3-2 Solving Systems Algebraically

Objective To solve linear systems algebraically

You can use the substitution method to solve a system of equations when it is easy to isolate one of the variables. After isolating the variable, substitute for that variable in the other equation. Then solve for the other variable.

not #

Got It? 1. What is the solution of the system of equations? $\begin{cases} x + 3y = 5 \\ -2x - 4y = -5 \end{cases}$

Substitution

① Isolate: $x + 3y = 5$

$\underline{-3y \quad -3y}$

$x = 5 - 3y$

② Substitute: $-2x - 4y = -5$

$-2(5 - 3y) - 4y = -5$

③ Solve: $-10 + \underbrace{6y - 4y} = -5$

$-10 + 2y = -5$

$\underline{+10 \qquad\quad +10}$

$\frac{2y}{2} = \frac{5}{2} \longrightarrow y = 2.5$

$x = 5 - 3(2.5)$

$x = -2.5$

$\boxed{(-2.5, 2.5)}$

Got It? 2. An online music company offers 15 downloads for \$19.75 and 40 downloads for \$43.50. Each price includes the same one-time registration fee. What is the cost of each download and the registration fee?

x = cost of each download
y = registration fee

$15x + y = 19.75$
$40x + y = 43.50$

Substitution

① Isolate:

$15x + y = 19.75$
$-15x \qquad -15x$

$y = 19.75 - 15x$

$y = 19.75 - 15(0.95)$
$y = 5.5$

② Substitute

$40x + y = 43.50$

$40x + 19.75 - 15x = 43.50$

③ Solve: $25x + 19.75 = 43.50$
$-19.75 \quad -19.75$

$\frac{25x}{25} = \frac{23.75}{25}$

$x = \boxed{\$0.95 \text{ per download}}$

$\boxed{\text{Registration } \$5.50}$

You can use the Addition Property of Equality to solve a system of equations. If you add a pair of additive inverses or subtract identical terms, you can eliminate a variable.

$\rightarrow +4x - 4x = 0 \leftarrow$ Eliminates x!

When you multiply each side of one or both equations in a system by the same nonzero number, the new system and the original system have the same solutions. The two systems are called **equivalent systems**. You can use this method to make additive inverses.

Elimination

Independent → One Solution (x, y)

★ Solving a system algebraically does not always provide a unique solution. Sometimes you get infinitely many solutions. Sometimes you get no solutions.

Dependent

Inconsistent

Elimination

N. Multiplying

Eliminate

Got It? 3. What is the solution of the system of equations? $\begin{cases} -2x + 8y = -8 \\ 5x - 8y = 20 \end{cases}$

Add. Inverses

Doesn't Matter which Equation

$$\begin{array}{r} -2x + \cancel{8y} = -8 \\ +\ 5x - \cancel{8y} = 20 \\ \hline 3x = 12 \end{array}$$

$$\frac{3x}{3} = \frac{12}{3}$$

$$x = 4$$

$$-2(4) + 8y = -8$$

$$\begin{array}{r} \cancel{-8} + 8y = -8 \\ \cancel{+8} \qquad\quad +8 \\ \hline 8y = 0 \end{array}$$

$$\frac{8y}{8} = \frac{0}{8}$$

$$y = 0$$

$$(4, 0)$$

Elimination - Both Eq's

Got It? 4 a. What is the solution of this system of equations? $\begin{cases} 3x + 7y = 15 & \cdot 5 \\ 5x + 2y = -4 & \cdot -3 \end{cases}$

Eliminated

$$\cancel{15x} + 35y = 75$$
$$+ \cancel{-15x} - 6y = 12$$

$$\frac{29y}{29} = \frac{87}{29}$$

$$y = 3$$

$$5x + 2(3) = -4$$
$$5x + 6 = -4$$
$$-6 \quad -6$$
$$\frac{5x}{5} = \frac{-10}{5}$$
$$x = -2$$

$$(-2, 3)$$

a) Elimation - One Eq.

Got It? **5.** What are the solutions of the following systems? Explain.

a. $\begin{cases} -x + y = -2 \\ 2x - 2y = 0 \end{cases}$ $\cdot\, 2$

b. $\begin{cases} 4x + y = 6 \\ 12x + 3y = 18 \end{cases}$ $\cdot\, -3$

Eliminated

Rewrite

$$\begin{array}{r} \cancel{-2x} + \cancel{2y} = -4 \\ + \quad \cancel{2x} - \cancel{2y} = 0 \\ \hline 0 = -4 \end{array}$$

Both x, y gone!

No Solution

Inconsistent

$$\begin{array}{r} -\cancel{12x} - \cancel{3y} = -18 \\ + \quad \cancel{12x} + \cancel{3y} = 18 \\ \hline 0 = 0 \end{array}$$

Infinitely Many Solution

Dependent

Inclass: p. 146 #14, 20, 38
Homework: p. 146 #11-41(odd)
Interactmath: #10, 13, 15, 19, 22, 24, 26, 33, 34, 37, 41