

Mr. Hollen

Algebra 2

Packets #17-21

Functions

A **relation** pairs inputs with outputs. When a relation is given as ordered pairs, the x -coordinates are inputs and the y -coordinates are outputs. A relation that pairs each input with *exactly one* output is a **function**.

Example 1 Determine whether each relation is a function. Explain.

a. $(-1, 3), (0, 3), (1, 3), (2, 1), (3, 1)$

Every input has exactly one output.

► So, the relation is a function.

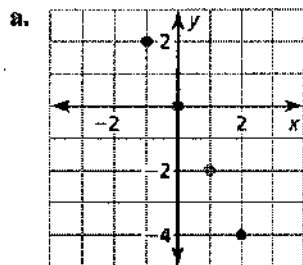
b. $(5, 1), (9, 8), (7, 5), (5, 4), (6, 3)$

The input 5 has two outputs, 1 and 4.

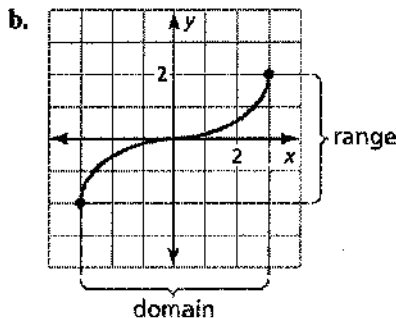
► So, the relation is *not* a function.

The **domain** of a function is the set of all possible input values. The **range** of a function is the set of all possible output values.

Example 2 Find the domain and range of the function represented by the graph.



► The domain is $-1, 0, 1, \text{ and } 2$.
The range is $-4, -2, 0, \text{ and } 2$.



► The domain is $-3 \leq x \leq 3$.
The range is $-2 \leq y \leq 2$.

Practice

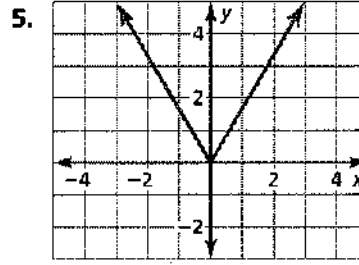
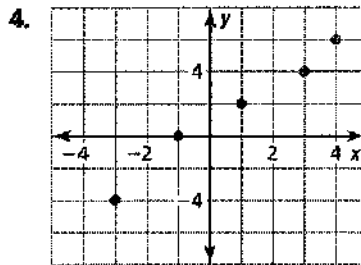
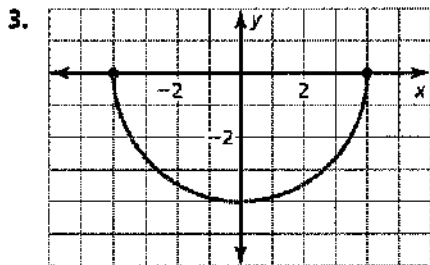
Check your answers at BigIdeasMath.com.

Determine whether the relation is a function. Explain.

1. $(2, -5), (3, -1), (4, 2), (5, -5), (6, 7)$



2. $(8, 5), (6, 0), (4, -7), (2, -4), (4, 7)$

Find the domain and range of the function represented by the graph.



Graphing Linear Functions

A **linear function** is a function whose graph is a nonvertical line. A linear function can be represented by a linear equation in two variables, $y = mx + b$, where m is the slope and b is the y -intercept. A **solution of a linear equation in two variables** is an ordered pair (x, y) that makes the equation true. The graph of a linear equation in two variables is the set of points (x, y) in a coordinate plane that represents all solutions of the equation. The points may be distinct or connected.

Discrete Domain	Continuous Domain
A discrete domain is a set of input values that consists of only certain numbers in an interval.	A continuous domain is a set of input values that consists of all numbers in an interval.
Example: Integers from 1 to 5	Example: All numbers from 1 to 5
	

Example 1 The linear function $y = 29.8x$ represents the number y of kilometers Earth travels in orbit around the Sun in x seconds. (a) Find the domain of the function. Is the domain discrete or continuous? Explain. (b) Graph the function using its domain.

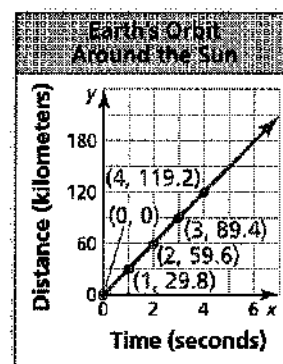
- a. Earth can travel in orbit for part of a second. The number x of seconds Earth travels in orbit can be any value greater than or equal to 0.

► So, the domain is $x \geq 0$, and it is continuous.

- b. Make an input-output table to find ordered pairs.

x	0	1	2	3	4
$y = 29.8x$	0	29.8	59.6	89.4	119.2

Plot the ordered pairs. Draw a line through the points starting at $(0, 0)$. Use an arrow to indicate that the line continues without end.



Practice

Check your answers at BigIdeasMath.com.

Copy and complete the table.

1.

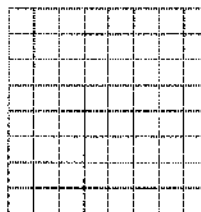
x	-2	-1	0	1	2
$y = x + 2$					

2.

x	-2	-1	0	1	2
$y = x - 7$					

3. **BOATING** A speed boat tour costs \$60 per ticket. There are 5 tickets left. The total cost y of the tickets is a function of the number t of tickets you buy.

- a. Find the domain of the function. Is the domain discrete or continuous? Explain.



- b. Graph the function using its domain.

Function Notation

A linear function can be written in the form $y = mx + b$. By naming a linear function f , you can also write the function using **function notation**.

$$f(x) = mx + b \quad \text{Function notation}$$

The notation $f(x)$ is another name for y . If f is a function, and x is in its domain, then $f(x)$ represents the output of f corresponding to the input x . You can use letters other than f to name a function, such as g or h .

Example 1 Evaluate the function for the given value of x .

a. $f(x) = 2x + 5; x = 7$

$$\begin{aligned} f(7) &= 2(7) + 5 && \text{Substitute 7 for } x. \\ &= 14 + 5 && \text{Multiply.} \\ &= 19 && \text{Add.} \end{aligned}$$

► When $x = 7$, $f(x) = 19$.

b. $g(x) = 4x - x^2; x = -3$

$$\begin{aligned} g(-3) &= 4(-3) - (-3)^2 && \text{Substitute } -3 \text{ for } x. \\ &= -12 - 9 && \text{Multiply.} \\ &= -21 && \text{Subtract.} \end{aligned}$$

► When $x = -3$, $g(x) = -21$.

Example 2 Determine whether the ordered pair is a solution of the equation.

a. $h(x) = 8 + x; (-6, 2)$

$$\begin{aligned} 2 &\stackrel{?}{=} 8 + (-6) && \text{Substitute } -6 \text{ for } x \\ &&& \text{and 2 for } h(x). \\ 2 &= 2 \quad \checkmark && \text{Add.} \end{aligned}$$

► So, $(-6, 2)$ is a solution.

b. $p(x) = |3x - 1|; (-2, -7)$

$$\begin{aligned} -7 &\stackrel{?}{=} |3(-2) - 1| && \text{Substitute } -2 \text{ for } x \\ &&& \text{and } -7 \text{ for } p(x). \\ -7 &\stackrel{?}{=} |-7| && \text{Evaluate.} \\ -7 &\neq 7 \quad \times && \text{Evaluate.} \end{aligned}$$

► So, $(-2, -7)$ is *not* a solution.

Practice

Check your answers at BigIdeasMath.com.

Evaluate the function for the given value of x .

1. $f(x) = x + 9; x = 8$

2. $g(x) = 6 - 5x; x = -1$

3. $h(x) = 4x + 3; x = 10$

4. $n(x) = -x - 4; x = -2$

5. $p(x) = -\frac{3}{4}x^2; x = 6$

6. $q(x) = x^2 - 11x; x = 4$

7. $k(x) = x^2 + 7x - 1; x = -3$

8. $h(x) = |3x - 8|; x = 1$

9. $f(x) = |x| + 2; x = -15$

Determine whether the ordered pair is a solution of the equation.

10. $f(x) = 3x + 5; (-1, 2)$

11. $h(x) = 7x - 2; (-3, -19)$

12. $g(x) = -x^2 + x + 5; (-5, 25)$

13. $n(x) = x^2 - 6x - 1; (4, -7)$

14. $h(x) = |x| - 14; (-4, 10)$

15. $p(x) = |-9x - 2|; (0, 2)$

16. **TICKETS** The function $C(x) = 49.5x + 19.5$ represents the cost (in dollars) of buying x concert tickets. How much does it cost to buy four tickets? How many tickets can you buy with \$465?

Zeros of Quadratic Functions

A **zero of a function** f is an x -value for which $f(x) = 0$. If a real number k is a zero of the function $f(x) = ax^2 + bx + c$, then k is an x -intercept of the graph of the function.

Example 1 Find the zeros of each function.

a. $f(x) = 9x^2 - 36$

Set $f(x)$ equal to 0. Then use square roots to solve for x .

$$9x^2 - 36 = 0$$

$$9x^2 = 36$$

$$x^2 = 4$$

$$x = \pm\sqrt{4}$$

$$x = \pm 2$$

► The zeros of the function are $x = -2$ and $x = 2$.

b. $f(x) = x^2 - 2x - 8$

Set $f(x)$ equal to 0. Then use factoring to solve for x .

$$x^2 - 2x - 8 = 0$$

$$(x - 4)(x + 2) = 0$$

$$x - 4 = 0 \quad \text{or} \quad x + 2 = 0$$

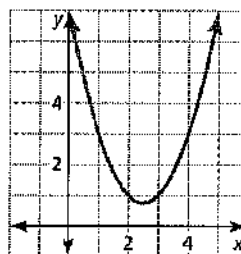
$$x = 4 \quad \text{or} \quad x = -2$$

► The zeros of the function are $x = -2$ and $x = 4$.

Example 2 Find the zeros of $f(x) = x^2 - 5x + 7$.

Set $f(x)$ equal to 0. Then use the Quadratic Formula to solve for x .

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(7)}}{2(1)} \\ &= \frac{5 \pm \sqrt{-3}}{2} \\ &= \frac{5 \pm i\sqrt{3}}{2} \end{aligned}$$



► The zeros of the function are $x = \frac{5}{2} + \frac{\sqrt{3}}{2}i$ and $x = \frac{5}{2} - \frac{\sqrt{3}}{2}i$.

Notice that the graph of f does not intersect the x -axis.

Practice

Check your answers at BigIdeasMath.com.

Find the zero(s) of the function.

1. $f(x) = 8x^2 + 32$

3. $f(x) = x^2 - 8x + 16$

5. $f(x) = 4(x + 5)(x - 1)$

7. $f(x) = 3x^2 + 12x + 15$

9. $f(x) = -(x + 1)^2 + 18$

2. $f(x) = -5x^2 + 40$

4. $f(x) = 4x^2 + 12x + 9$

6. $f(x) = -\frac{1}{2}x(x + 3)$

8. $f(x) = 2x^2 - x - 15$

10. $f(x) = (x - 7)^2 + 9$

#21

Name _____

Date _____

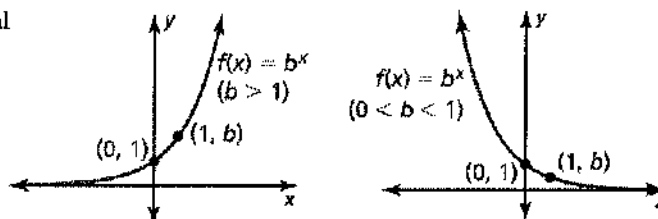
Exponential Functions

Graphing Exponential Functions

An **exponential function** is a nonlinear function of the form $y = ab^x$, where $a \neq 0$, $b \neq 1$, and $b > 0$.

- When $a > 0$ and $b > 1$, the function is an exponential growth function.
- When $a > 0$ and $0 < b < 1$, the function is an exponential decay function.

The graphs of the parent exponential functions $y = b^x$ are shown.

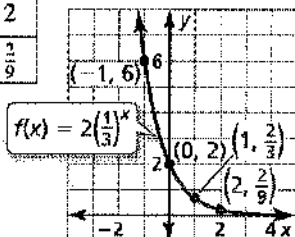


Example 1 Tell whether each function represents *exponential growth* or *exponential decay*. Then graph the function.

a. $f(x) = 2\left(\frac{1}{3}\right)^x$

Because $a = 2$ is positive and $b = \frac{1}{3}$ is greater than 0 and less than 1, the function is an exponential decay function. Use a table to graph the function.

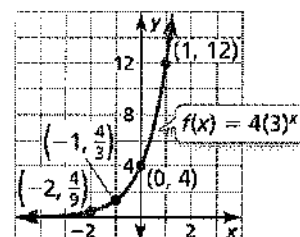
x	-1	0	1	2
y	6	2	$\frac{2}{3}$	$\frac{2}{9}$



b. $f(x) = 4(3)^x$

Because $a = 4$ is positive and $b = 3$ is greater than 1, the function is an exponential growth function. Use a table to graph the function.

x	-2	-1	0	1
y	$\frac{4}{9}$	$\frac{4}{3}$	4	12



Practice

Check your answers at BigIdeasMath.com.

Tell whether the function represents *exponential growth* or *exponential decay*.

1. $f(x) = \left(\frac{1}{4}\right)^x$

2. $f(x) = \left(\frac{4}{3}\right)^x$

3. $f(x) = 0.5(4)^x$

4. $f(x) = 3(0.75)^x$

5. $f(x) = 2(0.8)^x$

6. $f(x) = 5(2)^x$