$, graph the following by hand and check on your calculator:

(11) $f(x) = [\![x]\!] + 2$ $g(x) = [\![x + 2]\!]$.

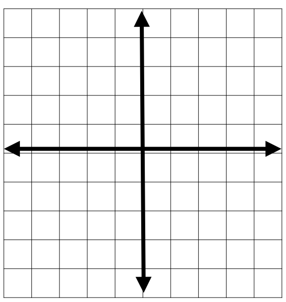


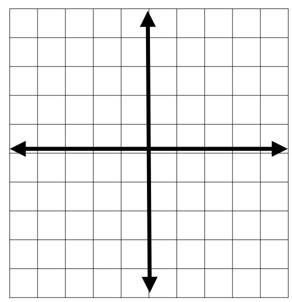


Explain the shift in each graph and how they differ. ____

____

(12) $f(x) = 2[\![x]\!]$ $g(x) = [\![2x]\!]$

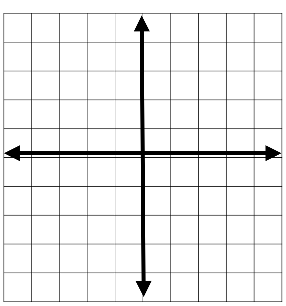


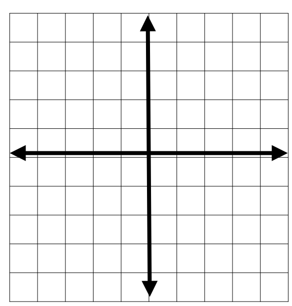


Explain the dilation in each graph and how they differ. ______________________________________

____

(13) $f(x) = -[\![x]\!]$ $g(x) = [\![-x]\!]$





Explain the rotation in these graphs and how they differ. ____________________________________

____

Real World Application of Step Functions

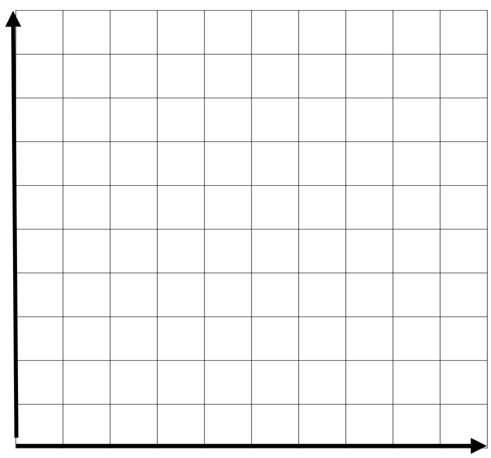
Prior to September, 2000, taxi fares from Washington DC to Maryland were described as follows: $2.00 up to and including ½ mile, $0.70 for each additional ½ mile increment.

(14) Describe the independent and dependent variables and explain your choices. ____________

____

____

(15) Graph the fares for the first 2 miles: *(Make sure to label the axes.)*



(16) Write the piecewise function for 0 to 2 miles.

$$f(x) = \begin{cases} \\ \\ \end{cases}$$

(17) Discuss why this is a step function and it is different from the greatest integer parent function $f(x) = \lfloor x \rfloor$.

____

____

____

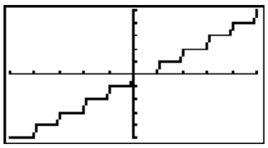
____

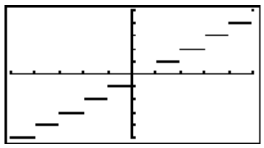
Unit 1, Activity 10, Greatest Integer Discovery Worksheet with Answers

Name______________________________________ Date________________________

Graphing the Greatest Integer Function

(1) On the graphing calculator, graph $y = \text{int}(x)$. *(Note: On the TI calculator, the greatest integer function is under MATH, NUM, 5: int(. In your calculator you will type $y_1 = \text{int}(x)$.)* If it looks like the first graph below, the calculator is in connected mode. Change the mode to dot mode under MODE, DOT. Trace to find the indicated values.





f(1)=__*1*__, *f*(1.8)=__*1*__, *f*(2)=__*2*__, *f*(2.1)=__*2*__.

Which sides of the line segments have open or closed dots? ***closed dots on left and open dots on right***

(2) Write a piecewise function for the graph above on the domain $-3 \le x \le 3$ and state the range.

$$f(x) = \begin{cases} 3 & \text{if } x = 3 \\ 2 & \text{if } 2 \le x < 3 \\ 1 & \text{if } 1 \le x < 2 \\ 0 & \text{if } 0 \le x < 1 \\ -1 & \text{if } -1 \le x < 0 \\ -2 & \text{if } -2 \le x < -1 \\ -3 & \text{if } -3 \le x < -2 \end{cases}$$

Range: _*{–3, –2, –1, 0, 1, 2, 3}*_

The above piecewise function is defined symbolically as $f(x) = \lfloor x \rfloor$ *and verbally as "the greatest integer less than or equal to x" or, in other words, a "round down" function. It is a step function, and the graph is said to have "jump discontinuities" at the integers.*

Evaluating Greatest Integer Expressions Evaluate the following:

(3) $\lfloor 7.1 \rfloor$ = ___*7*___ (4) $\lfloor 1.8 \rfloor$ = ___*1*___ (5) $\lfloor \pi \rfloor$ = ___*3*___

(6) $\lfloor -6.8 \rfloor$ = ___*–7*___ (7) $\lfloor -2.1 \rfloor$ = ___*–3*___ (8) $\lfloor 0 \rfloor$ = ___*0*___

Solving Greatest Integer Equations

Solve the following equations for *x* and write the answers in set notation:

(9) $\left\lfloor \frac{2x}{7} \right\rfloor = 1$

Solution: $\frac{7}{2} \le x < 7$

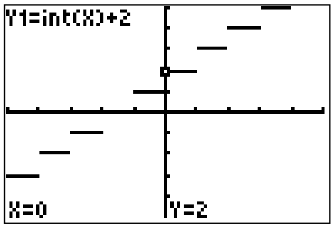
(10) $\lfloor 3x \rfloor = 12$

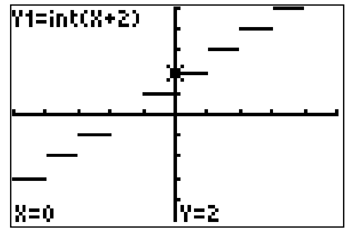
Solution: $4 \le x < \frac{13}{3}$

Unit 1, Activity 10, Greatest Integer Discovery Worksheet with Answers

Translating Graphs of Greatest Integer Functions Using what you learned about the translations of $y = a|x - h| + k$, graph the following by hand and check on your calculator:

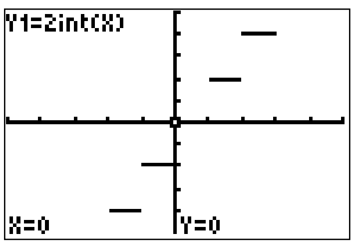
(11) $f(x) = \lfloor x \rfloor + 2$ $g(x) = \lfloor x + 2 \rfloor$.

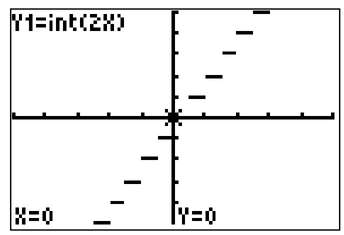




Explain the shift in each graph and how they differ. ***In f(x) the y values were shifted up 2 but in g(x) the x values were shifted to the left 2; however, the results were the same.***

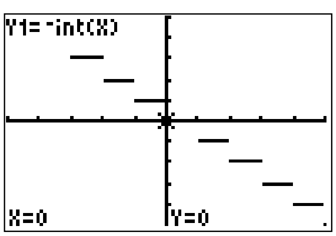
(12) $f(x) = 2\lfloor x \rfloor$ $g(x) = \lfloor 2x \rfloor$

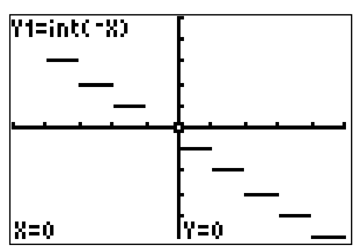




Explain the dilation in each graph and how they differ. ***The y-values of f(x) were multiplied by 2 resulting in a vertical stretch making the steps further apart. The x-values in g(x) were divided by 2 resulting in a horizontal compression making the steps narrower.***

(13) $f(x) = -\lfloor x \rfloor$ $g(x) = \lfloor -x \rfloor$





Explain the rotation in these graphs and how they differ. ***In f(x) the y-values are rotated through space around the x–axis, and in the g(x) the x-values are rotated through space around the y–axis.***

Unit 1, Activity 10, Greatest Integer Discovery Worksheet with Answers

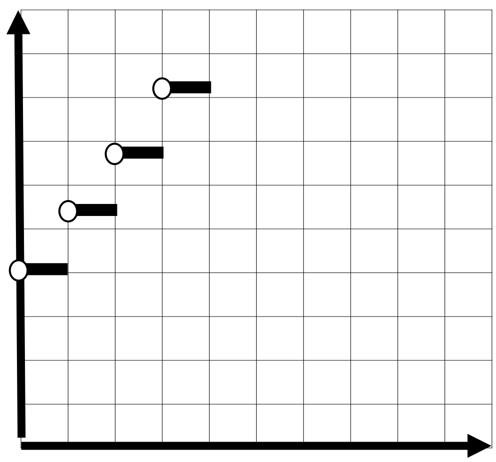
Real World Application of Step Functions

Prior to September, 2000, taxi fares from Washington DC to Maryland were described as follows: $2.00 up to and including ½ mile, $0.70 for each additional ½ mile increment.

(14) Describe the independent and dependent variables and explain your choices. ***The number of miles is the independent variable and the fare is the dependent variable because the fare depends on how far you travel.***

(15) Graph the fares for the first 2 miles: *(Make sure to label the axes.)*

$.50 units



½ mile units

(16) Write the piecewise function for 0 to 2 miles.

$$f(x)=\begin{cases} \$2.00 & if \quad 0 < x \le .5 \; mile \\ \$2.70 & if \quad .5 < x \le 1 \; mile \\ \$3.40 & if \quad 1 < x \le 1.5 \; miles \\ \$4.10 & if \quad 1.5 < x \le 2 \; miles \end{cases}$$

(17) Discuss why this is a step function and it is different from the greatest integer parent function.

This is a step function because it is made with horizontal line segments. It is different than the greatest integer function because it does not start at 0, jump discontinuities occur at every increment of ½ instead of 1, and the increments of y are 0.7 instead of 1. It also rounds up instead of rounding down meaning the closed dot is on the right side of the horizontal step instead of on the left.

Unit 1, Activity 10, Step Function Data Research Project

Name_____________________________________ Date_______________________

Objective: To find data on the Internet or in the newspaper that is conducive to creating a step function graph.

Materials: ½ piece of poster board, colored pencils, markers or crayons, data

Directions:

- ❖ Find data on the Internet or in newspapers or real-life situations that are indicative of step functions. Make sure to write down your source. Write the data in a clearly labeled table on the poster board.
- ❖ Using the data, draw a step function graph with the axis clearly labeled.
- ❖ Determine the piecewise equation for the step function including the domain in the equation. Specify the overall domain and range of the function.
- ❖ Write a real-world problem in which the data can be used to interpolate and extrapolate to solve another problem. Do the interpolation and extrapolation and find the correct answer. Discuss if this answer is realistic.

The following information must be on the front of the poster board: (everything must be in color – be creative)

1. Creative title (keep it clean) with poster neat, complete, readable, and decorated relative to the topic.
2. Data neatly presented in a clearly labeled table with source included.
3. Graph of the step function showing the x and y axes and units of measure.
4. Piecewise function with domain and range
5. Real-world word problem interpolating and extrapolating and solved correctly.
6. Your name, date and hour

Rubric : Step Function Data Research Project

Student: ______________________________ **Score:** ______________

CATEGORY & Sub Score	10–8	7–5	4–2	1
Data	Professional looking and accurate representation of the data in tables. Tables are labeled and titled. Source of information is included.	Accurate representation of the data in tables. Tables are labeled and titled. Source of information is included.	Accurate representation of the data in written form, but no tables are presented. Source of information is missing.	Data is not shown or is not step function data OR is inaccurate.
Step Function Graph	Clear, accurate step function graph is included and makes the research easier to understand. Graph is labeled neatly and accurately.	Graph is included and is labeled neatly and accurately.	Graph is included and is not labeled.	Needed graph is missing OR is missing important labels.
Piecewise Equation, Domain and Range	Piecewise equation with domain and range is accurate and symbolically correct.	Piecewise equation with domain and range is accurate but symbolically incorrect.	Piecewise equation is correct but domain and range are missing.	No piecewise equation is shown OR results are inaccurate or mislabeled.
Real World Problem	Real-life problem is included and typed and uses the function to interpolate <u>and</u> extrapolate and has correct answers.	Real-life problem is included typed or handwritten and uses the function to interpolate <u>or</u> extrapolate and has correct answers.	Real-life problem is written and uses the function to interpolate <u>or</u> extrapolate but answers are incorrect.	Real-life problem is handwritten but no interpolation or extrapolation.
Poster	Poster is neat, complete and creative and uses headings and subheadings to visually organize the material. Poster is decorated relative to the topic.	Poster is neat and complete and material is visually organized but not decorated relative to the topic.	Poster is neat but incomplete.	Poster is handwritten and looks sloppy and is incomplete

Unit 1, Activity 11, Composite Function Discovery Worksheet

Name____________________________________ Date______________________

Composite Functions in a Double Function Machine

(1) $f(x) = 3x + 7$ and $g(x) = -4x - 1$. Find $f(g(5))$ and $g(f(5))$ with the function machine.

Input	Machine 1		Machine 2	Output
$x = 5$	$g(x) = -4x-1$	________	$f(x) = 3x + 7$	________
$x = 5$	$f(x) = 3x + 7$	________	$g(x) = -4x - 1$	________

$f(g(5))$=______________ $g(f(5))$=__________________

(2) Use the following function machine to find a rule for $f(g(x))$ and $g(f(x))$.

Input	Machine 1		Machine 2	Output
$x = x$	$g(x) = -4x-1$	________	$f(x) = 3x + 7$	__________________
$x = x$	$f(x) = 3x + 7$	________	$g(x) = -4x - 1$	__________________

$f(g(x))$ =___________________ $g(f(x))$ =___________________

Finding Equations of Composite Functions and Graphing Them on the Calculator

- In order to graph the composition $f(g(x))$ on a graphing calculator, enter $g(x)$ into $y_1 = -4x - 1$ and turn it off so it will not graph. *(Note: To turn an equation off, use your left arrow to move the cursor over the = sign and press ENTER.)*
- Next, enter $f(x)$ into y_2 as follows $y_2 = 3(y_1) + 7$ and graph. *(Note: y_1 is under* VARS , *Y–VARS, 1: Function, 1: Y_1.)*
- Graph the answer to $f(g(x))$ from the function machine in #2 above in y_3, to see if they are the same graph.

(3) Practice with the polynomial functions: $f(x) = 2x + 1$ and $g(x) = 4x^2 + 3$. Find $f(g(x))$ and $g(f(x))$ and check on the calculator.

Input	Machine 1		Machine 2	Output
$x = x$	$g(x) = 4x^2 + 3$	________	$f(x) = 2x + 1$	__________________
$x = x$	$f(x) = 2x + 1$	________	$g(x) = 4x^2 + 3$	__________________

$f(g(x))$ =___________________ $g(f(x))$ =___________________

Unit 1, Activity 11, Composite Function Discovery Worksheet

Synthesis of Composite Functions: Use $f(x) = 3x^2 + 2$ to evaluate the functions in #4 and #5 and to create new functions in #6 – 8:

(4) $f(3)$ =_______________________________

(5) $f(a)$ =_______________________________

(6) $f(a + b)$ =____________________________

(7) $f(x) + h$ =____________________________

(8) $f(x + h)$ =_____________________________

(9) One of the most difficult compositions that is also very necessary for higher mathematics is finding $\frac{f(x+h)-f(x)}{h}$ which is called a difference quotient. Find the difference quotient for $f(x) = 3x^2 + 2$

(10) Find the difference quotient $\frac{g(x+h)-g(x)}{h}$ for $g(x) = -2x - 5$.

Composite Functions in a Table: Use the table to calculate the following compositions:

(11) $f(g(2))$ =____________________

(12) $g(f(2))$ =____________________

(13) $f(g(3))$ =____________________

(14) $g(f(3))$ =____________________

x	***f(x)***	***g(x)***
2	4	5
3	8	4
4	9	12
5	3	7

Decomposition of Composite Functions

Most functions are compositions of basic functions. Work backwards to determine the basic functions that created the composition.

	f(g(x))	***f(x)***	***g(x)***
(15)	$(x+4)^2+5$		
(16)	$\sqrt{x-4}$		
(17)	$(4x-1)^2$		
(18)	$\|x+2\|$		
(19)	$\llbracket x-2 \rrbracket + 4$		

Domain & Range of Composite Functions

Find the domains and composition *f(g(x))* to fill in the table below to discover the rule for the domain of a composite function:

	f(x)	***g(x)***	**Domain of *f(x)***	**Domain of *g(x)***	***f(g(x))***	**Domain of *f(g(x))***
(20)	$\sqrt{x-3}$	$x+1$				
(21)	$x+1$	$\sqrt{x-3}$				
(22)	$\frac{1}{x}$	$2x+4$				
(23)	$2x+4$	$\frac{1}{x}$				
(24)	$\sqrt{x}$	x^2				
(25)	x^2	$\sqrt{x}$				

(26) Develop a rule for determining the <u>domain of a composition</u>: ______________________________

____

____

Unit 1, Activity 11, Composite Function Discovery Worksheet with Answers

Name______________________________________ Date_______________________

Composite Functions in a Double Function Machine

(1) $f(x) = 3x + 7$ and $g(x) = -4x - 1$. Find $f(g(5))$ and $g(f(5))$ with the function machine.

x = 5	$g(x) = -4x - 1$	***–21***	$f(x) = 3x + 7$	***–56***
x = 5	$f(x) = 3x + 7$	***22***	$g(x) = -4x - 1$	***–89***

$f(g(5))=$ ***-56*** $g(f(5))=$ ***–89***

(2) Use the following function machine to find a rule for $f(g(x))$ and $g(f(x))$.

x = x	$g(x) = -4x - 1$	***–4x – 1***	$f(x) = 3x + 7$	***3(–4x–1) + 7***
x = x	$f(x) = 3x + 7$	***3x + 7***	$g(x) = -4x - 1$	***–4(3x + 7) – 1***

$f(g(x)) =$ ***–12x + 4*** $g(f(x)) =$ ***–12x –29***

Finding Equations of Composite Functions and Graphing Them on the Calculator

- In order to graph the composition $f(g(x))$ on a graphing calculator, enter $g(x)$ into $y_1 = -4x - 1$ and turn it off so it will not graph. *(Note: To turn an equation off, use your left arrow to move the cursor over the = sign and press ENTER.)*
- Next, enter $f(x)$ into y_2 as follows $y_2 = 3(y_1) + 7$ and graph. *(Note: y_1 is under* VARS , *Y–VARS, 1: Function, 1: Y_1.)*
- Graph the answer to $f(g(x))$ from the function machine in #2 above in y_3, to see if they are the same graph.

(3) Practice with the polynomial functions: $f(x) = 2x + 1$ and $g(x) = 4x^2 + 3$. Find $f(g(x))$ and $g(f(x))$ and check on the calculator.

x = x	$g(x) = 4x^2 + 3$	***$4x^2 + 3$***	$f(x) = 2x + 1$	***$2(4x^2 + 3) + 1$***
x = x	$f(x) = 2x + 1$	***$2x + 1$***	$g(x) = 4x^2 + 3$	***$4(2x + 1)^2 + 3$***

$f(g(x)) =$ ***$8x^2 + 7$*** $g(f(x)) =$ ***$16x^2 + 16x + 7$***

Unit 1, Activity 11, Composite Function Discovery Worksheet with Answers

Synthesis of Composite Functions: Use $f(x) = 3x^2 + 2$ to evaluate the function in #4 and #5 and to create a composite function in #6 – 8:

(4) $f(3) =$ ***29***

(5) $f(a) =$ ***$3a^2 + 2$***

(6) $f(a + b) =$ ***$3a^2 + 6ab + 3b^2 + 2$***

(7) $f(x) + h =$ ***$3x^2 + 2 + h$***

(8) $f(x + h) =$ ***$3x^2 + 6xh + 3h^2 + 2$***

(9) One of the most difficult compositions, that is also very necessary for higher mathematics, is finding $\frac{f(x+h)-f(x)}{h}$ which is called a difference quotient. Find the difference quotient for $f(x) = 3x^2 + 2$

Solution: $6x + 3h$

(10) Find the difference quotient $\frac{g(x+h)-g(x)}{h}$ for $g(x) = -2x - 5$.

Solution: -2

Composite Functions in a Table: Use the table to calculate the following compositions:

(11) $f(g(2)) =$ ***3***

(12) $g(f(2)) =$ ***12***

(13) $f(g(3)) =$ ***9***

(14) $g(f(3)) =$ ***cannot be determined***

x	*f(x)*	*g(x)*
2	4	5
3	8	4
4	9	12
5	3	7

Unit 1, Activity 11, Composite Function Discovery Worksheet with Answers

Decomposition of Composite Functions

Most functions are compositions of basic functions. Work backwards to determine the basic functions that created the composition.

	f(g(x))	***f(x)***	***g(x)***
(15)	$(x + 4)^2 + 5$	$x^2 + 5$	$x+4$
(16)	$\sqrt{x-4}$	$\sqrt{x}$	$x - 4$
(17)	$(4x - 1)^2$	x^2	$4x - 1$
(18)	$\lvert x + 2\rvert$	$\lvert x\rvert$	$x + 2$
(19)	$[\![x-2]\!]+4$	$[\![x]\!]+4$	$x - 2$

Domain & Range of Composite Functions

Find the domains and composition *f(g(x))* to fill in the table below to discover the rule for the domain of a composite function:

	f(x)	***g(x)***	**Domain of *f(x)***	**Domain of *g(x)***	***f(g(x))***	**Domain of *f(g(x))***
(20)	$\sqrt{x-3}$	$x + 1$	$x \geq 3$	*all reals*	$\sqrt{x-2}$	$x \geq 2$
(21)	$x + 1$	$\sqrt{x-3}$	all reals	$x \geq 3$	$\sqrt{x-3}+1$	$x \geq 3$
(22)	$\frac{1}{x}$	$2x + 4$	$x \neq 0$	*all reals*	$\frac{1}{2x+4}$	$x \neq -2$
(23)	$2x + 4$	$\frac{1}{x}$	*all reals*	$x \neq 0$	$\frac{2}{x}+4$	$x \neq 0$
(24)	$\sqrt{x}$	x^2	$x \geq 0$	*all reals*	$\sqrt{x^2}$	*all reals*
(25)	x^2	$\sqrt{x}$	*all reals*	$x \geq 0$	$\left(\sqrt{x}\right)^2$	$x \geq 0$

(26) Develop a rule for determining the domain of a composition: *To determine the domain of the composition f(g(x)), find the domain of g(x) and further restrict it for the composition f(g(x)). Note that the domain restrictions on f(x) have no consequences on the composition.*

Unit 1, Activity 12, Inverse Function Discovery Worksheet

Name_____________________________________ Date_______________________

Recognizing Inverse Functions: An inverse relation is defined as any relation that swaps the independent and dependent variables. Swap the variables in the relations below. Then determine whether the new inverse relation is also a function of *x* and explain why or why not.

(1) *the set of ordered pairs {(x, y) : (1, 2), (3, 5), (3, 6), (7, 5), (8, 2)}*
Swap the *x* and *y* and write a new set of ordered pairs: ______________________________

Is the new relation a function of *x*?________ Explain why or why not. ________________

____

(2) *the set of ordered pairs {(s, d) : (1, 1), (2, 4), (3, 9), (–1, 7), (–2, 4), (–3, 9)}*
Swap the *s* and *d* and write a new set of ordered pairs: ______________________________

Is the new relation a function of *s*?________ Explain why or why not. ________________

____

(3) *the relationship "x is a student of y"*
Write the words for the inverse relationship. ______________________________________

Is the new relation a function of *x*?________ Explain why or why not. ________________

____

(4) *the relationship " x is the biological daughter of mother y"*
Write the words for the inverse relationship. ______________________________________

Is the new relation a function of *x*?________ Explain why or why not. ________________

____

(5) *the equation* $2x + 3y = 6$
Swap the *x* and *y* and write a new equation: ______________________________________

Is the new relation a function of *x*?________ Explain why or why not. ________________

____

(6) *the equation* $x + y^2 = 9$
Swap the *x* and *y* and write a new equation: ______________________________________

Is the new relation a function of *x*?________ Explain why or why not. ________________

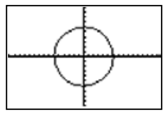
____

(7) *the equation* $y = x^2 + 4$
Swap the *x* and *y* and write a new equation: ____

Is the new relation a function of *x*?________ Explain why or why not. ________________

____

(8) the graph of the circle



If you swapped the *x* and *y*, what would the new graph look like? _____________________

Is the new graph a function of *x*?________ Explain why or why not. __________________

____

Answer the following questions:

(9) How can you look at a set of ordered pairs and determine if the inverse relation will be a function?

____

(10) How can you look at an equation and determine if the inverse relation will be a function?

____

(11) How can you look at a graph and determine if the inverse relation will be a function?

____

(12) How can you look at a verbal statement and determine if the inverse verbal relationship will be a function?

____

Defining Inverse Functions

(13) Complete the function machine for *f(g(x))* and *g(f(x))*, using the functions $f(x) = 2x + 4$ and $g(x) = ½\,x – 2$ to find *f(g(2))* and *g(f(2))*.

Solution:

$x = 2$	$f(x) = 2x+4$	________	$g(x) = ½\,x – 2$	________
$x = 2$	$g(x) = ½\,x – 2$	________	$f(x) = 2x+4$	________

f (g(2)) = __________________________ *g(f (2))* = ___________________________

Unit 1, Activity 12, Inverse Function Discovery Worksheet

- An **Inverse function** is defined in the following ways:
  - Symbolically ≡ If f is a function, then $f^{-1}(x)$ is the inverse function if $f(f^{-1}(x)) = f^{-1}(f(x)) = x$. *(Note: When the –1 exponent is on the f, it means inverse function and does not mean reciprocal.)*
  - Verbally ≡ When you compose a function with its inverse and the inverse with its function, the resulting rule is the identity function $y = x$.
  - Numerically ≡ The domain of a function is the range of the inverse function and vice versa.
  - Graphically ≡ A function and its inverse are symmetric to each other over the line $y = x$.

Finding Equations for Inverse Functions: Find the inverse of the following functions by replacing the $f(x)$ with y, interchanging x and y, and solving for y. Rename y as $f^{-1}(x)$, graph $f(x)$ and $f^{-1}(x)$ on the same graph, and find the domain and range of both.

(14) $f(x) = 3x + 4$

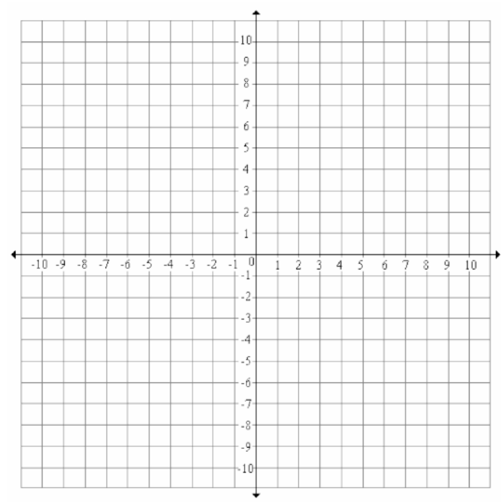
$f^{-1}(x) =$ ______________________________

Domain of $f(x)$: ______________________

Range of $f(x)$: ______________________

Domain of $f^{-1}(x)$: ______________________

Range of $f^{-1}(x)$: ______________________



(15) $f(x) = 2|x - 1|$ on the domain $x \leq 1$

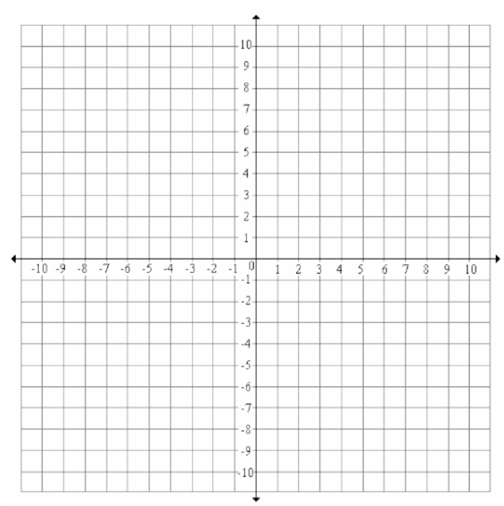
$f^{-1}(x) =$ ______________________________

Domain of $f(x)$: ______________________

Range of $f(x)$: ______________________

Domain of $f^{-1}(x)$: ______________________

Range of $f^{-1}(x)$: ______________________



Unit 1, Activity 12, Inverse Function Discovery Worksheet with Answers

Name_______________________________________ Date________________________

Recognizing Inverse Functions: An inverse relation is defined as any relation that swaps the independent and dependent variables. Swap the variables in the relations below. Then determine whether the new inverse relation is also a function of *x* and explain why or why not.

(1) *the set of ordered pairs {(x, y) : (1, 2), (3, 5), (3, 6), (7, 5), (8, 2)}*
Swap the *x* and *y* and write a new set of ordered pairs: *{(y, x) : (2,1), (5, 3), (6, 3), (5, 7), (2, 8)}*

Is the new relation a function of *x*?__no___ Explain why or why not. _________________

the number 2 maps onto 1 and 8 and 5 maps onto 3 and 7

(2) *the set of ordered pairs {(s, d) : (1, 1), (2, 4), (3, 9), (–1, 7), (–2, 4), (–3, 9)}*
Swap the *s* and *d* and write a new set of ordered pairs: *{(d, s) : (1,1), (4,2), (9,3), (7,–1), (4,–2), (9,–3)},*

Is the new relation a function of *s*?___*yes*___ Explain why or why not. _________________

every input has only one output

(3) *the relationship "x is a student of y"*
Write the words for the inverse relationship. *"y is a teacher of x"*

Is the new relation a function of *x*?__*no*___ Explain why or why not. _________________

a teacher can have more than one student

(4) *the relationship " x is the biological daughter of mother y"*
Write the words for the inverse relationship. *"y is the biological mother of daughter x"*

Is the new relation a function of *x*?__*no*___ Explain why or why not. _________________

a mother can have many daughters

(5) *the equation* $2x + 3y = 6$
Swap the *x* and *y* and write a new equation: $2y + 3x = 6$,

Is the new relation a function of *x*?__*Yes*__ Explain why or why not. _________________

for every x there is only one y

(6) *the equation* $x + y^2 = 9$
Swap the *x* and *y* and write a new equation: $y + x^2 = 9$

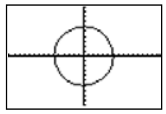
Is the new relation a function of *x*?__*Yes*__ Explain why or why not. _________________

for every x there is only one y

(7) *the equation* $y = x^2 + 4$
Swap the *x* and *y* and write a new equation: $x = y^2 + 9$
Is the new relation a function of *x*? ***no*** Explain why or why not.

two y values for each x

(8) *the graph of the circle*



If you swapped the *x* and *y*, what would the new graph look like? ***same as the original relation***
Is the new graph a function of *x*? ***no*** Explain why or why not.

two y values for each x

Answer the following questions:

(9) How can you look at a set of ordered pairs and determine if the inverse relation will be a function?

Looking at ordered pairs, neither the independent nor the dependent variables can be repeated

(10) How can you look at an equation and determine if the inverse relation will be a function?

neither the x nor the y can be raised to an even power

(11) How can you look at a graph and determine if the inverse relation will be functions?

neither a vertical nor a horizontal line can intersect the graph at two points.

(12) How can you look at a verbal statement and determine if the inverse verbal relationship will be a function?

neither the independent not the dependent variable may be repeated

Defining Inverse Functions

(13) Complete the function machine for *f(g(x))* and *g(f(x))*, using the functions $f(x) = 2x + 4$ and $g(x) = ½\, x – 2$ to find *f(g(2))* and *g(f(2))*.

Input	Machine	Output	Machine	Output
$x = 2$	$f(x) = 2x+4$	***8***	$g(x) = ½\, x – 2$	***2***
$x = 2$	$g(x) = ½\, x – 2$	***–1***	$f(x) = 2x+4$	***2***

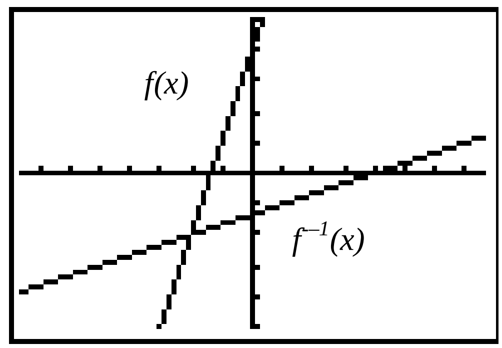
f(g(2)) = ***2*** *g(f(2))* = ***2***

- **Inverse function** is defined in the following ways:
  - Symbolically ≡ If *f* is a function, then $f^{-1}(x)$ is the inverse function if $f(f^{-1}(x)) = f^{-1}(f(x)) = x$. *(Note: When the –1 exponent is on the f, it means inverse function and does not mean reciprocal.)*
  - Verbally ≡ When you compose a function with its inverse and the inverse with its function, the resulting rule is the identity function $y = x$.
  - Numerically ≡ The domain of a function is the range of the inverse function and vice versa.
  - Graphically ≡ A function and its inverse are symmetric to each other over the line $y = x$.

Finding Equations for Inverse Functions: Find the inverse of the following functions by replacing the $f(x)$ with y, interchanging x and y, and solving for y. Rename y as $f^{-1}(x)$, graph $f(x)$ and $f^{-1}(x)$ on the same graph, and find the domain and range of both.

(14) $f(x) = 3x + 4$

$$f^{-1}(x) = \frac{1}{3}x - \frac{4}{3}$$



Domain of $f(x)$: *all reals*

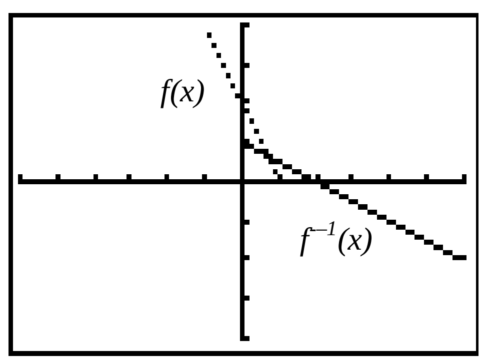
Range of $f(x)$: *all reals*

Domain of $f^{-1}(x)$: *all reals*

Range of $f^{-1}(x)$: *all reals*

(15) $f(x) = 2|x - 1|$ on the domain $x \le 1$

$f^{-1}(x)$ = **$-½x + 1$** *on the domain $x > 0$*



Domain of $f(x)$: $(-\infty, 1]$

Range of $f(x)$: $[0, \infty)$

Domain of $f^{-1}(x)$: $[0, \infty)$

Range of $f^{-1}(x)$: $(-\infty, 1]$